Lecture 2

- <u>Quantising the laser field</u>

- <u>A quantum formalism for continuous laser beams</u>

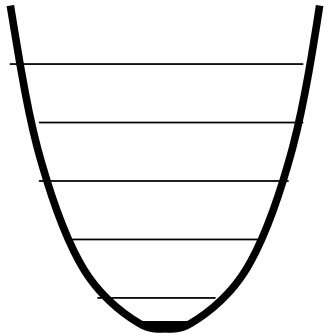
Quantisation of a laser beam

Start with the quantisation of one mode:

Consider the EM field => standing wave
Energy of the field: $\varepsilon_0 E_x^2(z,t) + 1/\mu 0\, H_y^2(z,t)$

=> operators E and H => possible but complex

use physics of the simple harmonic oscillator



=> Excitation of particle in harmonic potential
=> equally spaced Energy eigenvalues
=> ground state = 1/2 h ν

Number or Fock state

eigenstate $|n\rangle$ eigenvalue $h\nu\,(n+1/2)$ of the operator $\hat{a}^\dagger \hat{a}$

$$\hat{a}^\dagger \hat{a}\,|n\rangle = n\,|n\rangle = n\,|n\rangle$$

groundstate $|0\rangle$ cannot be lowered further

$$\hat{a}|n\rangle = \sqrt{n}\,|n-1\rangle \qquad \hat{a}^\dagger|n\rangle = \sqrt{(n+1)}\,|n+1\rangle$$

create all possible eigen states by $|n\rangle = (\hat{a}^\dagger)^n / \sqrt{n!}\;|0\rangle$

number states are orthogonal and complete

photon number is certain

$$\langle \Delta n^2\rangle = Vn = \langle n|\,\Delta n^2\,|n\rangle = \langle n|\,n^\dagger n\,|n\rangle - \langle n|n|n\rangle^2 = n^2\langle n|n\rangle - n^2\langle n|n\rangle = 0$$

useful where n is small => γ-rays, single photon optics
not useful for a laser where n is large

Coherent state |a>

We like: Eigenstate of operator â , clearly defined energy
Minimum uncertainty in intensity and phase
Closest approximation to ideal laser beam

Requires: Displacement operator D(α) that describes changes in the photon number via detection or loss:

$D(\alpha) = \exp(\alpha \hat{a} - \alpha^* \hat{a}^\dagger)$ $|\alpha\rangle = D(\alpha)\, |0\rangle$

it can be shown that $\hat{a}\, |\alpha\rangle = \alpha\, |\alpha\rangle$ α^2 **= photon number**

Expanding over number states $|a\rangle = \Sigma\, \alpha^n/\sqrt{n!}\, \exp(-1/2|\alpha|^2)\, |n\rangle$

coherent states are not exactly orthogonal

$|\langle\alpha_1|\alpha_2\rangle|^2 = |\langle 0|\, D^\dagger(\alpha_1)\, D(\alpha_2)\, |0\rangle|^2 = \exp(-|\alpha_1 - \alpha_2|^2)$

