4 Lectures on
Quantum Optics
with photons and continuous laser beams

by Hans-A. Bachor

Australian Centre of Excellence for
Quantum Atom Optics
Canberra, Australia

More details can be found in:
A guide to experiments in quantum optics
H-A. Bachor & T.C. Ralph, VCH-Wiley 2004
Lecture 1

- Overview of the concepts and ideas

  - Classical model for laser beams & applications

  - Define quantum optics
A mode of light

Intensity $I$
Direction $z$
Size $w_0$
Polarisation $P$
Frequency $\nu$
Phase $\Phi$ (relative to second mode)

Information is sent in the form of modulation
of any one of these parameters
An electromagnetic wave can be described by the harmonic function at the Optical frequency \( \nu \) and the dimensionless complex amplitudes \( \alpha(r,t) \)

\[
E(r,t) \sim [\alpha(r,t) \exp(i \ 2\pi \nu \ t) + \alpha^*(r,t) \exp(-i \ 2\pi \nu \ t)] \ \mathbf{p} \ (r,t)
\]

Phase is an important concept expanding the complex amplitude into
The magnitude \( \alpha_0(r,t) \) and the phase \( \phi(r,t) \)

\[
\alpha(r,t) = \alpha_0(r,t) \ \exp \left( i \ \phi(r,t) \right)
\]

The spatial distribution of the phase \( \phi(r,t) \), or wavefront, determines the shape of the wave; plane wave: \( \phi(r,t) = k \cdot r \), \( \alpha(z) = a_0 \ \exp( i \ k \ z) \)  
spherical wave: \( \alpha(r,t) = \alpha_0/r \ \exp \left( i \ k \ r \right) \)
Quadrature amplitudes

We can describe the same wave using quadrature amplitudes $X_1$ and $X_2$.

$$\mathbf{E}(r,t) \sim [X_1(r,t) \cos (2\pi vt) + X_2(r,t) \sin (2\pi vt)] \ p (r,t)$$

With the definition for $X_1$ and $X_2$:

$$X_1(r,t) = \alpha(r,t) + \alpha^*(r,t) \quad X_2(r,t) = l [ \alpha(r,t) - \alpha^*(r,t)]$$

Each wave can be represented by a wave in a phasor diagram:
Phasor diagrams

In a phasor diagram each complex amplitude is represented by a vector. (phase space representation)

A beam with fluctuating magnitude and phase will provide quadratures that lie within an uncertainty area.
Gaussian beam

The shape and the total energy of a Gaussian beam remains fixed, but the beam broadens. The shape is preserved. Lenses and mirrors transform the Gaussian size and wavefront. (paraxial approximation)

This is the ideal TEM\(_{00}\) output mode from a laser or the mode created inside a cavity. In reality a beam has imperfections. These can be expressed as higher order modes. TEM\(ij\).

Mode-matching refers to overlapping two beams with the same mode, that means the same size and mode curvature. Interference with high fringe visibility requires mode-matching.
Modulation

Amplitude modulation  AM

\[ \alpha(t) = \alpha_0 \left( 1 - \frac{M}{2} \left( 1 - \cos (2\pi \Omega_{\text{mod}} t) \right) \right) \exp (i 2\pi \nu_L t) \]

\[ = \alpha_0 \left( 1 - \frac{M}{2} \right) \exp (i 2\pi \nu_L t) \]

\[ + \alpha_0 \frac{M}{4} \left[ \exp (i 2\pi (\nu_L + \Omega_{\text{mod}}) t) + \exp (i 2\pi (\nu_L - \Omega_{\text{mod}}) t) \right] \]

Phase or frequency modulation  FM

\[ \alpha(t) = \alpha_0 \exp \left( i M \cos (2\pi \Omega_{\text{mod}} t) \right) \exp (i 2\pi \nu_L t) \]

\[ = \alpha_0 \left\{ 1 - \frac{M^2}{4} + \ldots \right\} \exp (i 2\pi \nu_L t) \]

\[ + i \left( \frac{M}{2} + \ldots \right) \left[ \exp (i 2\pi (\nu_L + \Omega_{\text{mod}}) t) + \exp (i 2\pi (\nu_L - \Omega_{\text{mod}}) t) \right] \]

\[ - \left( \frac{M^2}{8} + \ldots \right) \left[ \exp (i 2\pi (\nu_L + 2 \Omega_{\text{mod}}) t) - \exp (i 2\pi (\nu_L - 2 \Omega_{\text{mod}}) t) \right] + \ldots \]
Graphical presentation of the sidebands

