Lecture 3

- Example of transfer functions
- Laser intensity noise
- Photo detection
Beamsplitter in the Photon picture:

(a) Coherent or Poissonian input

\[ N_{\text{Ain}} \]

\[ P(n) \]

Remains Poissonian
Beamsplitter in the Photon picture

Turns into Poissonian

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Beam splitter: Quantum calculation I

**Vacuum state**

\[
\begin{align*}
    & \delta A_r = \sqrt{\varepsilon} \delta A_{in} + \sqrt{(1-\varepsilon)} \delta A_u \\
    & \delta A_t = \sqrt{(1-\varepsilon)} \delta A_{in} - \sqrt{\varepsilon} \delta A_u \\
\end{align*}
\]

Input and output operators have to obey the commutation relationship for Bosons

\[
[\delta A_i, \delta A_j^\dagger] = \delta_{ij}, \quad [\delta A_i, \delta A_j] = 0
\]

We find that for the fluctuations we cannot ignore the mode \(\delta A_u\).

If we ignored it the commutators for \(\delta A_r\) and \(\delta A_t^\dagger\) would no longer work. For example, we want \([\delta A_r, \delta A_t^\dagger] = 0\). If we set \(\delta A_r = \sqrt{\varepsilon} \delta A_{in}\) and \(\delta A_t = \sqrt{1-\varepsilon} \delta A_{in}\), we would get \([\delta A_r, \delta A_t^\dagger] = \sqrt{(\varepsilon (1-\varepsilon))} [\delta A_{in}, \delta A_{in}^\dagger] = \sqrt{(\varepsilon (1-\varepsilon))}\) which is not correct.

We have to include \(\delta A_u\), the vacuum state makes a contribution to the noise.
Beam splitter: Quantum calculation II

Recipe: determine \( V1(\Omega) = \langle \delta X1(\Omega) \delta X1^*(\Omega) \rangle \)

\[
\delta X1_r(\Omega) = \sqrt{\epsilon} \left( \delta A_{in}^\dagger(\Omega) + \delta A_{in}(\Omega) \right) + \sqrt{1-\epsilon} \left( \delta A_{u}^\dagger(\Omega) + \delta A_{u}(\Omega) \right)
\]

\[
\langle \delta X1_r(\Omega) \delta X1_{r}^*(\Omega) \rangle = \\
\epsilon \langle \left( \delta A_{in}^\dagger(\Omega) + \delta A_{in}(\Omega) \right)^2 \rangle + (1-\epsilon) \langle \left( \delta A_{u}^\dagger(\Omega) + \delta A_{u}(\Omega) \right)^2 \rangle + \sqrt{\epsilon (1-\epsilon)} \left\{ \langle \delta A_{in}^\dagger(\Omega) \delta A_{u}(\Omega) \rangle + \langle \delta A_{in}(\Omega) \delta A_{u}^\dagger(\Omega) \rangle \right\} + \ldots
\]

where the terms in the \{\} bracket all cancel.

Result: \( V1_r = \epsilon \ V1_{in} + (1-\epsilon) \ V1_u \) and \( V1_t = (1-\epsilon) \ V1_{in} + \epsilon \ V1_u \)

since \( V1_u = 1 \) we get \( (V1_r - 1) = \epsilon \ (V1_{in} - 1) \).
that means the difference between \( V1_{in} \) and the quantum noise limit gets reduced.
\( V1_r \) is closer to the QNL than \( V1_{in} \). We say: A vacuum beam gets mixed in.

We can interpret the effect of a beam splitter as adding a part \((1-\epsilon)\) of the vacuum beam.
Effect on signal to noise ratio

The signal to noise ratio is given by

\[ \text{SNR} = \frac{(V_{\text{signal}}(\Omega) - V_{\text{quantum noise}}(\Omega))}{V_{\text{quantum noise}}(\Omega)} \]

and for \( V_{\text{quantum noise}}(\Omega) = 1 \) and for signals sufficiently large we simply have:

\[ \text{SNR}_{\text{out}} = \varepsilon \text{SNR}_{\text{in}} \]

for a quantum noise limited system with large signal

This applies to all situations where we have attenuation, such as absorption losses, diffraction, quantum inefficiencies, mode matching errors, .......

