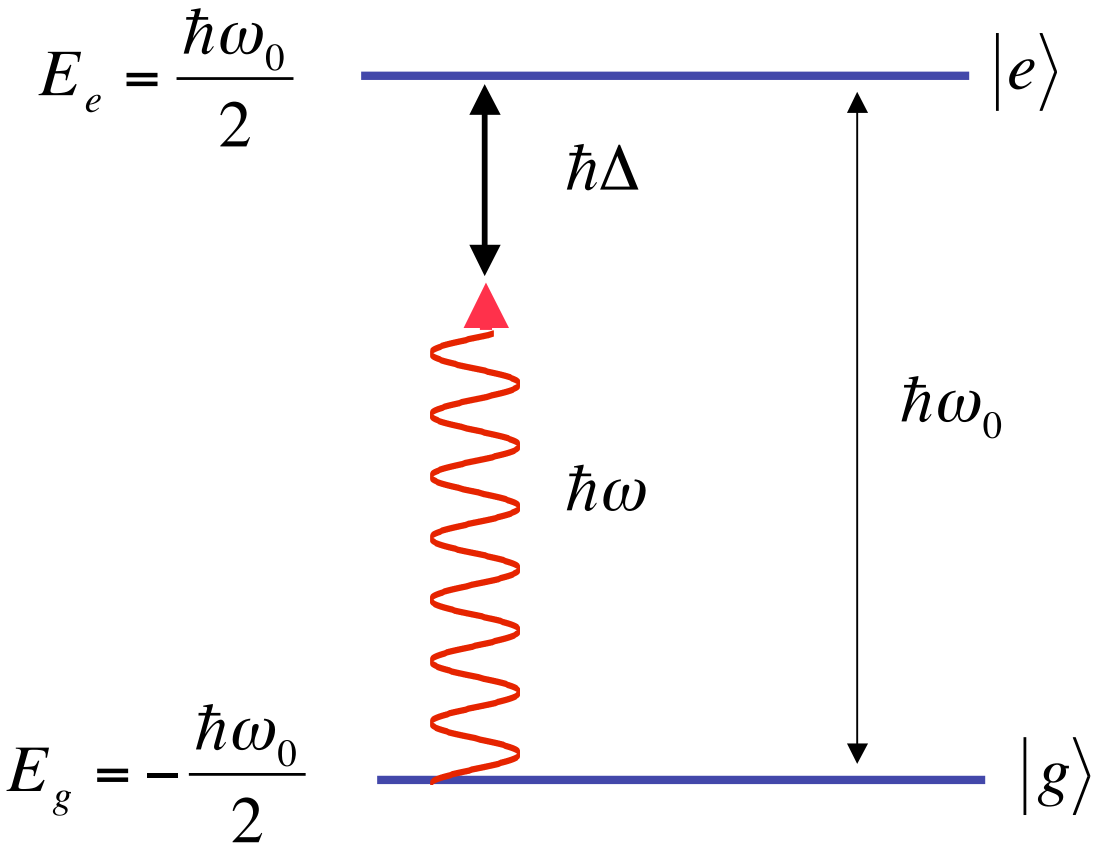
The Two Level Atom



$$\boldsymbol{H_A = \frac{\hbar\omega_0}{2}\{\, |e\rangle\langle e| - |g\rangle\langle g|\,\}} + \vec{\wp}\bullet\vec{E}\cos\omega t\{\, |e\rangle\langle g| + |g\rangle\langle e|\,\}$$

$$\vec{\wp} = q\langle e|\vec{r}|g\rangle$$

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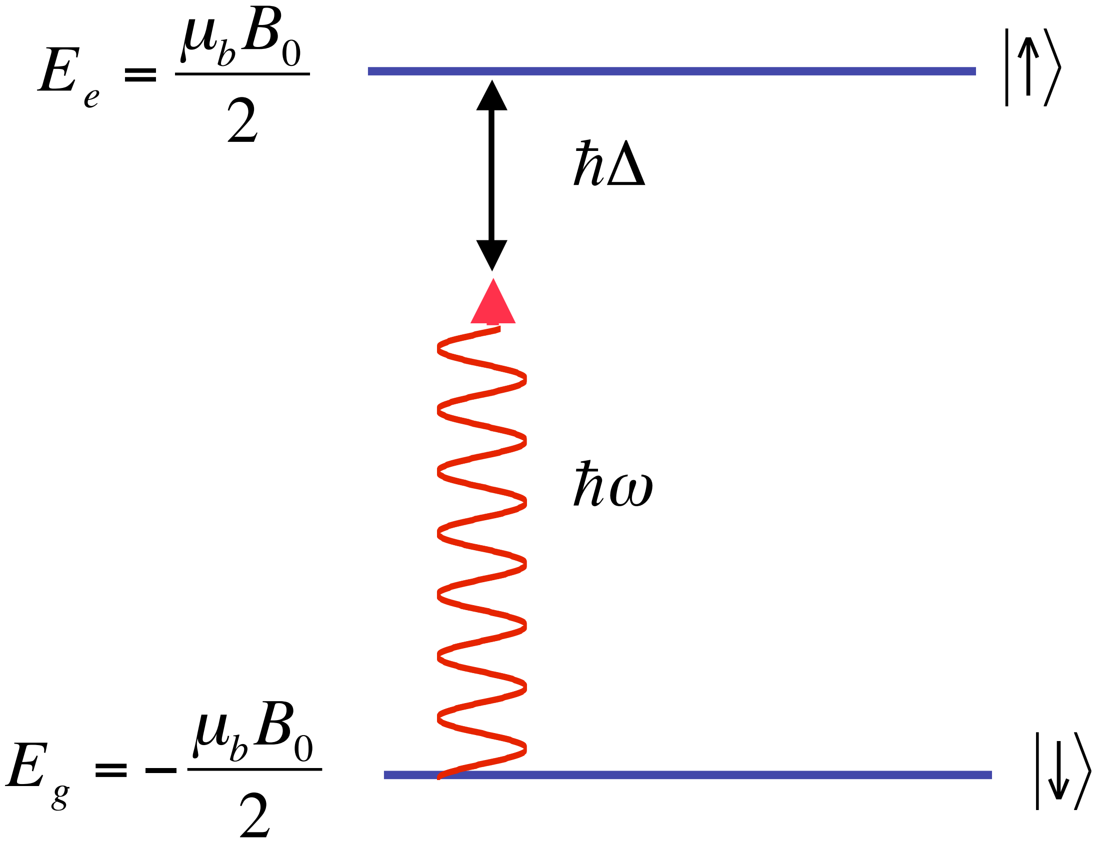

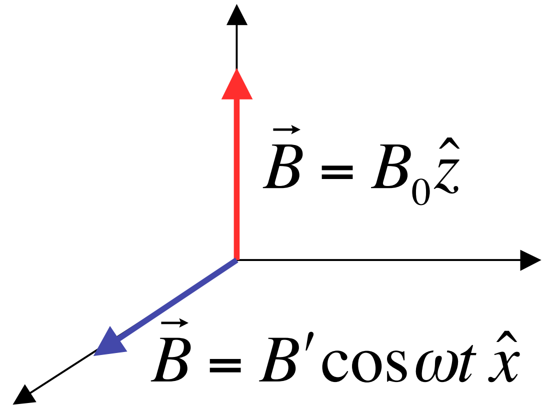

$$H_A = \frac{\mu_b B_0}{2}\left\{ |\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow| \right\} + \frac{\mu_b B'}{2}\cos\omega t \left\{ |\uparrow\rangle\langle\downarrow| + |\downarrow\rangle\langle\uparrow| \right\}$$