Both types of modulation (AM and FM) produce sidebands. For a laser at optical frequency $\nu_L$ and a modulation frequency $\Omega_{\text{mod}}$ these are at $\nu_L \pm \Omega_{\text{mod}}$. 
Example of a noise spectrum showing noise at many frequencies and two modulations at $\Omega_1$ and $\Omega_2$. This plot has a logarithmic y scale and the signal to noise ratio (SNR) can be read of directly if the modulation depth $M(\Omega) \gg \text{Var}(I(\Omega))$. 
Quantum Optics 0. order

Processes: spontaneous & stimulated emission and absorption

Light as an electromagnetic wave and atoms are quantised

\[ E_2 - E_1 = h \nu \]
\[ \Delta E_1 + \Delta E_2 = h \Delta \nu \]

Lifetimes \( \tau \) of atoms are given by dipole moments

Find these as the solutions of the Schrödinger equation of the atom
Beams of photons

Photon flux

Pulses

Poisson distribution

\[ V(n) = n_{\text{ave}} \]
Entangled photons

Pair generation in a nonlinear crystal

Correlated clicks

Quantum correlations between the photons in all properties: time, polarisation, …
Quantum noise in communication

\[ \text{Photon flux} \rightarrow \text{Photo current } i + \delta i (t) \]

\[ i + \delta i (t) \]

\[ t \]
Sending information

Photon flux

Photo current $i + \delta i (\Omega)$

Observe beat signals

Example AM modulation when both sidebands are in phase
Photons & laserbeams: what we observe

**Single Photon**

- Clicks
- Information: yes/ no within $\Delta t$
- Correlations between two detectors

**Laserbeams**

- Photocurrent
- Information: Modulation & Noise
- Correlations between two photocurrents
Quantum Optics 1. level

Photon statistics
  Arrival times
  Poissonian
  Bunching
  Anti-bunching

Application:
  Q. - cryptography

Quantum Noise limit
  QNL
  Quantum noise in photo current
  Limit to signal to noise ratio
  SNR reduces with power
  Limits to opt. Instruments (shot noise limit)
Quantum Noise: Real spectrum of a laser

![Graph showing quantum noise and signal with detection frequency Ω in MHz.](image)
Quantum noise in communication

Photon flux

Photo current $i + \delta i (\Omega)$

Observe beat signals

quantum noise

$= V (i) = i_{ave}$
Special properties of quantum noise

Absorption signal is limited by quantum noise

Classical noise is cancelled
Quantum noise is added
Quantum Optics

2. Level

Measurements below the QNL

noise $< \text{QNL}$ $\iff$ squeezed light

Measurements without noise penalty

Quantum non demolition experiments

QND

Generate one photon at a time

( number or Fock states )

$\implies$ elusive single photon source
Quantum Optics 3. level:

Entanglement
Two modes which allow information to be (perfectly) inferred

Pairs of photons

Two squeezed beams

Scientific goals:

Teleportation of information
Quantum logic ???
Transfer of entanglement light $\leftrightarrow$ atoms ???
A complete experiment ... many losses
# Experiment versus Theory

| $|a_1\rangle$, $|a_1\rangle$ | $H$ | $c|a_1\rangle + c_2|a_1\rangle + c_3|v\rangle$ |
|--------------------------|-----|----------------------------------|
| States                   | operators | Operator & vacuum state |
| Beams & modulation       | Experiment Linear & nonlinear components | Complex state Prob. of det & correlation |
| intensity, phase         | Loss | output beams |
| Signals & noise at $\Omega$ | output beams | Detectors electronics |
| Coherent states          | Coherent states | Coherent states |
| quadrature operators     | Quantum Transfer Function | Variances at $\Omega$ |
| Variances at $\Omega$    | $V_{1_{in}}(\Omega)$, $V_{2_{in}}(\Omega)$ | $V_{out}$=1 |
| $\alpha^2$, $\Phi$, $\delta X_1(\Omega)$, $\delta X_2(\Omega)$ | $\alpha_{out}^2$, $\Phi_{out}$, $V_{1_{out}}$, $V_{2_{out}}$ | $V_{in}(\Omega)$ |

12/13/04