From EPR objections (intuitions) to quantum information: amazing entanglement

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A review on Bell's inequalities: AA quant-ph/0402001

Entanglement and the new quantum revolution*

The first quantum revolution (1900-)

- Conceptual (wave particle duality): understanding matter, radiation
- Technological: transistor (IC), laser: information based society

A new quantum revolution (1964-)

- Quantum mechanics applied to single objets:
  - Microscopic: experiments on single electron, ion, photon, atom: quantum jumps, quantum Monte-Carlo
  - Macro(meso)scopic ? Josephson junctions, BEC
- Entanglement recognized as a different extraordinary quantum property (as advocated by Einstein): Bell's inequalities
- A new technological revolution??? (quantum information)

* AA, « John Bell and the second quantum revolution »: Introduction to 2nd edition of « Speakable and Unspeakable in Quantum Mechanics », John Bell, Cambridge University Press (2004)

From EPR objections (intuitions) to quantum information: amazing entanglement

Part 1: EPR paper (1935), Bell's theorem (1965), experimental tests (1972-): entanglement as a conceptual question (quantum non locality)

Part 2: Quantum cryptography, entanglement on demand, quantum computing: entanglement as a resource for quantum information

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From EPR to tests of Bell's inequalities: entanglement as a conceptual question

The point of view of a naive experimentalist*

- Einstein-Podolsky-Rosen correlations
  The Einstein-Bohr debate (1935-1955)
- Bell's theorem (1965)
  From epistemology back to physics
- Experimental tests: a brief review (1972-2002)
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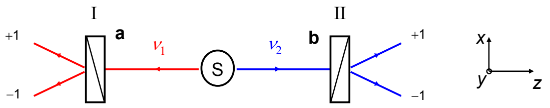
EPR original question: Can quantum mechanics be considered complete?

Is it possible (necessary?) to explain the probabilistic character of Quantum Mechanics predictions with underlying supplementary parameters (hidden variables)?

It is suggested by the Einstein-Podolsky-Rosen argument, but denied by Bohr (1935)

Bell's theorem allows one to give an experimental answer.

Einstein-Podolsky-Rosen GedankenExperiment with photons correlated in polarization

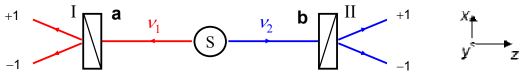


Measurements of linear polarization of ν_1 along **a** and of linear polarization of ν_2 along **b** : results +1 or –1

⇒ Probabilities of detection in channels +1 or –1 of polarizer I and in channels +1 or –1 of polarizer II (in orientations **a** and **b**).

EPR situation : entangled state $|\Psi(\nu_1,\nu_2)\rangle = \frac{1}{\sqrt{2}}\{|x,x\rangle + |y,y\rangle\}$

Einstein-Podolsky-Rosen GedankenExperiment with photons correlated in polarization



•Photons in the entangled state: $\left|\Psi(\nu_1,\nu_2)\right\rangle = \frac{1}{\sqrt{2}}\left\{\left|x,x\right\rangle + \left|y,y\right\rangle\right\}$

•Quantum Mechanics predictions:

$P_+(\mathbf{a}) = P_-(\mathbf{a}) = \frac{1}{2}$; $P_+(\mathbf{b}) = P_-(\mathbf{b}) = \frac{1}{2}$ Single results random

$$P_{++}(\mathbf{a},\mathbf{b}) = P_{--}(\mathbf{a},\mathbf{b}) = \frac{1}{2}\cos^2(\mathbf{a},\mathbf{b})$$

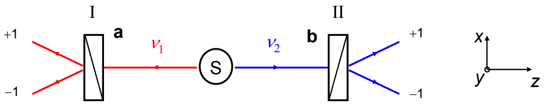
$$P_{+-}(\mathbf{a},\mathbf{b}) = P_{-+}(\mathbf{a},\mathbf{b}) = \frac{1}{2}\sin^2(\mathbf{a},\mathbf{b})$$