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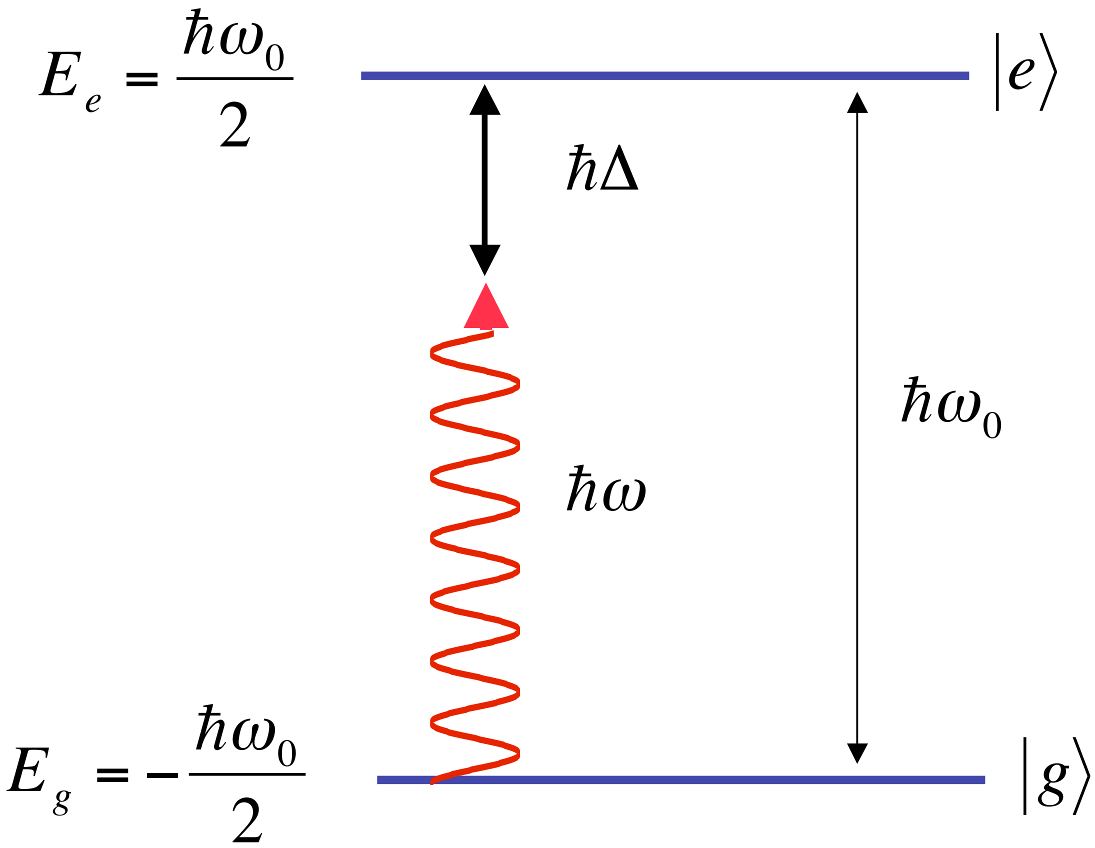


$$\frac{\hbar\omega}{2}\{|e\rangle\langle e| - |g\rangle\langle g|\} + \frac{\hbar\Delta}{2}\{|e\rangle\langle e| - |g\rangle\langle g|\} + \vec{\wp}\bullet\vec{E}\cos\omega t\{|e\rangle\langle g| + |g\rangle\langle e|\}$$

Schrödinger Picture

$$H = H_0 + V$$

$$i\hbar \frac{\partial}{\partial t} |\Psi\rangle_S = (H_0 + V)|\Psi\rangle_S$$

Unitary Transformation

$$|\Psi\rangle_I = e^{\frac{iH_0 t}{\hbar}} |\Psi\rangle_S$$

$$V_I = e^{\frac{iH_0 t}{\hbar}} V_S e^{\frac{-iH_0 t}{\hbar}}$$

Interaction Picture

$$i\hbar \frac{\partial}{\partial t} |\Psi\rangle_I = V_I |\Psi\rangle_I$$

Interaction Picture Reality Check

Commutators

$$\left[\hat{A}_S, \hat{B}_S\right] = \hat{C}_S$$

$$\hat{A}_I = \exp\left(iH_0 t/\hbar\right)\hat{A}_s \exp\left(-iH_0 t/\hbar\right) \quad \hat{B}_I = \exp\left(iH_0 t/\hbar\right)\hat{B}_s \exp\left(-iH_0 t/\hbar\right)$$

$$\left[\hat{A}_I, \hat{B}_I\right] = \hat{C}_I$$

Expectation Values

$$\left\langle \Psi_S \right| \hat{A}_S \left| \Psi_S \right\rangle$$

$$= \underbrace{\left\langle \Psi_S \right| \exp(-iH_0 t/\hbar)}_{\left\langle \Psi_I \right|} \underbrace{\exp(iH_0 t/\hbar)\hat{A}_S \exp(-iH_0 t/\hbar)}_{A_I} \underbrace{\exp(iH_0 t/\hbar)\left| \Psi_S \right\rangle}_{\left| \Psi_I \right\rangle}$$

$$= \left\langle \Psi_I \right| \hat{A}_I \left| \Psi_I \right\rangle$$

Two Level Atom in the (an) Interaction Picture

$$\underbrace{\frac{\hbar\omega}{2}\{|e\rangle\langle e| - |g\rangle\langle g|\}}_{H_0} + \underbrace{\frac{\hbar\Delta}{2}\{|e\rangle\langle e| - |g\rangle\langle g|\} + \vec{\wp}\bullet\vec{E}\cos\omega t\{|e\rangle\langle g| + |g\rangle\langle e|\}}_{V}$$

$$V_I = \frac{\hbar\Delta}{2}\{\,|e\rangle\langle e| - |g\rangle\langle g|\,\} + \frac{\hbar\,\Omega}{2}\,\{\,|e\rangle\langle g| + |g\rangle\langle e|\,\}$$

$$V_I = \frac{\hbar}{2}\begin{pmatrix} \Delta & \Omega \\ \Omega & -\Delta \end{pmatrix} \qquad \Omega = \frac{\vec{\wp}\bullet\vec{E}}{\hbar} = Rabi\ \ frequency$$

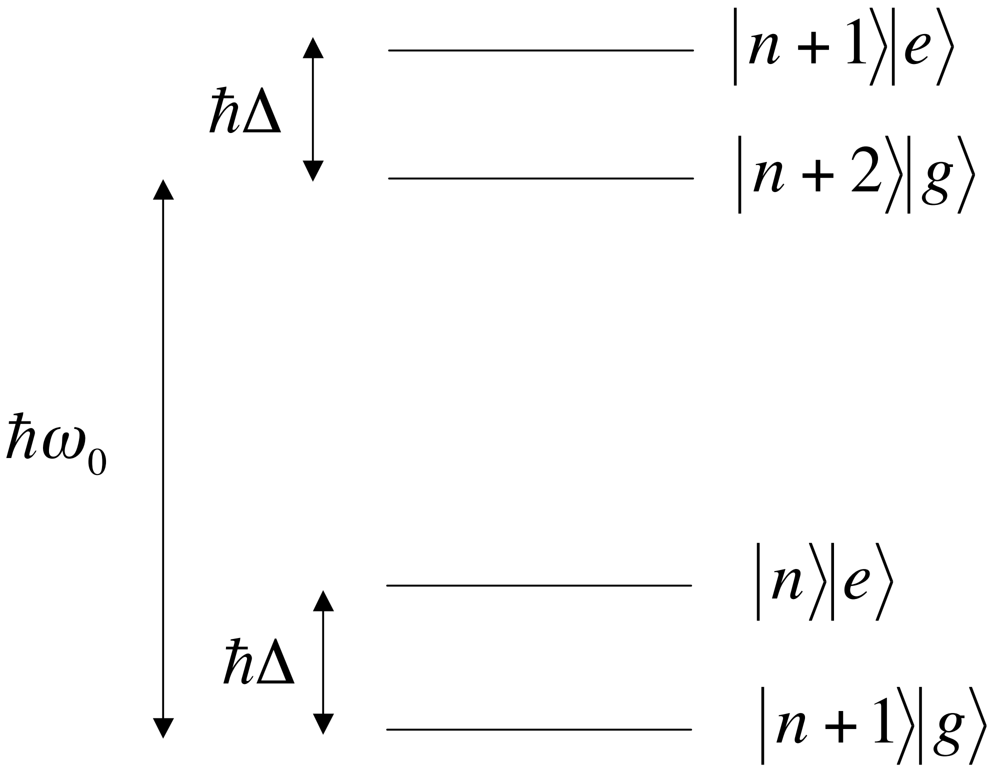
Full Quantum Model

$$H = \underbrace{\hbar\omega\left(a^{+}a + 1/2\right)}_{H_F} + \underbrace{\frac{\hbar\omega_0}{2}\left\{\,|e\rangle\langle e| - |g\rangle\langle g|\,\right\}}_{H_A} + \underbrace{\hbar\Omega_v\left(a|e\rangle\langle g| + a^{+}|g\rangle\langle e|\right)}_{H_{AF}}$$

