Excercises

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Warm Up Exercises

i) The selection rules on total angular momentum for an electric dipole transition are $\Delta F = 0, \pm 1$. If $F$ is the total angular momentum of the atom and if photons carry angular momentum $\hbar$, how is $\Delta F = 0$ possible?

ii) What is the selection rule on $\Delta L$? Why?

iii) In an applied magnetic field, the $M_F = 2$ state has the highest energy in the $F=2$ manifold. In the $F=1$ manifold, the $M_F = -1$ state has the highest energy. Why are the two manifolds different?
Calculate the scattering cross section from a driven dipole

\[ m \ddot{x} + \alpha \dot{x} + \frac{1}{2} m \omega^2 x^2 = E_0 e^{i \omega t} \]

Solve the equation of motion above and calculate the power dissipated by the dipole in terms of \( \alpha \). Set this equal to the power radiated by the dipole to determine \( \alpha \). Then use:

\[ \sigma = \frac{P_{\text{radiated}}}{I_{\text{incident}}} = \frac{2}{c \varepsilon_0 E_0^2} P_{\text{radiated}} = \frac{3 \lambda^2}{2\pi} \approx 10^{-13} m^2 \]
i) A cloud of $^{87}\text{Rb}$ atoms 5 mm in size is released from a MOT and expands freely. What is the timescale governing expansion?

ii) In one expansion time, you wish to optically pump the atoms to the $M_F=2$ state. What laser power should you use? Make reasonable assumptions.
Determine the minimum bias field needed to avoid spin flips in a trap of frequency $\omega$ using the same formalism we used for adiabatic following at an avoided crossing. Compare with the classical formula we developed in lecture. $\left(V \cdot \nabla\right)B \ll \gamma B^2$
i) Develop a model to determine the optimum RF power to use in RF evaporation as a function of temperature and trap parameters. Model evaporation using your knowledge of a two level atom in an RF field (do not use adiabatic following at an avoided crossing to answer this part of the question.)
Dressed States and Avoided Crossings

\[ |1\rangle = \frac{1}{\sqrt{2}} [|\uparrow\rangle + |\downarrow\rangle] \]

\[ |2\rangle = \frac{1}{\sqrt{2}} [|\uparrow\rangle - |\downarrow\rangle] \]

\[ \hbar \Omega = \mu_b B' \]

i) Repeat the previous problem above using adiabatic following at an avoided crossing and compare with your previous result.

ii) Use the adiabatic theorem to make an estimate of the rate that the frequency of the cut should be reduced to drive evaporation.
Model an atom in a light field as a driven harmonic oscillator and determine the light potential. Compare with the result above that we discussed in class.

\[ V(\vec{r}) = \frac{\hbar \Omega^2(\vec{r})}{4|\Delta|} \]