Superradiant phonon scattering from a hydrodynamic vortex

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We have shown that sound waves scattered from a hydrodynamic vortex may be amplified [1]. Such superradiant scattering follows from the physical analogy between spinning black holes and hydrodynamic vortices [2].

Sound waves are linear perturbations of the velocity potential $\phi$. We considered a barotropic, inviscid, irrotational fluid, and assumed the unperturbed fluid flow was a vortex centred on the origin of our cylindrical coordinate system. We considered cylindrical wave solutions of the form $\phi(t, r, \theta, z) = \psi(t, r)e^{-im\theta}$, with angular wavenumber $m$. Assuming that the square of the speed of sound is proportional to the density, as is the case for a BEC, the density may be eliminated from the sound wave equation, which then becomes

$$\frac{\partial^2 \psi}{\partial t^2} - 2\frac{mv_\theta}{r}\frac{\partial \psi}{\partial t} - \frac{1}{r}\frac{\partial}{\partial r} \left( r^2 \frac{\partial \psi}{\partial r} \right) + \frac{m^2}{r^2} \left( c^2 - v_\theta^2 \right) \psi = 0. \quad (1)$$

where $c$ is the unperturbed fluid’s speed of sound, and $v_\theta$ is the polar component of the unperturbed fluid’s flow speed. For irrotational flow, $v_\theta = \alpha/r$, for some constant $\alpha$. In order to present analytical calculations we used the density profile

$$\rho(r) = \rho_\infty \left[ \frac{(r - r_0)/\sigma}{2 + [(r - r_0)/\sigma]^2} \right]^2. \quad (2)$$

This is similar to the charge $l = 1$ vortex density profile for a BEC, but with the scale length given by the free parameter $\sigma$, rather than by the healing length $\chi$. For single frequency waves of the form $\psi(t, r) = R(r)e^{i\omega t}$ we found that an incoming wave may scatter into an outgoing reflected wave and an ingoing transmitted wave. We found that superradiance occurs for $\omega < m\alpha/r_0^2$.

We are currently extending this analysis beyond the hydrodynamic approximation.

Figure. Wavepacket propagation from numerical solution of the wave equation. The real part of the wave packet is plotted versus the dimensionless modified radial tortoise coordinate $\tilde{r}_*^\alpha$, defined in reference [1]. (a) the wavepacket is propagating towards the vortex. (b) the wavepacket has just split into reflected and transmitted parts. (c) the amplified reflected wavepacket.

References