Compare With Semiclassical

$$V_I = \frac{\hbar\Delta}{2}\left\{\,|e\rangle\langle e| - |g\rangle\langle g|\,\right\} + \frac{\hbar\,\Omega}{2}\left\{\,|e\rangle\langle g| + |g\rangle\langle e|\,\right\}$$

$$H_F + H_A = \hbar\omega\left(a^+ a + 1/2\right) + \frac{\hbar\omega_0}{2}\left\{\,|e\rangle\langle e| - |g\rangle\langle g|\,\right\}$$



$$H = \frac{\hbar}{2}\begin{pmatrix} \Delta & 0 \\ 0 & -\Delta \end{pmatrix}$$

$$\hbar\omega\left(a^{+}a+1/2\right)+\frac{\hbar\omega_0}{2}\left\{\,|e\rangle\langle e|-|g\rangle\langle g|\,\right\}+\hbar\Omega_v\left(a|e\rangle\langle g|+a^{+}|g\rangle\langle e|\right)$$

Basis states

$$|n+1\rangle|g\rangle$$

$$|n\rangle|e\rangle$$

$$H=\frac{\hbar}{2}\begin{pmatrix}\Delta & \Omega_v\sqrt{n+1}\\ \Omega_v\sqrt{n+1} & -\Delta\end{pmatrix}$$

Compare Full Quantum and Semi-classical

Full quantum

$$H = \frac{\hbar}{2}\begin{pmatrix} \Delta & \Omega_v\sqrt{n+1} \\ \Omega_v\sqrt{n+1} & -\Delta \end{pmatrix}$$

Semi-classical

$$V_I = \frac{\hbar}{2}\begin{pmatrix} \Delta & \Omega \\ \Omega & -\Delta \end{pmatrix}$$

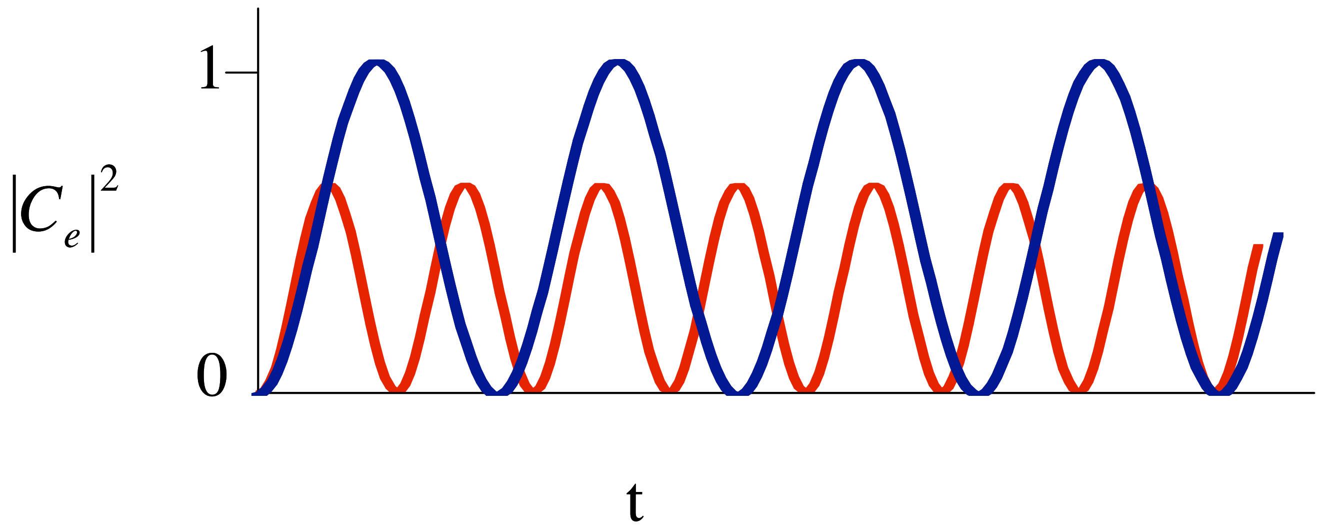
Two Level Atom in the (an) Interaction Picture

$$\left|\Psi(t)\right\rangle = C_e(t)\left|e\right\rangle + C_g(t)\left|g\right\rangle$$

$$\begin{pmatrix} \dot{C}_e \\ \dot{C}_g \end{pmatrix} = -\frac{i}{2}\begin{pmatrix} \Delta & \Omega \\ \Omega & -\Delta \end{pmatrix}\begin{pmatrix} C_e \\ C_g \end{pmatrix}$$

Solving for the dynamics of the two level atom with boundary Condition Cg(0) =1, Ce(0) = 0 gives:

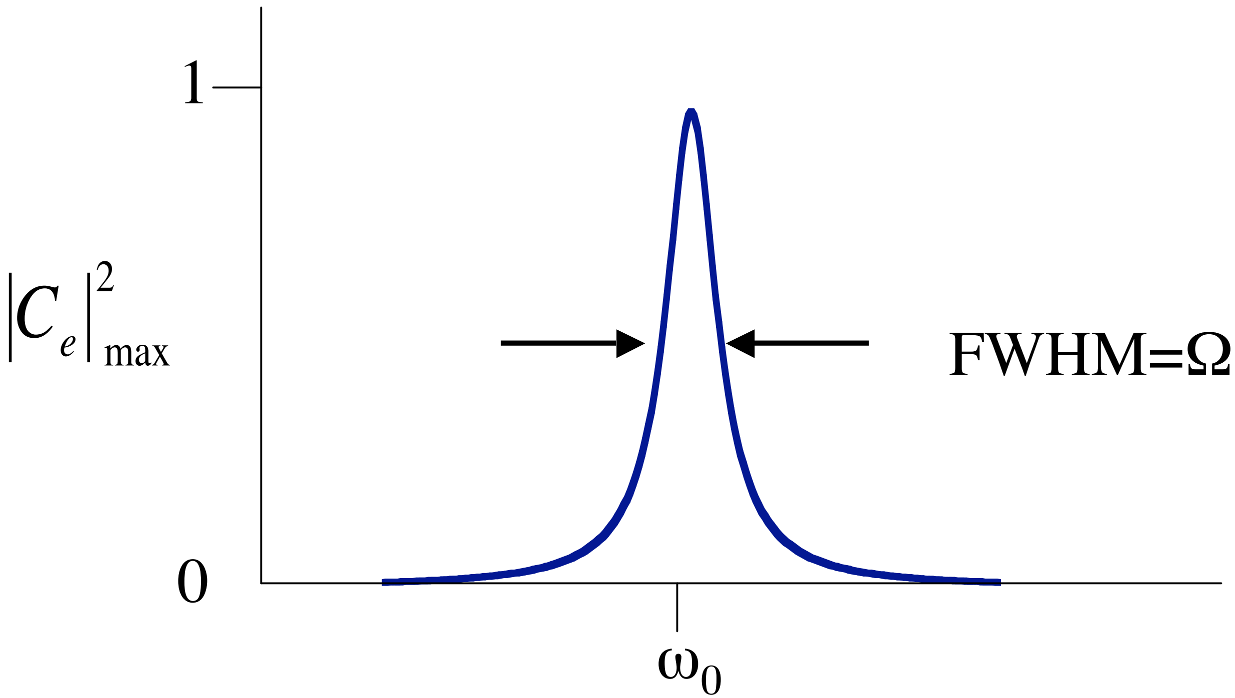
$$\left|C_e\right|^2 = \frac{\Omega^2}{\Omega^2 + \Delta^2} \sin^2\left(\sqrt{\Omega^2 + \Delta^2}\ t\right)$$