Strong correlations

$$(\mathbf{a},\mathbf{b}) = 0 \Rightarrow P_{++} = P_{--} = \frac{1}{2}$$

$$P_{+-} = P_{-+} = 0$$

Einstein-Podolsky-Rosen GedankenExperiment with photons correlated in polarization



•Photons in the EPR entangled state: $\left|\Psi(\nu_1,\nu_2)\right\rangle = \frac{1}{\sqrt{2}}\left\{\left|x,x\right\rangle + \left|y,y\right\rangle\right\}$

• Quantum Mechanics predicts a strong polarization correlation

Correlation coefficient $E(\mathbf{a},\mathbf{b}) = P_{++} + P_{--} - P_{+-} - P_{-+}$

$$E_{\text{MQ}}(\mathbf{a},\mathbf{b}) = \cos 2(\mathbf{a},\mathbf{b}) \quad \Rightarrow \quad E_{\text{MQ}}(0) = 1 \; ; \; E_{\text{MQ}}(90) = -1$$

How to understand the EPR correlations? How to make an image?

• Derive it from the calculation algorithm?

Global (straightforward) calculation:

$$P_{++}(\mathbf{a},\mathbf{b}) = \left|\left\langle +_{\mathbf{a}}, +_{\mathbf{b}} \middle| \Psi(\nu_1,\nu_2)\right\rangle\right|^2$$

Hard to make a picture in real space:

- $\left|\Psi(\nu_1,\nu_2)\right\rangle$ is a global 2-particles wave vector
- calculation done in an abstract space, without direct correspondence in real space

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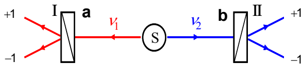
How to understand the EPR correlations? How to make an image?



2-steps calculation (standard QM)

• 1rst step: measure at polarizer I

⇒ result +1 (pol. along a) ⇒ projection of the state vector

or $\Rightarrow P_{+_{\mathbf{a}}}\left|\Psi(\nu_1,\nu_2)\right\rangle = \left|+_{\mathbf{a}},+_{\mathbf{a}}\right\rangle$ Photons polarized along **a**

⇒ result −1 (pol. perp to **a**)

• 2nd step: measure at polarizer II $\Rightarrow P_{++}(\mathbf{a},\mathbf{b}) = \frac{1}{2}\cos^2(\mathbf{a},\mathbf{b})$

⇒ result -1 (pol. perp. to **a**) ⇒ projection of the state vector

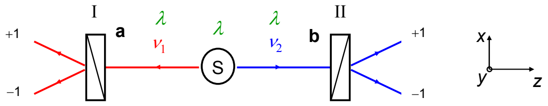
$\Rightarrow P_{-_{\mathbf{a}}}\left|\Psi(\nu_1,\nu_2)\right\rangle = \left|-_{\mathbf{a}},-_{\mathbf{a}}\right\rangle$ Photons polarized perp to **a**

What a picture! Polarization of ν_2 instantaneously affected by the result of measurement on ν_1 … which is far away.

Can't we try a less bizarre image?

Classical explanation for correlations between distant results of measurements

- Common property λ of both particles of the same pair
- λ randomly determined in S at the emission time



Simple image, but…

completes the formalism of quantum mechanics:
supplementary parameters λ (« hidden variables »)

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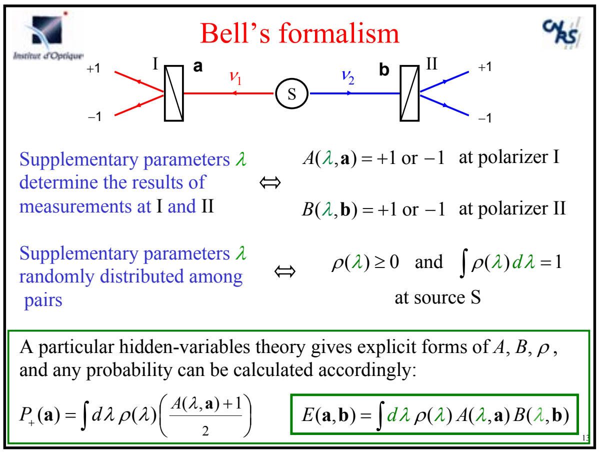


