Relative Phase States

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Studies of phase dependent phenomena in both Bose-Einstein condensates and quantum optics are hindered because phase has at least three different meanings [1]. The introduction of phase as eigenvalues of a linear Hermitian phase operator is the most objective approach [1], and such an operator can be defined for BEC following the method of Pegg and Barnett [2] for EM fields.

For the case of a two mode BEC with mode annihilation operators \(\hat{a}, \hat{\bar{a}}, \hat{b}, \hat{\bar{b}}\) and spatial mode functions \(\phi_a(\mathbf{r}), \phi_b(\mathbf{r})\) basis states \(|n_a\rangle, |n_b\rangle\) involving \(n_a, n_b\) bosons in the modes can be used to define relative phase eigenstates \(|\theta_p\rangle\) for the \(N = n_a + n_b\) boson system, where \(\theta_p = p(2\pi/(N + 1))\), \(p = -N/2, -N/2 + 1, ..., +N/2\) is a quasi-continuum of \(N + 1\) equispaced phase eigenvalues, and from which the Hermitian relative phase operator \(\hat{\Theta}\) is then defined. We have

\[
|\theta_p\rangle = \frac{1}{\sqrt{N + 1}} \sum_{k=-N/2}^{N/2} \exp(i k \theta_p) |N/2 - k\rangle_a |N/2 + k\rangle_b \quad \hat{\Theta} = \sum_p \theta_p |\theta_p\rangle \langle \theta_p| \tag{1}
\]

The relative phase eigenstate has several interesting properties. Firstly, it is a state with maximal mode entanglement [3] for the \(a, b\) sub-systems, so is of interest in quantum information. Secondly, it is a fragmented state [4], since there are two natural orbitals with macroscopic occupancy. For large \(N\) the natural orbitals obtained from the first order quantum correlation function are \(\chi_{\pm}(\mathbf{r}) = (\exp(i \theta_p/2)\phi_a^\dagger(\mathbf{r}) \pm \exp(-i \theta_p/2)\phi_b^\dagger(\mathbf{r}))/\sqrt{2}\), with occupancies \((\frac{1}{2} \pm \frac{\delta}{2})N\). For fragmented states generalized mean field theories [5] are required. Thirdly, the relative phase eigenstate is a spin squeezed state. Spin operators along \(\langle \hat{J}_z \rangle\) and perpendicular \(\langle \hat{J}_x, \hat{J}_y \rangle\) to the Bloch vector may be defined by \(\hat{J}_z = \hat{S}_z, \hat{J}_y = \hat{S}_y \sin \theta_p + \hat{S}_x \cos \theta_p, \hat{J}_x = \hat{S}_x \cos \theta_p - \hat{S}_y \sin \theta_p\), where \(\hat{S}_x = (\hat{b}^\dagger \hat{a} + \hat{a}^\dagger \hat{b})/2, \hat{S}_y = (\hat{b}^\dagger \hat{a} - \hat{a}^\dagger \hat{b})/2i, \hat{S}_z = (\hat{b}^\dagger \hat{b} - \hat{a}^\dagger \hat{a})/2\) are the usual Schwinger operators. For large \(N\) the Bloch vector is \(\langle \hat{J}_z \rangle = 0, \langle \hat{J}_x \rangle = 0, \langle \hat{J}_y \rangle = \frac{\delta}{N} \approx 0.392N\), which is in the equatorial plane with azimuthal angle \(\phi = 2\pi - \theta_p\), and inside the Bloch sphere of radius \(N/2\) - another indicator of fragmentation. For large \(N\) the fluctuations \(\langle \Delta \hat{J}_x^2 \rangle = \langle \hat{J}_x^2 \rangle - \langle \hat{J}_x \rangle^2\) in the Bloch vector components are found to be \(\delta \hat{J}_x \approx \sqrt{1/12N} \approx 0.289N, \delta \hat{J}_y \approx 1.30, \delta \hat{J}_z \approx \sqrt{(1/6 - \pi^2/64)}N \approx 0.112N\). As \(\langle \hat{J}_z \rangle / 2 \approx 0.196N\) we see that \(\delta \hat{J}_x, \delta \hat{J}_y \approx 0.375N\) is greater than \(\langle \hat{J}_x \rangle / 2\), consistent with the Heisenberg uncertainty principle. However, although \(\hat{J}_x\) is unsqueezed, the other perpendicular component \(\hat{J}_y\) is highly squeezed, with a fractional fluctuation \(\delta \hat{J}_y / \langle \hat{J}_x \rangle \) of order \(1/N\). The relative phase state could be of interest in Heisenberg limited interferometry [6].

Finally, even though no proposal yet exists for preparing a BEC in a relative phase eigenstate, relative phase eigenstates are a valuable theoretical concept for describing behaviour in BEC interferometry experiments, such as the Dunningham and Burnett [7] proposal for Heisenberg limited interferometry in two mode BEC.

References