This makes the experiments so challenging.
Interferometer: Classical calculation

Intensity at the output is the standard fringe pattern

\[ I_1 (\Delta \phi) = I_{in} \cos^2 (\Delta \phi / 2) \]
\[ I_2 (\Delta \phi) = I_{in} \sin^2 (\Delta \phi / 2) \]

Visibility

\[ \text{Vis} = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}} \]

\[ \alpha_i = \sqrt{1/2} \alpha_{in} \]
\[ \alpha_{ll} = \sqrt{1/2} \alpha_{in} \]
\[ \alpha_1 = \sqrt{1/2} \alpha_i \exp(i \Delta \phi) - \sqrt{1/2} \alpha_{ll} \]
\[ \alpha_1 = \sqrt{1/2} \alpha_i \exp(i \Delta \phi) + \sqrt{1/2} \alpha_{ll} \]
Interferometer: Quantum calculation

\[
\delta A_l = \sqrt{1/2} \delta A_{\text{in}} - \sqrt{1/2} \delta A_u
\]
\[
\delta A_{\parallel} = \sqrt{1/2} \delta A_{\text{in}} + \sqrt{1/2} \delta A_u
\]
\[
\delta A_1 = \sqrt{1/2} \delta A_l - \exp(i \Delta \phi) \sqrt{1/2} \delta A_{\parallel}
\]
\[
\delta A_2 = \sqrt{1/2} \delta A_l + \exp(i \Delta \phi) \sqrt{1/2} \delta A_{\parallel}
\]
\[
\delta A_l = \sqrt{1-\varepsilon} \varepsilon \delta A_{\text{in}} - \varepsilon \sqrt{1-\varepsilon} \exp(i \Delta \phi) \delta A_{\text{in}}
\]
\[
+ \sqrt{1-\varepsilon} \varepsilon \delta A_u + \varepsilon \sqrt{1-\varepsilon} \exp(i \Delta \phi) \delta A_u
\]

Determine \( V_{11}(\Omega) = \langle \delta X_{11}(\Omega) \delta X_{11}^*(\Omega) \rangle \)

For \( \varepsilon = 1/2 \)

\[
V_{11}(\Omega) = \cos^2(\Delta \phi/2) V_{1\text{in}}(\Omega) + \sin^2(\Delta \phi/2) V_{2u}(\Omega)
\]

This is like a variable attenuator.
Cavity: Quantum calculation

Cavity on resonance $\Delta = 0$, that means cavity is locked to the laser frequency.

Total loss $\kappa = \kappa_1 + \kappa_2 + \kappa_{\text{loss}}$ and small losses are important here.

\[
d \frac{\delta a_{\text{cav}}}{dt} = -\kappa \delta a_{\text{cav}} + \sqrt{2}\kappa_1 \delta A_{\text{in}} + \sqrt{2}\kappa_2 \delta A_{\text{u}} + \sqrt{2}\kappa_{\text{loss}} \delta A_{\text{loss}}
\]

\[
d \frac{\delta X_{1\text{cav}}}{dt} = -\kappa \delta X_{1\text{cav}} + \sqrt{2}\kappa_1 \delta X_{1\text{in}} + \sqrt{2}\kappa_2 \delta X_{1\text{u}} + \sqrt{2}\kappa_{\text{loss}} \delta X_{1\text{loss}}
\]

Boundary conditions $\delta X_{1\text{out}} = \sqrt{2}\kappa_2 \delta X_{1\text{cav}} - \delta X_{1\text{u}}$ and $\delta X_{1\text{refl}} = \sqrt{2}\kappa_1 \delta X_{1\text{cav}} - \delta X_{1\text{in}}$

$\delta X_{1\text{out}} = \sqrt{4}\kappa_1\kappa_2 \delta X_{1\text{in}} + \sqrt{4}\kappa_1\kappa_{\text{loss}} \delta X_{1\text{loss}} + (2\kappa_1 - \kappa - i2\pi\Omega) \delta X_{1\text{u}} / (\kappa - i2\pi\Omega)$

With $f\{df/dt\} = 2\pi\Omega f\{f(t)\}$

$V_{1\text{out}}(\Omega) = (4\kappa_1\kappa_2 V_{1\text{in}}(\Omega) + 4\kappa_1\kappa_{\text{loss}} V_{1\text{loss}} + [(2\kappa_1 - \kappa)^2 + (2\pi\Omega)^2] V_{1\text{u}}) / (\kappa^2 + (2\pi\Omega)^2)$

Transfer function is frequency dependent. Cavity needs to be locked to the laser.

Different behaviour inside and outside the cavity linewidth.
Transfer of signals

The cavity transmits the low frequencies but reflects the high frequencies. This applies to signals (V1 >1) and to squeezing (V1 <1).