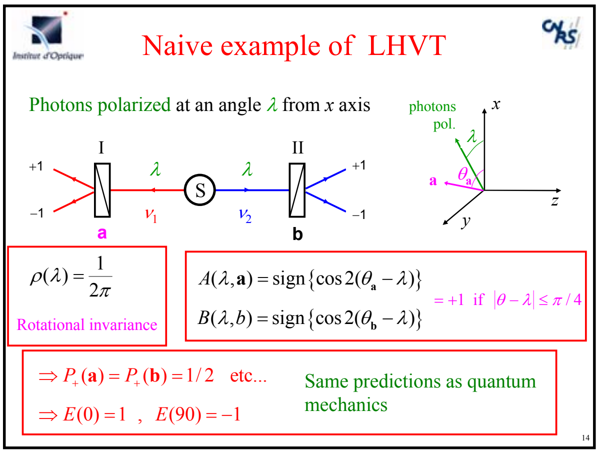
Bell's formalism

Supplementary parameters λ determine the results of measurements at I and II $\Leftrightarrow$ $A(\lambda,\mathbf{a}) = +1 \text{ or } -1$ at polarizer I; $B(\lambda,\mathbf{b}) = +1 \text{ or } -1$ at polarizer II

Supplementary parameters λ randomly distributed among pairs $\Leftrightarrow$ $\rho(\lambda) \geq 0$ and $\int \rho(\lambda)\, d\lambda = 1$ at source S

A particular hidden-variables theory gives explicit forms of A, B, ρ, and any probability can be calculated accordingly:

$$P_+(\mathbf{a}) = \int d\lambda\, \rho(\lambda) \left(\frac{A(\lambda,\mathbf{a})+1}{2} \right) \qquad \boxed{E(\mathbf{a},\mathbf{b}) = \int d\lambda\, \rho(\lambda)\, A(\lambda,\mathbf{a})\, B(\lambda,\mathbf{b})}$$



Naive example of LHVT

Photons polarized at an angle λ from x axis

$$\rho(\lambda) = \frac{1}{2\pi}$$

Rotational invariance

$$A(\lambda,\mathbf{a}) = \operatorname{sign}\{\cos 2(\theta_{\mathbf{a}} - \lambda)\}$$

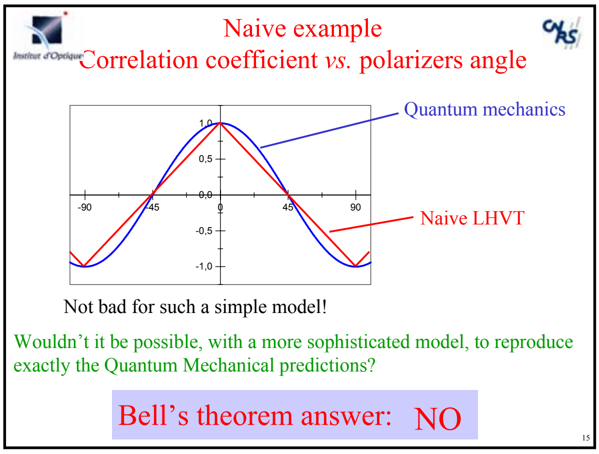
$$B(\lambda,b) = \operatorname{sign}\{\cos 2(\theta_{\mathbf{b}} - \lambda)\}$$

$= +1$ if $|\theta - \lambda| \leq \pi/4$

$\Rightarrow P_+(\mathbf{a}) = P_+(\mathbf{b}) = 1/2$ etc...

$\Rightarrow E(0) = 1 \;,\; E(90) = -1$

Same predictions as quantum mechanics

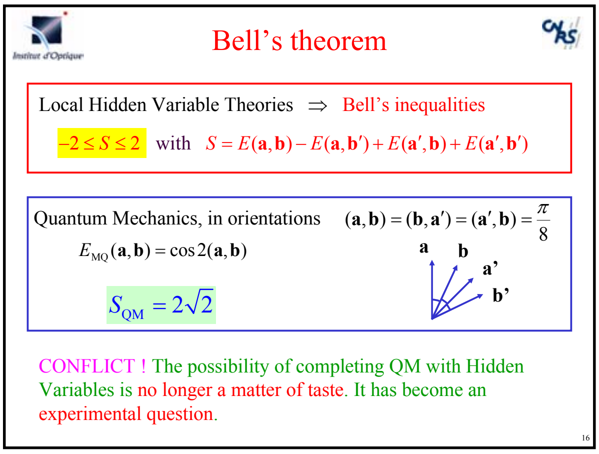


Naive example
Correlation coefficient *vs.* polarizers angle

Not bad for such a simple model!