R. Glauber Phys.Rev.131, 2766 (1962)

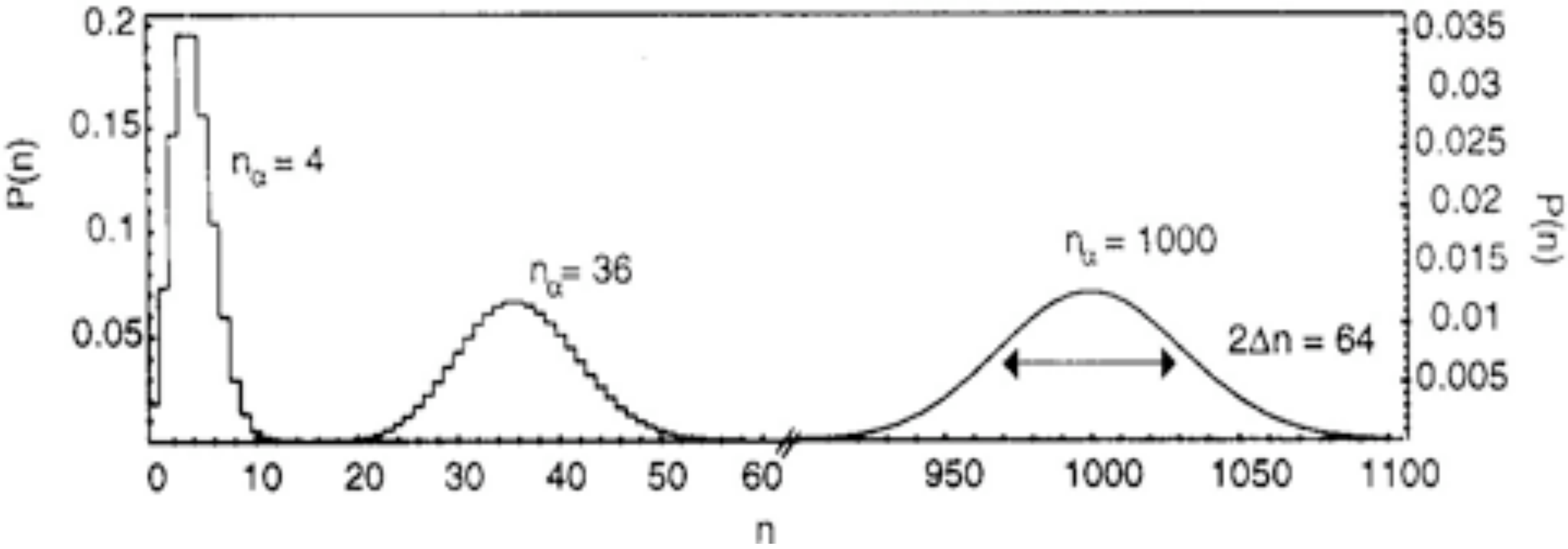
Photon number distribution

$P\alpha(n) = |\langle n|\alpha\rangle|^2 = |\alpha|^2 / n! \quad \exp(-|\alpha|^2)$

For larger α => Gaussian distribution of width

$\Delta n^2 = n_\alpha$

$Vn = n_\alpha$



Poissonian distribution of photon number for 3 coherent states

$n_\alpha = 4$ $n_\alpha = 36$ $n_\alpha = 1000$

Phase operator

We want to describe the phase of the light, but there is No direct operator, only approximation. For details see:
D.Pegg, S.M.Barnett J.of Mod. Optics 44, 225 (1997)

use $\hat{a} = (x1 + i\, x2) / 2$

x1 = quadrature amplitude
$x1 = \hat{a} + \hat{a}^\dagger$

x2 = quadrature phase
$x2 = -i\,(\hat{a} - \hat{a}^\dagger)$

x1 and x2 are hermitian <=> uncertainty principle applies

$\langle \Delta x1^2 \rangle \langle \Delta x2^2 \rangle = 1$ minimum uncertainty (pure state)

and $\langle \Delta x1^2 \rangle = \langle \Delta x2^2 \rangle$ symmetric uncertainty

Cavity modes and propagating beams

In side the cavity we have the operators

$\hat{a} = (x1 + i\,x2)/2$ with $\hat{a}^\dagger \hat{a}$ in units of photons

Outside a cavity with a freely propagating beam we have:

$A = (X1 + i\,X2)/2$ with $A^\dagger A$ in units of photons per sec

X1 = quadrature amplitude **X2 = quadrature phase**

$X1 = A + A^\dagger$ **$X2 = -i\,(A - A^\dagger)$**

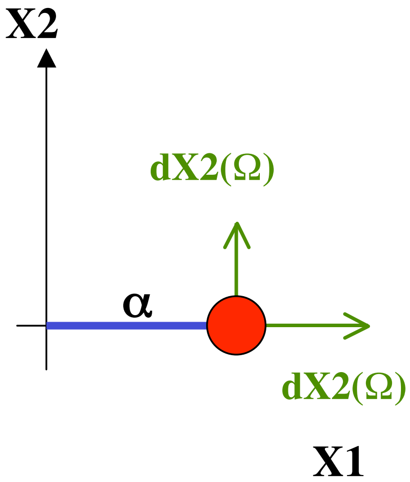
Note: more recently the notation is $X^+ = X1$, $X^- = X2$

X1 and X2 are hermitian <=> uncertainty principle applies

$\langle \Delta X1^2 \rangle \langle \Delta X2^2 \rangle = 1$ minimum uncertainty (pure state)

and $\langle \Delta X1^2 \rangle = \langle \Delta X2^2 \rangle$ symmetric uncertainty

Linearisation



Fluctuations are small =>
linearise around fixed value of a

$A = \alpha + \delta A(t)$ **(α always real, along X1 axis)**

photon flux $N = A^\dagger(t)\, A(t) = (\alpha + \delta A^\dagger(t))\,(\alpha + \delta A(t))$

$= \alpha^2 + \alpha\,\delta A^\dagger(t) + \alpha\,\delta A(t) + \delta A^\dagger(t)\,\delta A(t)$

$= \alpha^2 + \alpha\,\delta X1(t)$

we find expectation values such as

$N = \langle\alpha| N |\alpha\rangle = \alpha^2$

$VN = \langle\alpha| A^\dagger A A^\dagger A |\alpha\rangle - \langle\alpha| A^\dagger A |\alpha\rangle^2$ **(we measure)**

$V1 = \langle\alpha| X1^\dagger X1 |\alpha\rangle - \langle\alpha| X1 |\alpha\rangle^2 = VN/N$ **(we calculate)**

Spectral operators

The information is contained in certain frequency bands

$$A(\Omega) = \alpha\ \delta(0) + \delta A(\Omega)$$

$$\delta X1\ (\Omega) = \delta A(\Omega) + \delta A^{\dagger}(\Omega), \qquad \delta X2\ (\Omega) = -i\ (\delta A(\Omega) - \delta A^{\dagger}(\Omega))$$

$$N = \alpha^2\ \delta(0) + \alpha\ \delta X1\ (\Omega)$$

$$VN\ (\Omega) = \alpha^2 \langle\alpha|\ \delta X1\ (\Omega)^2\ |\alpha\rangle = \alpha^2\ V1\ (\Omega)$$

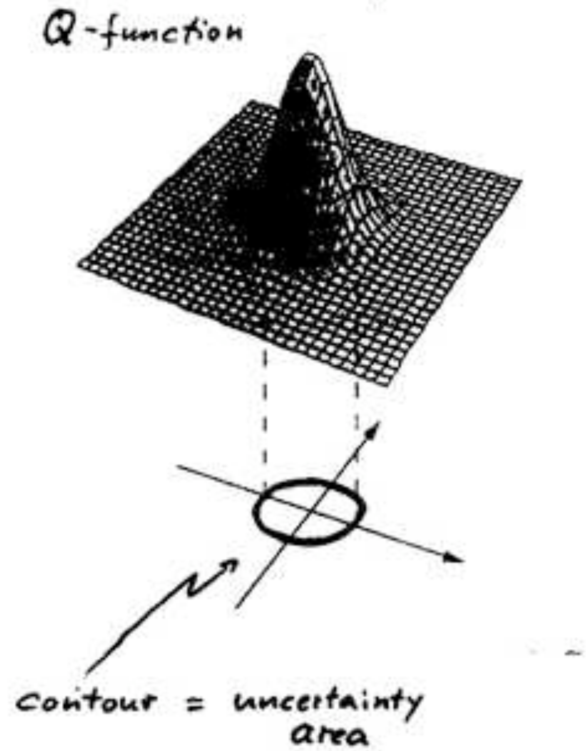
for quantum noise limited beam V1 =1

All probability distribution functions, in particular the Wigner Functions, are Gaussian. The variance is the only value required, no higher order momentsare possible.