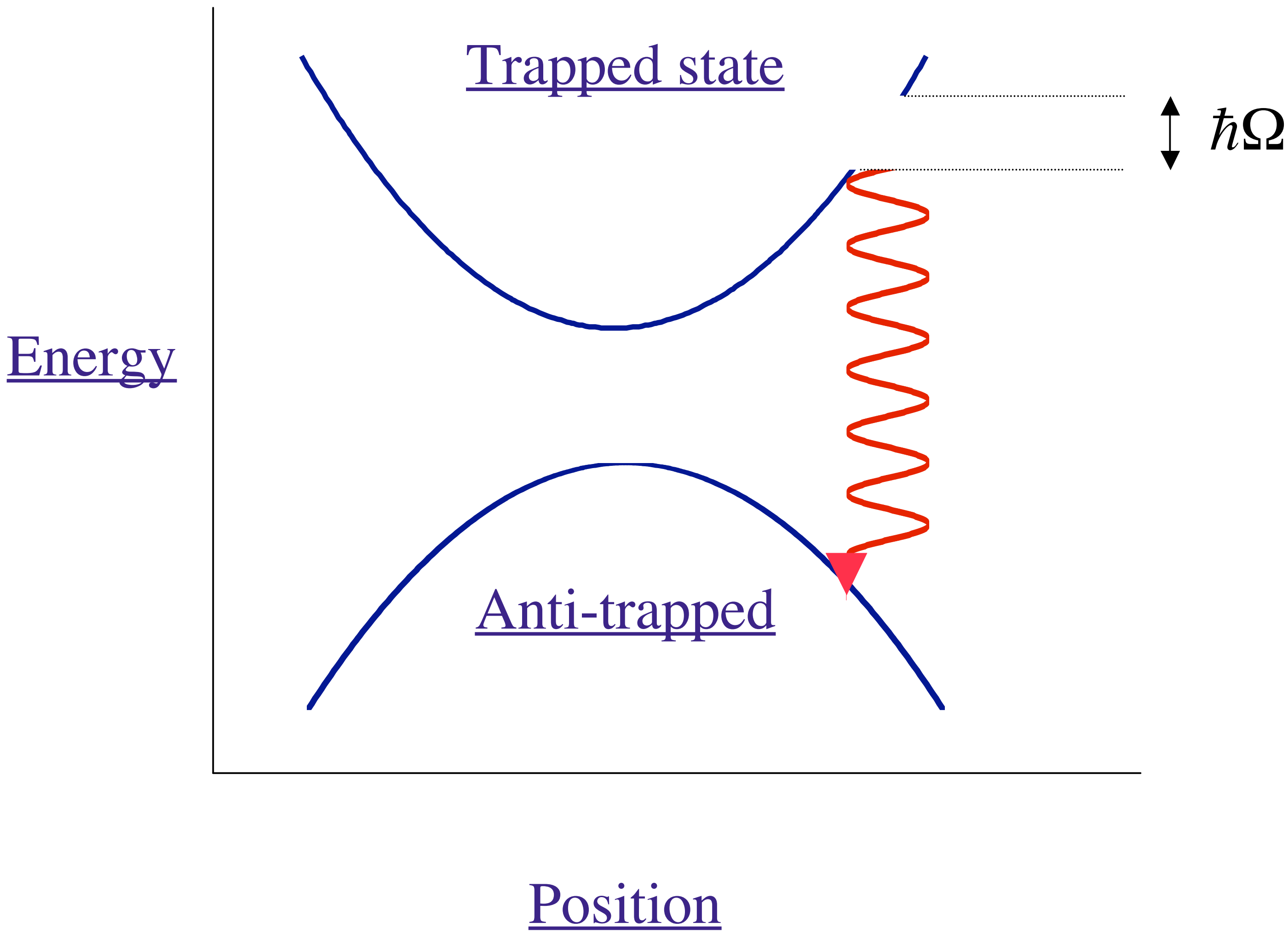
(question 2)

Solving for the dynamics of the two level atom with boundary Condition Cg(0) =1, Ce(0) = 0 gives:

$$|C_e|^2 = \frac{\Omega^2}{\Omega^2 + \Delta^2} \sin^2\left(\sqrt{\Omega^2 + \Delta^2}\ t\right)$$



Simple discussion of RF evaporation



Semi-classical Dressed States

$$V_I = \frac{\hbar}{2}\begin{pmatrix} \Delta & \Omega \\ \Omega & -\Delta \end{pmatrix}$$

This Hamiltonian has eigenvalues

$$E = \pm\frac{\hbar}{2}\sqrt{\Omega^2 + \Delta^2}$$

$$E \approx \pm\frac{\hbar}{2}\Delta\left[1 + \frac{1}{2}\left(\frac{\Omega}{\Delta}\right)^2\right]$$

And eigenstates…..

$$|1\rangle = \cos\theta|e\rangle + \sin\theta|g\rangle \qquad E = \frac{\hbar}{2}\sqrt{\Omega^2 + \Delta^2}$$

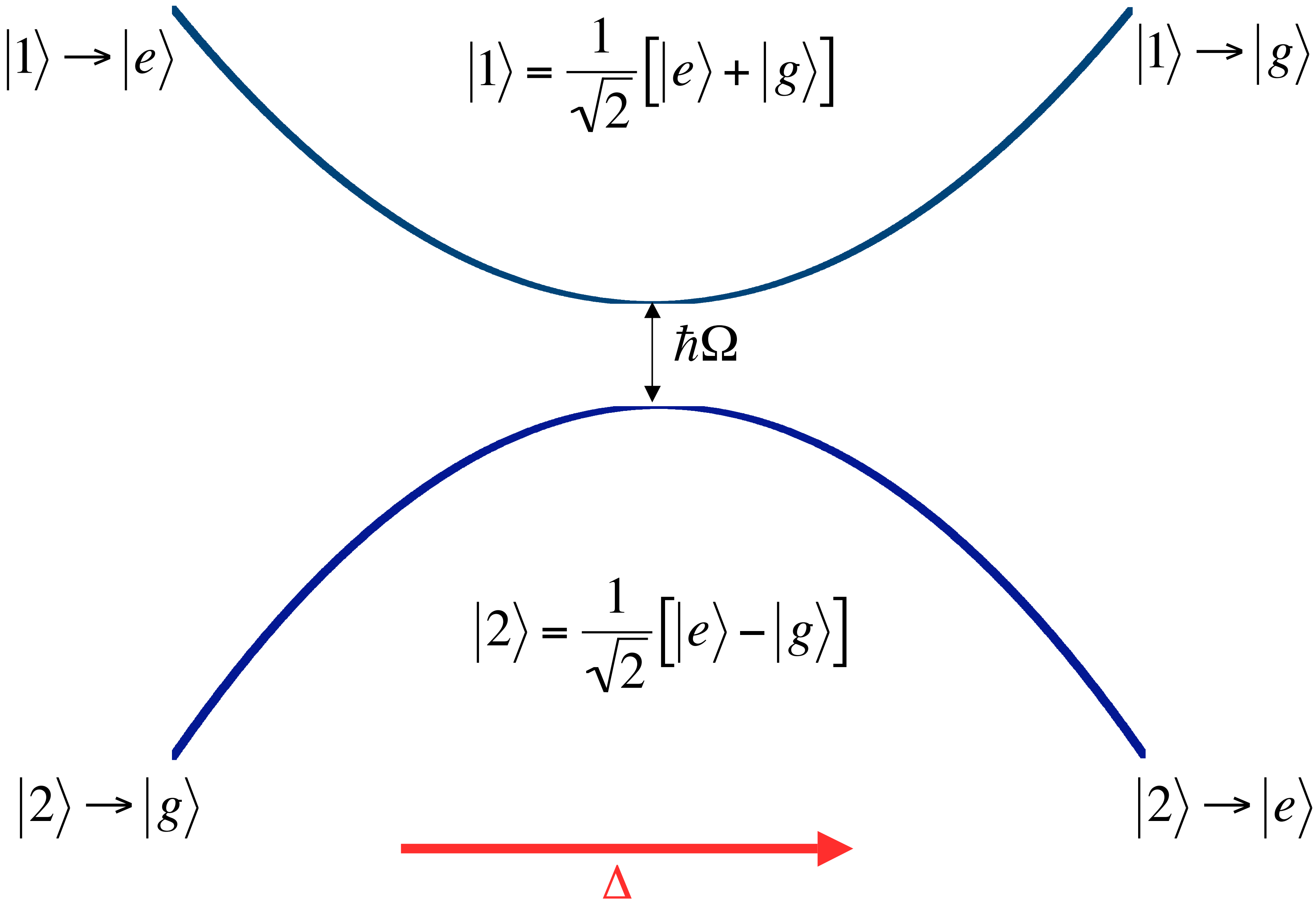
$$|2\rangle = \sin\theta|e\rangle - \cos\theta|g\rangle \qquad E = -\frac{\hbar}{2}\sqrt{\Omega^2 + \Delta^2}$$

$$\cos 2\theta = -\frac{\Delta}{\sqrt{\Omega^2 + \Delta^2}} \qquad \sin 2\theta = \frac{\Omega}{\sqrt{\Omega^2 + \Delta^2}}$$