Wouldn't it be possible, with a more sophisticated model, to reproduce exactly the Quantum Mechanical predictions?

Bell's theorem answer: NO



Bell's theorem

Local Hidden Variable Theories $\Rightarrow$ Bell's inequalities

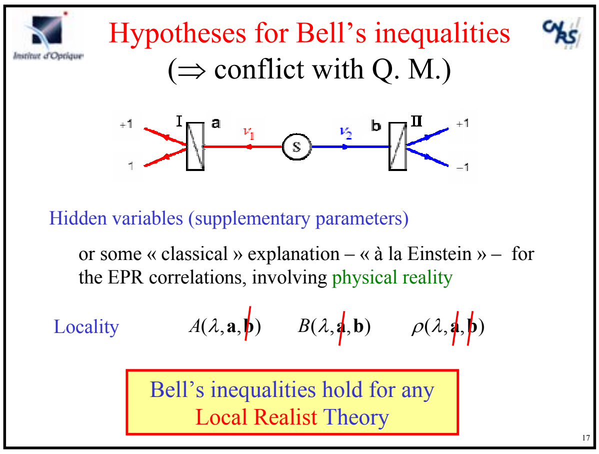
$-2 \leq S \leq 2$ with $S = E(\mathbf{a},\mathbf{b}) - E(\mathbf{a},\mathbf{b}') + E(\mathbf{a}',\mathbf{b}) + E(\mathbf{a}',\mathbf{b}')$

Quantum Mechanics, in orientations $(\mathbf{a},\mathbf{b}) = (\mathbf{b},\mathbf{a}') = (\mathbf{a}',\mathbf{b}) = \dfrac{\pi}{8}$

$$E_{\text{MQ}}(\mathbf{a},\mathbf{b}) = \cos 2(\mathbf{a},\mathbf{b})$$

$$S_{\text{QM}} = 2\sqrt{2}$$

CONFLICT ! The possibility of completing QM with Hidden Variables is no longer a matter of taste. It has become an experimental question.



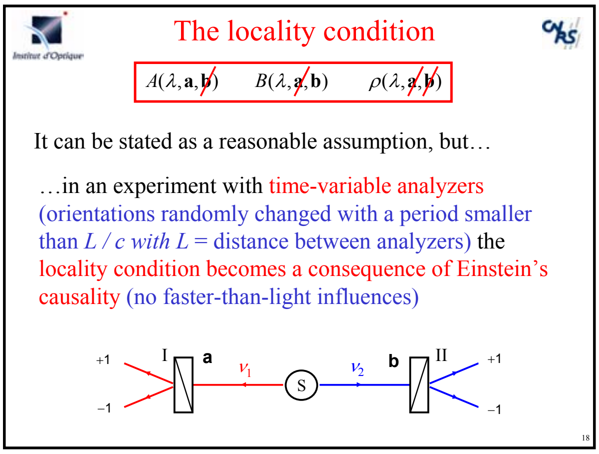
Hypotheses for Bell's inequalities
($\Rightarrow$ conflict with Q. M.)