If all inputs are at the QNL all the outputs are at the QNL. This is a good test.
The model for a laser: the energy level and processes and all the possible noise sources

Quantitative effect of loss
The laser noise equations

\[ V_{1\text{las}}(\Omega) = \frac{1}{[((2\pi \Omega_{\text{RRO}})^2 - (2\pi \Omega)^2)^2 + (2\pi \Omega)^2 \gamma_L^2} \]

\[
\{ 1 + 4\kappa_m^2 (2\pi \Omega)^2 + \gamma_L^2 \} - 8 \kappa_m G_{32} \alpha^2 \gamma_L V_{\text{vac}} \\
+ 2 \kappa_m G_{32}^2 \alpha^2 \Gamma V_{\text{pump}} \\
+ (2 \kappa_m G_{32}^2 \alpha^2 \gamma_t J_3 V_{\text{spont}} \\
+ 2 \kappa_m G_{32} (\gamma_t + \Gamma)^2 + (2\pi \Omega)^2 V_{\text{dipole}} \\
+ 4\kappa_m \kappa_l (2\pi \Omega)^2 + \gamma_L^2 \} V_{\text{losses}} \}
\]

with \( \kappa_m \) the resonator mirror, \( G_{32} \) rate constant, \( \gamma_L \) = combined rate, \( \Gamma \) = pump rate etc..... these are all parameters which can be determined independently for each laser.

The main point: the noise spectrum is a linear combination of the various input losses.

This model predicts a complex noise spectrum.
Noise of a Nd:YAG laser

Laser noise spectrum from the quantum transfer functions previous page. The spectrum peaks around $\Omega_{RRO}$, the resonance frequency for the relaxation oscillation. The pump noise (ii) dominates at very low frequencies. Total noise, for a QNL pump is given by line (vi). At high frequencies the laser is QNL. This is the best a conventional laser can achieve.
Noise of a Nd:YAG laser

Experimental test of laser noise:
(i) experiment, large pump noise
(ii) theory, large pump noise
(iii) experiment, low pump noise
(iv) theory, low pump noise

below RRO: laser noise comes from the pump
Above RRO laser can be quantum noise limited
Summary

We have a technique for predicting the noise spectrum for our experiment.

This can include passive components (mirrors, lenses,...) as well as active components (lasers, amplifiers,.....)

Once resonant systems (cavities) are involved the quadratures X1 and X2 will mix, that is we are getting a rotation is phase space.

Many lasers can be QNL, at least a high detection frequencies.

The coherent state is a perfect description of the idealised laser.

Real lasers can reach the pure coherent state for some detection frequencies.
Direct detection

\[ V1_{\text{las}}(\Omega) \]

Detector quantum efficiency \( \eta \)

Amplifier gain \( g_{\text{el}}(\Omega) \)

noise \( p_{\text{el}}(\Omega) \)

Spectrum Analyser
Displays  \( P_i(\Omega) \)

We need experimental calibration techniques to verify \( P_i, \text{QNL}(\Omega) \)

\[ V1_{\text{las}}(\Omega) = P_i(\Omega) / P_i, \text{QNL}(\Omega) \]
Quantum Noise: Real spectrum of laser signal

![Graph showing quantum noise and signal]

- **Signal**
- **Quantum noise**
- **Electrical pickup**
- **Detection frequency $\Omega$ [MHz]**

(i) Laser on
(ii) Laser off
Detection efficiency

The quantum efficiency varies with the material of the detector. The main materials are Si for visible light and InGaAs for light between 1000-2000 nm. The only fast photon counting detectors are so far made from Si. For fast CW detection we can use both Si and InGaAs. Practical efficiencies are 85% to 90% at 550-750 nm and at 1060 nm.
The noise contributions at various stages of the detection and amplification process. The amplifier generates gain noise, and the electronics adds thermal (or Johnson) noise. This illustrates the decline in the ratio of signal (downwards) to noise (upwards).
A real photo detector circuit

The effect of a spectrum analyser

A typical display from the spectrum analyser fluctuations and a signal and also pickup of stray radiation. The apparatus stores all settings.
The effect of a spectrum analyser

The spectrum analyser displays the variance of the fluctuations and a signal.

The actual noise level depends on the Resolution Bandwidth (RBW). We cannot make it too large because we are averaging over many spectral components of the light. The display can be made smoother by using a lower Video Bandwidth (VBW). However, this averages in time and the experiment has to be very stable.
Calibration: Balanced detector

\[ V_{1\text{vac}}(\Omega) \]

\[ V_{1\text{las}}(\Omega) \]

\[ \frac{V_{1\text{las}}(\Omega)}{50} \]

\[ \frac{50}{50} \]

\[ +/ - \]

\[ \frac{50}{50} \]

\[ \text{signal and q-noise} \]

\[ \text{only q-noise} \]

\[ V_{1\text{las}}(\Omega) = \frac{V_i(\text{+})}{V_i(-)} \]

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Balanced homodyne detector

\[ V_{\theta}(\Omega) = \sin^2(\theta) V_{1\text{ in}}(\Omega) + \cos^2(\theta) V_{2\text{ in}}(\Omega) \]

For \( \alpha_{\text{lo}} >> \alpha_{\text{in}} \)

we can observe both quadratures, one at a time. Calibrate by blocking \( V_{1\text{in}} \).