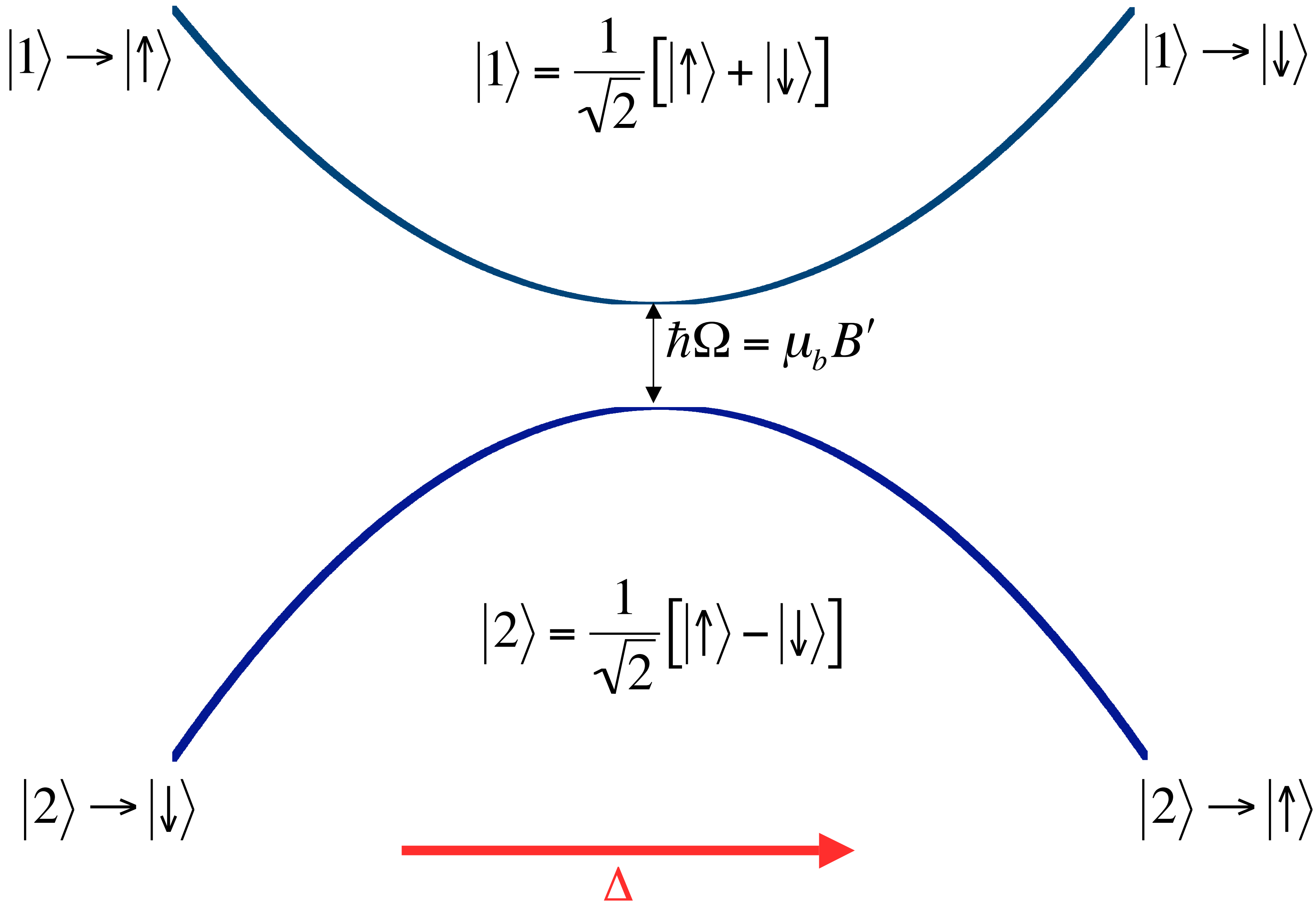
$$\Delta \to -\infty \qquad \cos\theta = 1 \quad \sin\theta = 0 \qquad |1\rangle \to |e\rangle \qquad |2\rangle \to |g\rangle$$

$$\Delta \to +\infty \qquad \cos\theta = 0 \quad \sin\theta = 1 \qquad |1\rangle \to |g\rangle \qquad |2\rangle \to |e\rangle$$