Hidden variables (supplementary parameters)

or some « classical » explanation – « à la Einstein » – for the EPR correlations, involving physical reality

Locality $\quad A(\lambda,\mathbf{a},\cancel{\mathbf{b}}) \quad B(\lambda,\cancel{\mathbf{a}},\mathbf{b}) \quad \rho(\lambda,\cancel{\mathbf{a}},\cancel{\mathbf{b}})$

Bell's inequalities hold for any Local Realist Theory



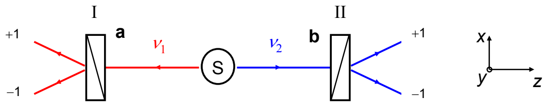
The locality condition

$A(\lambda,\mathbf{a},\cancel{\mathbf{b}}) \quad B(\lambda,\cancel{\mathbf{a}},\mathbf{b}) \quad \rho(\lambda,\cancel{\mathbf{a}},\cancel{\mathbf{b}})$

It can be stated as a reasonable assumption, but…

…in an experiment with time-variable analyzers (orientations randomly changed with a period smaller than L/c with L = distance between analyzers) the locality condition becomes a consequence of Einstein's causality (no faster-than-light influences)

Einstein-Podolsky-Rosen GedankenExperiment with variable polarizers



•Photons in the EPR entangled state: $$|\Psi(\nu_1,\nu_2)\rangle = \frac{1}{\sqrt{2}}\{|x,x\rangle + |y,y\rangle\}$$

Quantum Mechanics

$$E_{\mathrm{MQ}}(\mathbf{a},\mathbf{b}) = \cos 2(\mathbf{a},\mathbf{b})$$

← CONFLICT →

Bell's inequalities
(Einstein's local realism)

$$-2 \le S \le 2$$

$$S = E(\mathbf{a},\mathbf{b}) - E(\mathbf{a},\mathbf{b}') + E(\mathbf{a}',\mathbf{b}) + E(\mathbf{a}',\mathbf{b}')$$

Bell's theorem

Some predictions of Quantum Mechanics (in EPR situations) can not be mimicked by a « reasonable classical-like model » in the spirit of Einstein's ideas.

What about nature ?

When Bell's theorem appeared, there was no experimental result available for testing Bell's inequalities vs. Quantum Mechanics.

Couldn't it be that the violation of Bell's inequalities indicates a limit of the validity of Quantum Mechanics ?

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First experiments with EPR pairs

Experiments with γ photons (0.5 MeV) produced in positronium desintegration

Experiments with protons (proton scattering on a target)

Agreement with QM, but not a test of Bell's inequalities

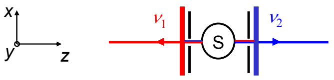
There are no polarizers (apparatus with 2 outcome).The polarization is inferred from a Compton scattering, by use of a QM calculation.

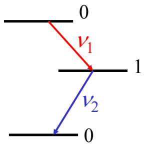
Visible photons EPR pairs produced in some atomic radiative cascades

Polarizers do exist for visible photons

EPR pairs produced in radiative cascades (Clauser, Horne, Shimony, Holt, 1969)

$$J = 0 \rightarrow J = 1 \rightarrow J = 0$$





$$|\Psi(\nu_1,\nu_2)\rangle = \frac{1}{\sqrt{2}}\{|x,x\rangle + |y,y\rangle\}$$

Any computable extra effect (finite solid angle, hyperfine structure…) leads to a decrease or even a cancellation of the conflict between Bell's inequalities and Quantum Mechanics.

The experiment must be as ideal as possible

First experiments with visible photons produced in atomic radiative cascades

1^{rst} generation

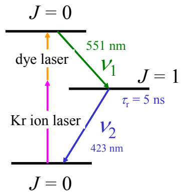
- Clauser & Freedman (Berkeley, 1972)
  ^{40}Ca 200 hours M. Q.
- Holt & Pipkin (Harvard, 1973)
  ^{200}Hg 200 hours B. I.

2^{nd} generation (laser excitation ***)

- Fry & Thompson (Texas A&M, 1976)
  ^{200}Hg 80 mn M. Q.

In these experiments, single channel polarizers : indirect reasoning, auxiliary calibrations required.