We can measure VN experimentally and calculate V1.

We deduce everything from measurements of VN.

Ouasi Probabilitv distribution



Q - function

$Q\Psi(\alpha) = |\langle \alpha | \Psi \rangle|^2 / \pi$

If $|\psi\rangle$ is another coherent state $|\beta\rangle$

$Q\beta(\alpha) = \exp(-|\alpha-\beta|^2) / \pi$

which is a 2 dim Gaussian

Quantum noise C.W.Gardiner Springer 1991
Quantum optics D.F.Walls, G.J.Milburn Springer 1994
Quantum optics M.O.Scully, M.S.Zubairy Cambridge 1997

Wigner Function

Wigner - function, which is preferred in theory papers.

$$W\Psi(\alpha) = | \langle \alpha | D(\alpha_1)\, \Pi\, D(\alpha_2) | \Psi \rangle |^2 / \pi$$

Π is the Parity operator

$$\Pi (c_0 |0\rangle + c_1 |1\rangle + c_2 |2\rangle + ..) = (c_0 |0\rangle - c_1 |1\rangle + c_2 |2\rangle + ..)$$

If $|\psi\rangle$ is another coherent state $|\beta\rangle$

$$W\beta(\alpha) = 2 \exp(- 2 |\alpha-\beta|^2) / \pi$$

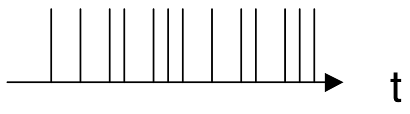
which also is a 2 dim Gaussian

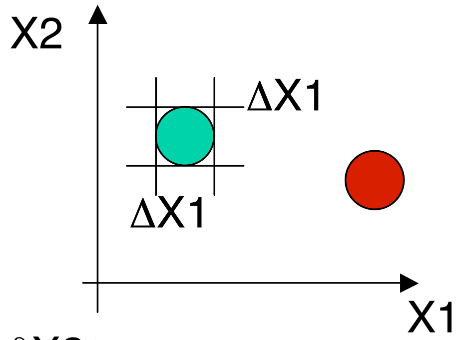
For nonclassical light the Wigner function has negative components.

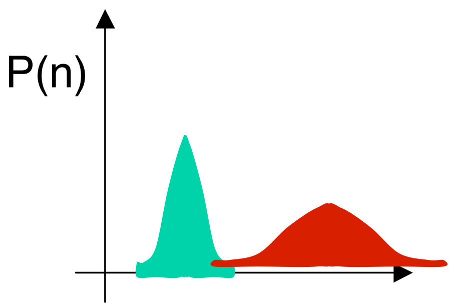
Quantum noise C.W.Gardiner, Springer 1991

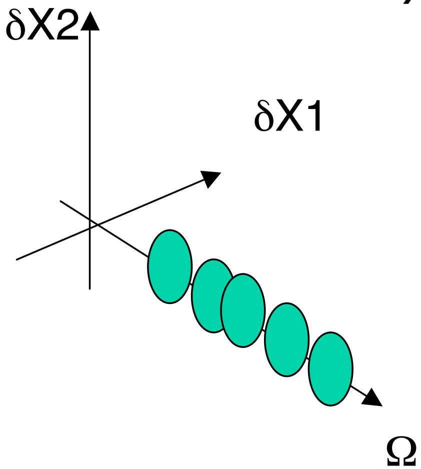
Representation of coherent states

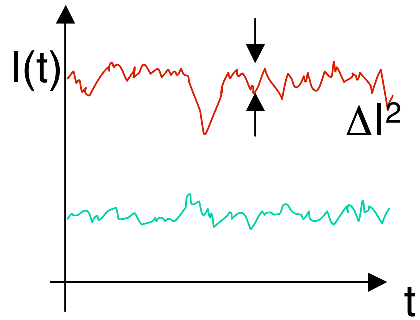
All these images are equivalent:

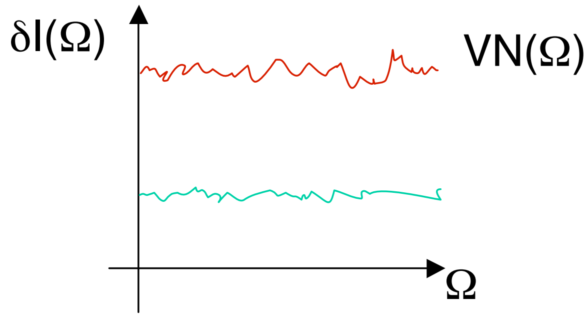


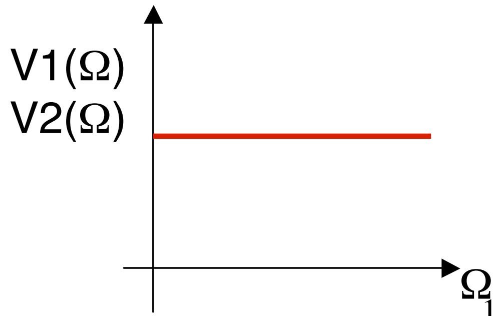












Transfer function

We can show that for all experiments we have:

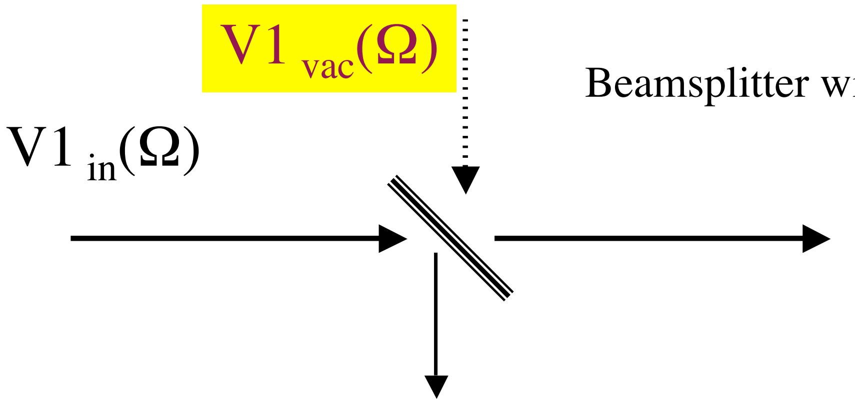
$$V1_{out}(\Omega) = c1\ V1_{in1}(\Omega) + c2\ V1_{in2}(\Omega) + c3\ V1_{in3}(\Omega) + $$
$$+ d1\ V2_{in1}(\Omega) + d2\ V2_{in2}(\Omega) + d3\ V2_{in3}(\Omega) + \ldots\ \ldots$$

$$V2_{out}(\Omega) = c1\ V2_{in1}(\Omega) + c2\ V2_{in2}(\Omega) + c3\ V2_{in3}(\Omega) + \ldots$$
$$+ d1\ V1_{in1}(\Omega) + d2\ V1_{in2}(\Omega) + d3\ V1_{in3}(\Omega) + \ldots\ \ldots$$

And for all systems without resonances
(that means no optical cavities) => di = 0,

This means V1 and V2 are independent:
no rotation of the quadratures.

Simplest example: beamsplitter



Beamsplitter with intensity reflection η

$V1_{out}(\Omega)$

$$V1_{out}(\Omega) = \eta\, V1_{in}(\Omega) + (1-\eta)\, V1_{vac}(\Omega)$$

All the difference to classical waves is in the vacuum beam

$$V1_{vac}(\Omega) = 1$$