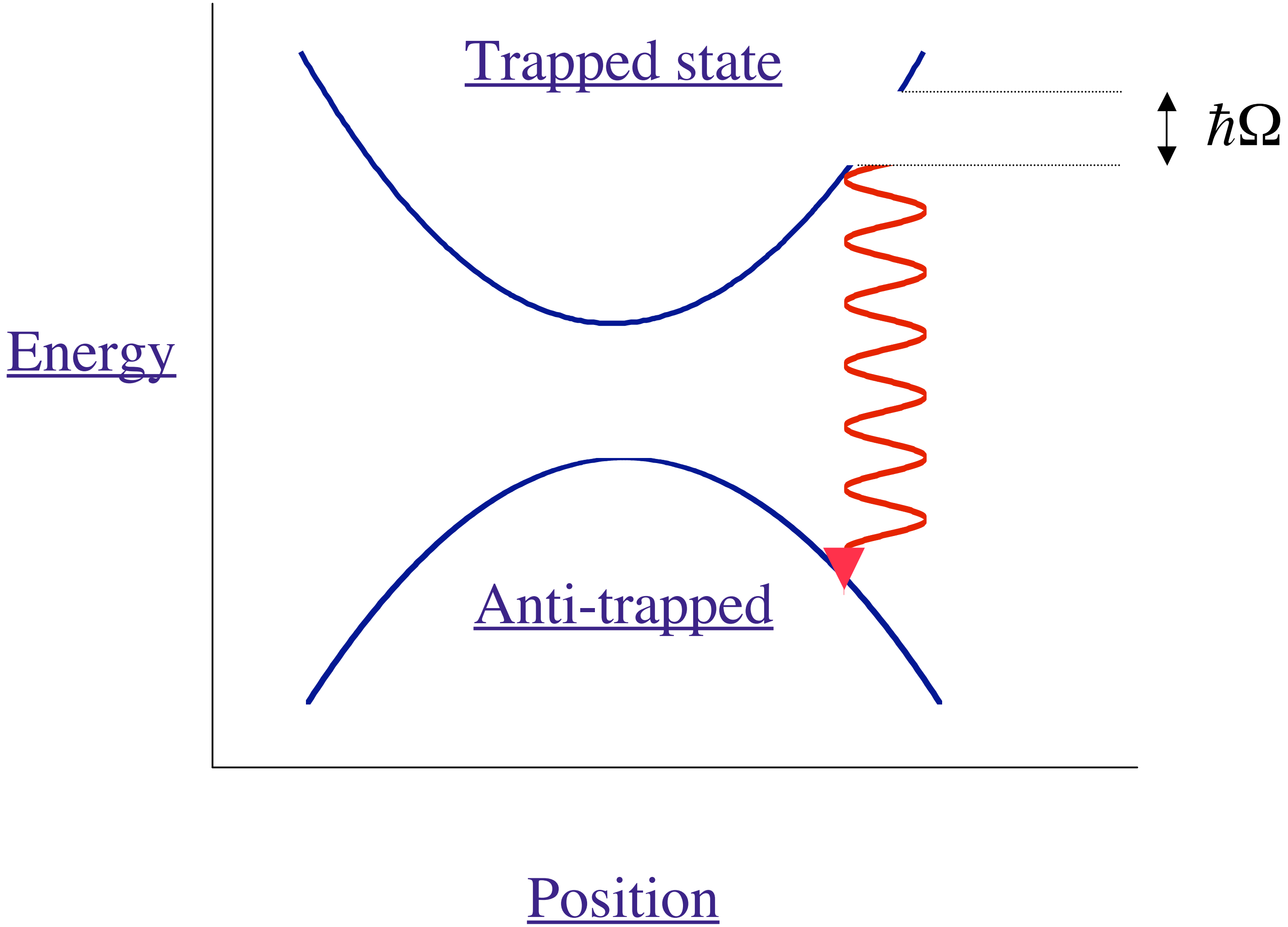
Dressed States and Avoided Crossings



Dressed States and Avoided Crossings

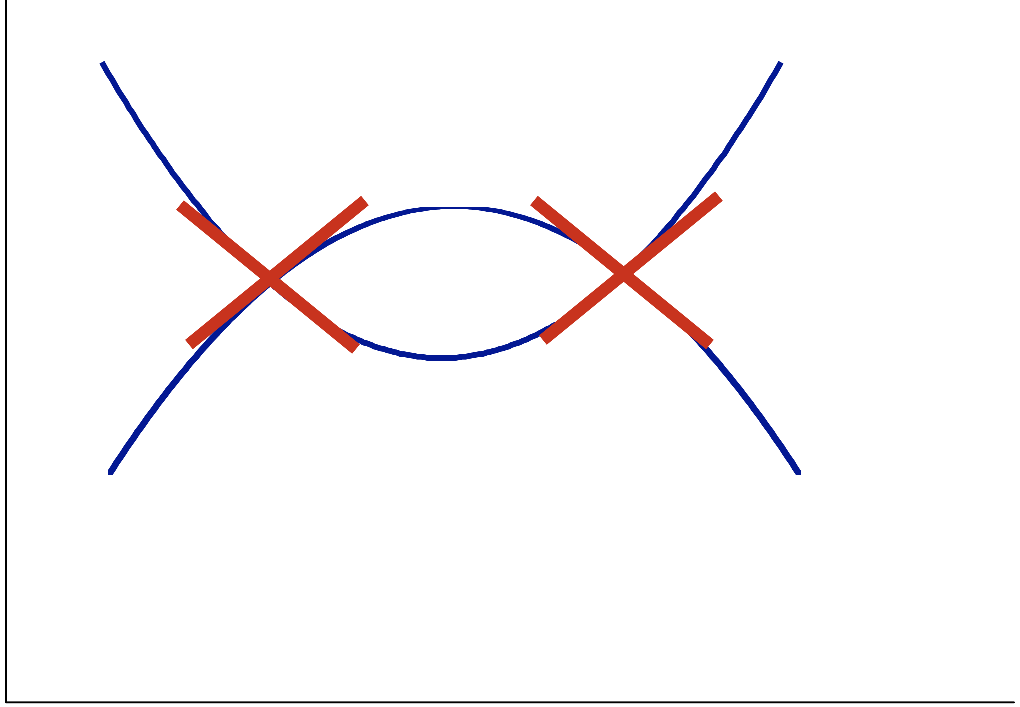


Schroedinger Picture



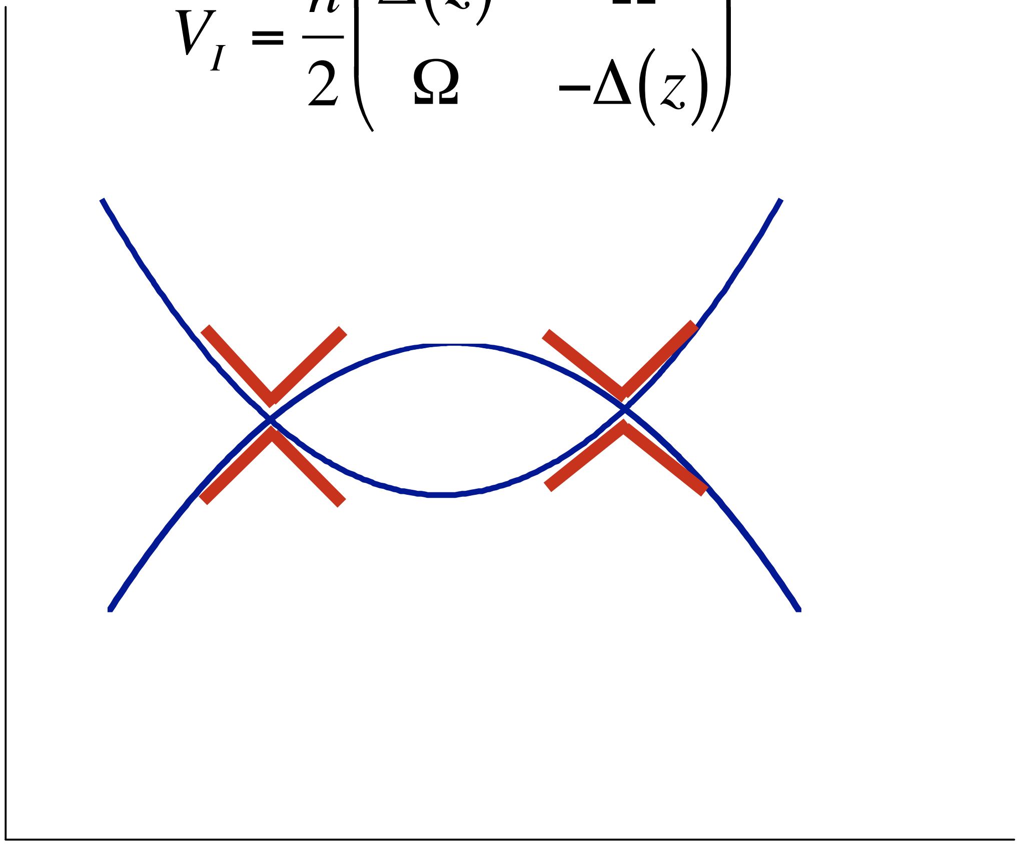
Interaction Picture

$$V_I = \frac{\hbar}{2}\begin{pmatrix} \Delta(z) & 0 \\ 0 & -\Delta(z) \end{pmatrix}$$