Orsay source of entangled photons



$4p^2\,{}^1S_0 - 4s4p\,{}^1P_1 - 4p^2\,{}^1S_0$

radiative cascade in calcium 40

Already used in Berkeley experiment

New : 2 photon laser excitation

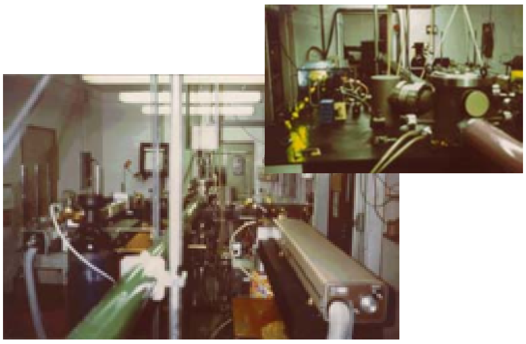
⇒ Selective excitation (isotope, level)

⇒ Small source : 0.5 x 0.05 mm^2

⇒ Optimum cascade rate (4 x 10^7 s^{-1}) easily achieved

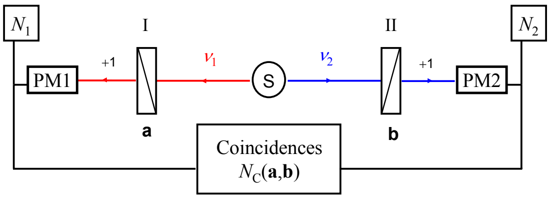
1% accuracy on coincidence rate in 100 s

The Orsay source of entangled photons



Experiment with 1- channel polarizers

AA, P. Grangier, G. Roger, PRL 1981

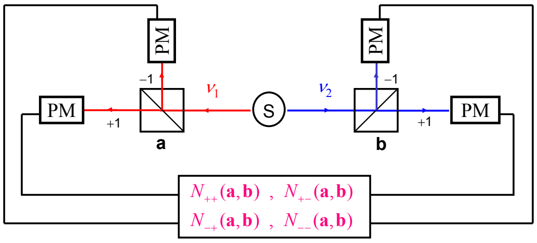


High grade pile of plates polarizers, but only one channel (+1)

- Excellent agreement with QM.
- Violation of Bell's inequalities by 9 σ
- No change in the results with polarizers at a distance (6 m) larger than the coherence length of ν_2 (1.5 m)

Experiment with 2- channels polarizers

AA, P. Grangier, G. Roger, PRL 1982

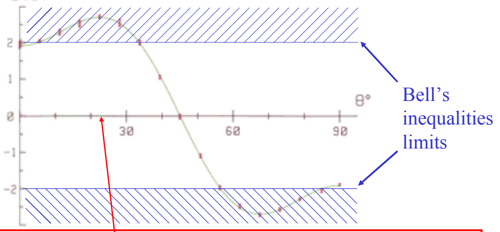


$N_{++}(\mathbf{a},\mathbf{b})$, $N_{+-}(\mathbf{a},\mathbf{b})$
$N_{-+}(\mathbf{a},\mathbf{b})$, $N_{--}(\mathbf{a},\mathbf{b})$

Fourfold coincidence system: the 4 coincidence rates are measured during the same run ⇒ coefficient of correlation

$$E(\mathbf{a},\mathbf{b}) = \frac{N_{++}(\mathbf{a},\mathbf{b}) - N_{+-}(\mathbf{a},\mathbf{b}) - N_{-+}(\mathbf{a},\mathbf{b}) + N_{--}(\mathbf{a},\mathbf{b})}{N_{++}(\mathbf{a},\mathbf{b}) + N_{+-}(\mathbf{a},\mathbf{b}) + N_{-+}(\mathbf{a},\mathbf{b}) + N_{--}(\mathbf{a},\mathbf{b})}$$

Experiment with 2- channels polarizers



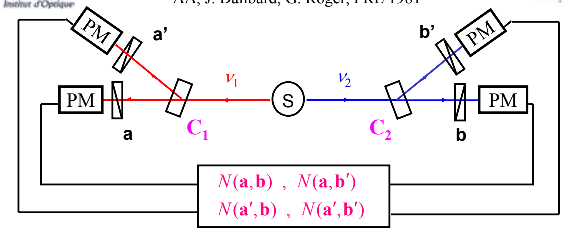
For $\theta = (\mathbf{a},\mathbf{b}) = (\mathbf{b},\mathbf{a}') = (\mathbf{a}',\mathbf{b}) = \frac{\pi}{8}$ $S_{\text{exp}}(\theta) = 2.697 \pm 0.015$