Interaction Picture

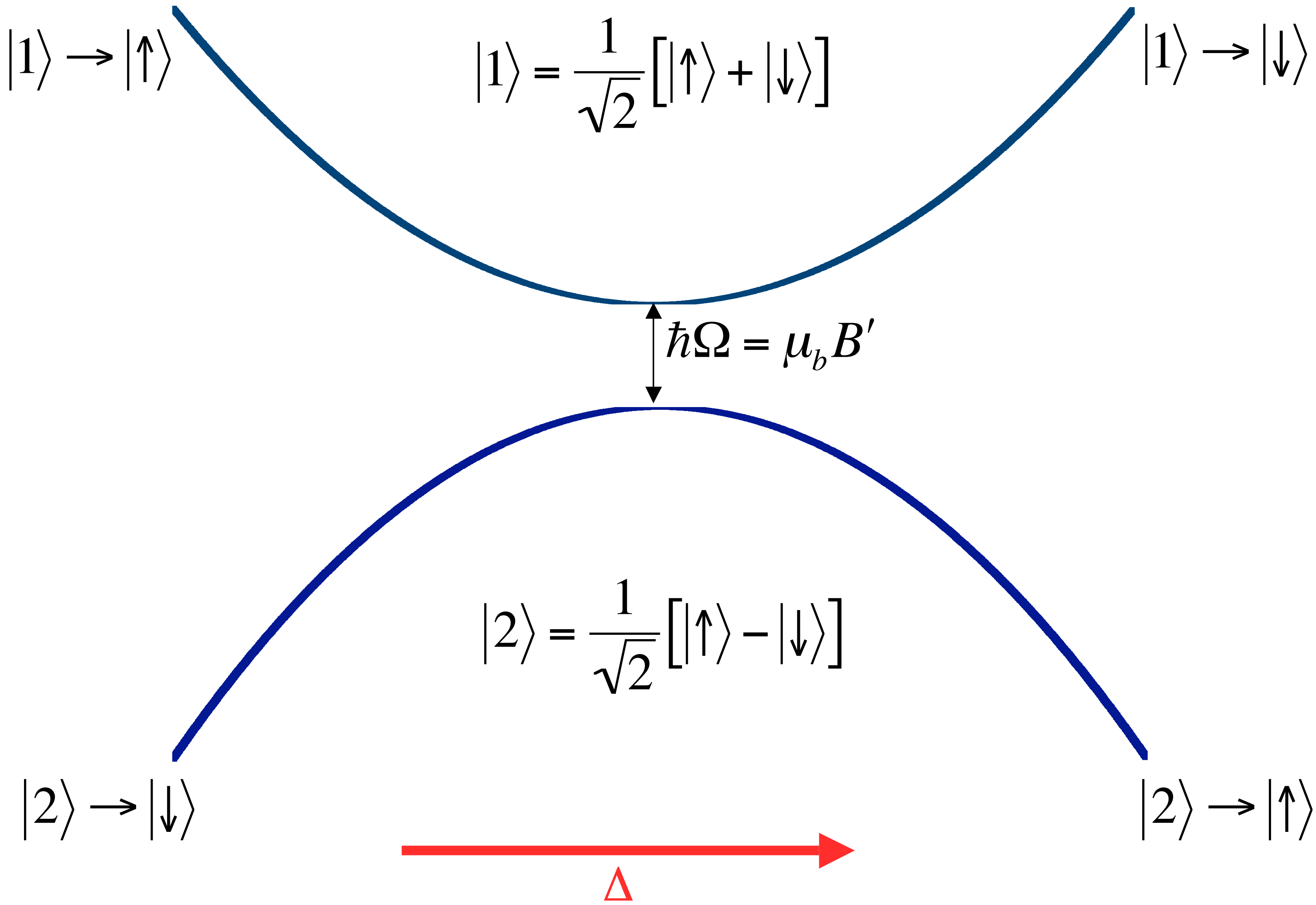
$$V_I = \frac{\hbar}{2}\begin{pmatrix} \Delta(z) & \Omega \\ \Omega & -\Delta(z) \end{pmatrix}$$



Energy

Position (and detuning)

Dressed States and Avoided Crossings



Landau Zener and Adiabatic Following

$$V_I = \frac{\hbar}{2}\begin{pmatrix} \Delta(t) & \Omega \\ \Omega & -\Delta(t) \end{pmatrix}$$

$$|\psi(t)\rangle = C_1(t)|1\rangle + C_2(t)|2\rangle$$

Probablity of a non-adiabatic transition

$$P = \exp(-2\pi\Gamma)$$

$$\Gamma = \frac{\Omega^2}{(d\Delta/dt)_{t_c}}$$

Light Potentials

$$|1\rangle = \cos\theta|e\rangle + \sin\theta|g\rangle \qquad E = \frac{\hbar}{2}\sqrt{\Omega^2 + \Delta^2}$$

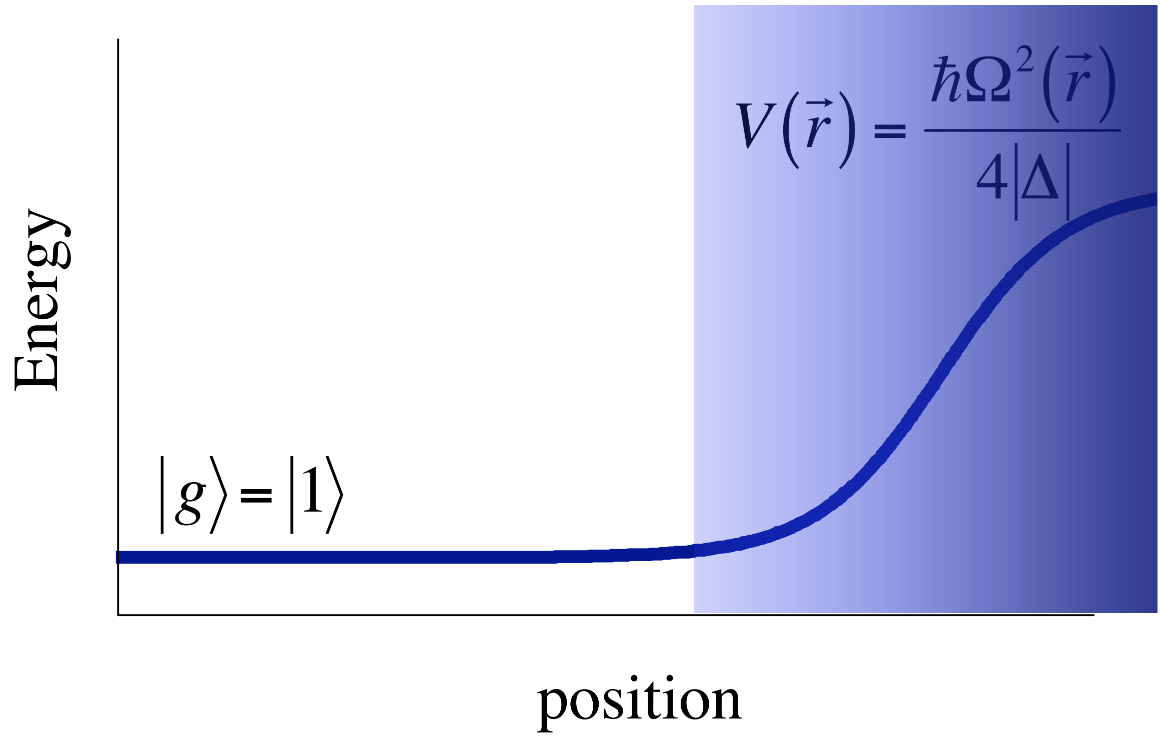
$$|2\rangle = \sin\theta|e\rangle - \cos\theta|g\rangle \qquad E = -\frac{\hbar}{2}\sqrt{\Omega^2 + \Delta^2}$$

$$\cos 2\theta = -\frac{\Delta}{\sqrt{\Omega^2 + \Delta^2}} \qquad \sin 2\theta = \frac{\Omega}{\sqrt{\Omega^2 + \Delta^2}}$$

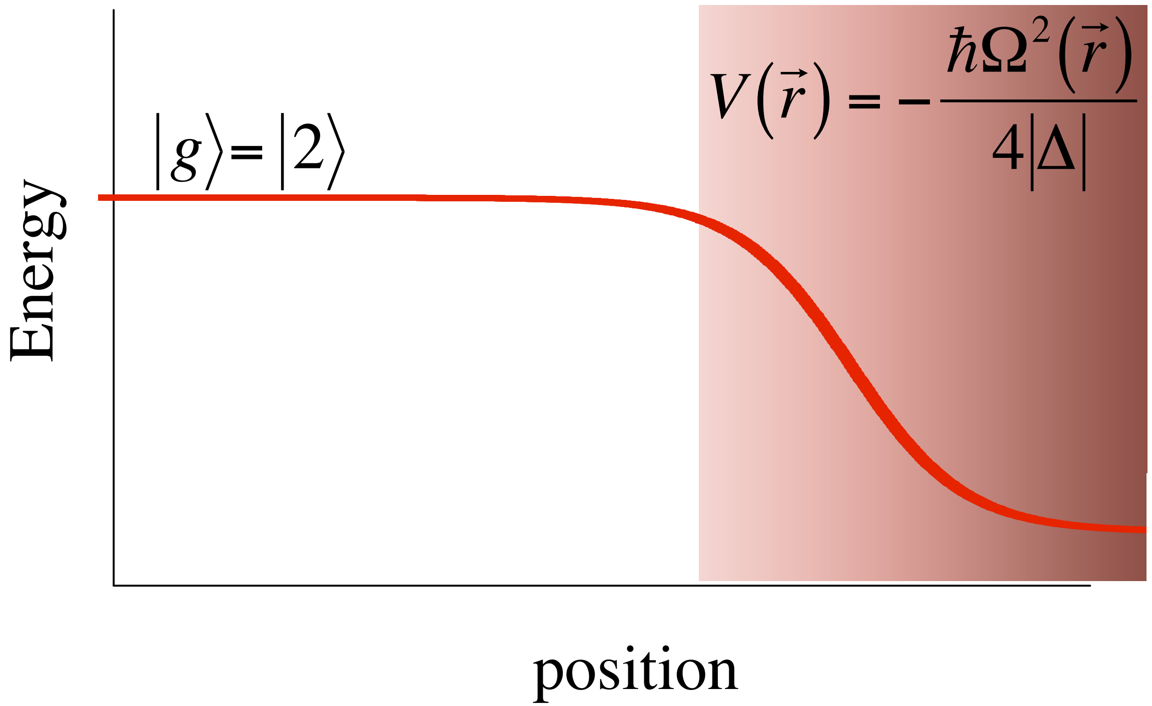
$$\Delta > 0 \quad \Omega \to 0 \qquad \cos\theta = 0 \quad \sin\theta = 1 \qquad |1\rangle \to |g\rangle$$