Violates Bell's inequality (S ≤ 2) by > 40 σ

No auxiliary calibration necessary.
Excellent agreement with Q.M. $S_{\text{QM}} = 2.70$

Experiment with optical switches

AA, J. Dalibard, G. Roger, PRL 1981



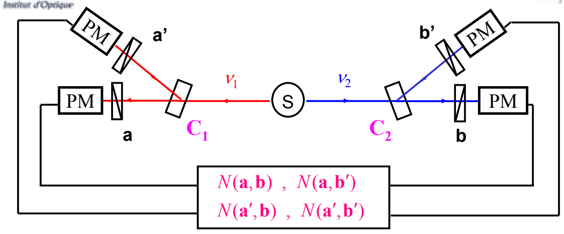
$N(\mathbf{a},\mathbf{b})$, $N(\mathbf{a},\mathbf{b}')$
$N(\mathbf{a}',\mathbf{b})$, $N(\mathbf{a}',\mathbf{b}')$

Each switch redirects the photon towards one of two polarizers in different orientations: equivalent to a single polarizer rapidly rotated from an orientation to the other one.

Switching period: 10 ns << $C_1C_2 / c = 40$ ns

Spacelike separated events

Experiment with optical switches



In the 1982 Orsay experiment, each switch C_1 and C_2 worked in a quasi-periodic way, not truly random.

But the two switches were driven by two different generators, drifting independently.

Experiment with optical switches: results

Reduced signal (limited aperture of the switches)

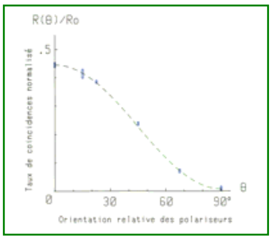
⇒ Averaging necessary (15 hours)

Violation of the relevant Bell's inequality $\delta \leq 0$

$\delta_{\mathrm{exp}} = 0.064 \pm 0.01$

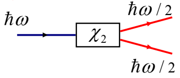
Good agreement with QM :

$\delta_{\mathrm{QM}} = 0.059$



Towards the ideal experiment

- Perrie et al. (1985): pair of UV photons (metastable deuterium desexcitation)
- 4th generation: entangled photons by parametric splitting



Alley, Mandel, Rarity, Martiensen, Kimble, Gisin, Zeilinger (super source by Kwiat, Weinfurter et al.)

- Perfect correlation: violation of BI by 100 σ (Innsbruck 1998)
- Other observables: time / energy; position /momentum
- Use of optical fibers
  - ⇒ large distances (Malvern, Geneva)
  - ⇒ experiments with active random polarizers (Innsbruck 1998)

    Strong enforcement of the locality condition
- Implemented in laboratory classes at Institut d'Optique: $S_{\mathrm{typ}} > 2.5$

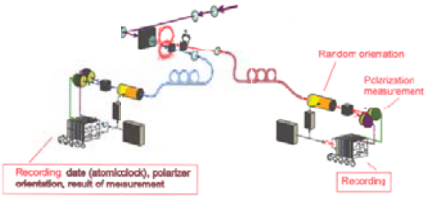
Geneva experiment

Use of optical fibers of the commercial telecom network

Non locality at more than 10 km…

Innsbruck experiment

Experiment with randomly reoriented polarizers



Correlation between distant (400 m) measurements determined a posteriori

Strong violation of Bell's inequalities, agreement with QM

Towards the ideal experiment

- 4th generation *bis*: massive particles pairs
  - Rydberg atoms and RF photons (ENS Paris 2000)
  - Trapped ions (Boulder, 2000)

    ⇒ experiments with 100% detection efficiency

    closure of the "detection loophole"
- 4th generation *ter*: continuous variables
  - Quadratures of macroscopic light beams (Caltech, Orsay, Canberra)

    ⇒ 99 % detection efficiency; sophisticated schemes

    ⇒ locality condition « easy » to enforce
- Ultimate experiment (200?)

  detection loophole closed and locality enforced

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Violation of Bell's inequalities: what can we conclude? (conceptual issue)

- Failure of local realism à la Einstein: quantum non locality
- Accept negative probabilities (???)
- Is it a real problem ?