$$E \approx \frac{\hbar}{2}|\Delta|^2\left[1 + \frac{1}{2}\left(\frac{\Omega}{\Delta}\right)^2\right] \qquad V(\vec{r}) = \frac{\hbar\Omega^2(\vec{r})}{4\Delta}$$

Light Potentials (blue detuning)



Light Potentials (red detuning)



Coherence and the Density Operator

$$|\Psi\rangle = C_e|e\rangle + C_g e^{i\delta}|g\rangle$$

$$\langle\Psi|A|\Psi\rangle = |C_e|^2\langle e|A|e\rangle + |C_g|^2\langle g|A|g\rangle + \underbrace{2\mathrm{Re}\left[C_e C_g\langle g|A|e\rangle\right]\cos\delta - 2\mathrm{Im}\left[C_e C_g\langle g|A|e\rangle\right]\sin\delta}_{Interference\ terms}$$

Consider a mixed state where the probability of finding an atom in state $|\xi_i\rangle$ is P_i

$$\langle A\rangle = \sum_i P_i\langle\xi_i|A|\xi_i\rangle = \sum_j\sum_i P_i\langle\xi_i|A|\phi_j\rangle\langle\phi_j|\xi_i\rangle = \sum_j\langle\phi_j|\underbrace{\left(\sum_i P_i|\xi_i\rangle\langle\xi_i|\right)}_{\hat{\rho}}|\phi_j\rangle$$

Density operator → $\hat{\rho}$

Density Operator Dynamics

$$\frac{\partial \hat{\rho}}{\partial t} = \frac{\partial}{\partial t} \sum_i P_i |\xi_i\rangle\langle\xi_i|$$

$$\frac{\partial \hat{\rho}}{\partial t} = \frac{-i}{\hbar}\left[H, \hat{\rho}\right]$$

$$\langle A \rangle = Tr\left(\hat{\rho} A\right)$$

For an arbitrary pure state of our two level atom

$$|\Psi\rangle = C_e|e\rangle + C_g|g\rangle$$

$$\hat{\rho} = |\psi\rangle\langle\psi| = |C_e|^2|e\rangle\langle e| + C_e C_g^*|e\rangle\langle g| + C_g C_e^*|g\rangle\langle e| + |C_g|^2|g\rangle\langle g|$$

Calculating time evolution using the equation on the previous page
And taking matrix elements:

$$\dot{\rho}_{ee} = -\gamma\rho_{ee} + \frac{i}{2}\left(\Omega^*\rho_{eg} - \Omega\rho_{ge}\right)$$

$$\dot{\rho}_{gg} = \gamma\rho_{ee} + \frac{i}{2}\left(\Omega\rho_{ge} - \Omega^*\rho_{eg}\right)$$

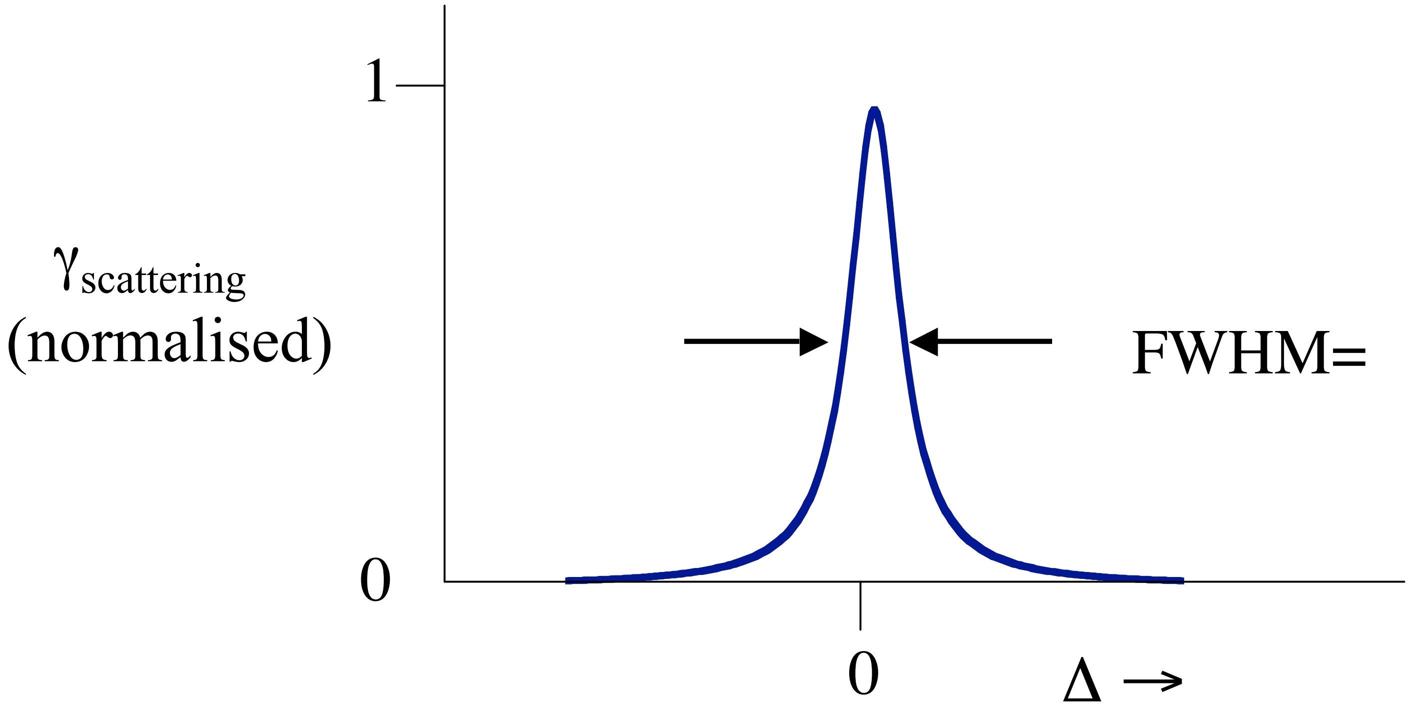
$$\dot{\rho}_{eg} = -\frac{\gamma}{2}\rho_{eg} - i\Delta\rho_{eg} + \frac{i\Omega}{2}\left(\rho_{ee} - \rho_{gg}\right)$$

$$\dot{\rho}_{ge} = -\frac{\gamma}{2}\rho_{ge} + i\Delta\rho_{ge} + \frac{i\Omega^*}{2}\left(\rho_{gg} - \rho_{ee}\right)$$

Steady State Solutions: $\dot{\rho} = 0$

$$scattering\ rate = \gamma\rho_{ee} = \left(\frac{s_0}{1+s_0}\right)\frac{\gamma/2}{1+\left(2\Delta/\gamma'\right)^2} \qquad \gamma' = \gamma\sqrt{1+s_0}$$

$$s_0 = \frac{\Omega^2}{\gamma^2+\Delta^2}$$



We can manipulate cold atoms with light potentials by detuning the lasers far off resonance and raising the intensity. As the potential scales as I/Δ and the scattering rate scales as I/Δ^2, an arbitrarily deep (or high) potential can be achieved with an arbitrarily low scattering rate. This is the basis of optical traps optical spoons etc.