Failure of Einstein's local realism

If the EPR conclusion was not correct then one should…

> « - either drop the need of the independence of the physical realities present in different parts of space
>
> - or accept that the measurement of S_1 changes (instantaneously) the real situation of S_2 »
>
> A. Einstein

The violation of Bell's inequalities show that we must renounce

Einstein's local realism:

- independence of physical realities of separated (in a relativistic sense) subsystems;
- no faster than light connection

Not really independent hypotheses

Quantum non locality

Quantum non locality

We must abandon Einstein's local realism:

- independence of physical realities of separated (in a relativistic sense) subsystems;
- no faster than light connection

Do NOT conclude that one can use entanglement to send faster than light signals

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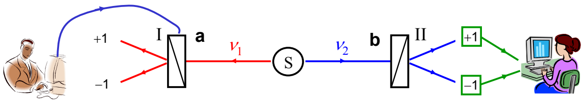
No faster than light signaling with EPR entangled pairs



Arthur changes the setting of polarizer I from **a** to **a'**: can Beatrice instantaneously observe a change on its measurements at II ?

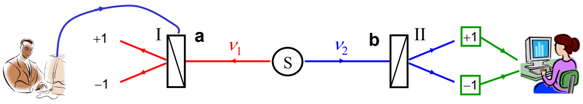
Single detections: $P_+(\mathbf{b}) = P_-(\mathbf{b}) = 1/2$ No information about **a**

Joint detections: $P_{++}(\mathbf{a},\mathbf{b}) = P_{--}(\mathbf{a},\mathbf{b}) = \frac{1}{2}\cos^2(\mathbf{a},\mathbf{b})$ etc.

Instantaneous change !

Faster than light signaling ?

No faster than light signaling with EPR entangled pairs



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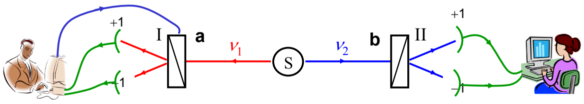
Joint detections: $P_{++}(\mathbf{a},\mathbf{b}) = P_{--}(\mathbf{a},\mathbf{b}) = \frac{1}{2}\cos^2(\mathbf{a},\mathbf{b})$ etc.

Instantaneous change ! Faster than light signaling ?

To measure $P_{++}(\mathbf{a},\mathbf{b})$ Beatrice must compare her results to the results at I: the transmission of these results from I to Beatrice is done on a classical channel, not faster than light.

cf. role of classical channel in quantum teleportation.

So there is no problem ?



View *a posteriori* onto the experiment:

During the runs, Arthur and Beatrice carefully record the time and result of each measurement.

After completion of the experiment, they meet and compare their data…

… and they find that $P_{++}(\mathbf{a},\mathbf{b})$ had changed instantaneously when Arthur had changed his polarizers orientation…

Non locality is there, but it cannot be used for « practical telegraphy »

Quantum non locality

- Is it a real problem ?

« It has not yet become obvious to me that there is no real problem. I cannot define the real problem, therefore I suspect there's no real problem, but I am not sure there is no real problem. So that's why I like to investigate things. »

R. Feynman

Int. Journ. of Theoret. Phys. 21, 467 (1982)*

* A paper about… quantum computers!

From EPR objections (intuitions) to quantum information: amazing entanglement

Part 1: EPR paper (1935), Bell's theorem (1965), experimental tests (1972-): entanglement as a conceptual question (quantum non locality)

Part 2: Quantum cryptography, entanglement on demand, quantum computing: entanglement as a resource for quantum information

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Entanglement as a resource for quantum information

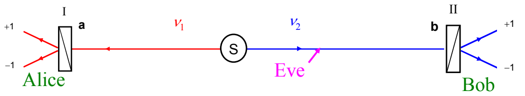
The properties of a pair of entangled photons are more than the sum of the properties of the individual photons (failure of Enstein's local realism)

At the root of new concepts in information theory (« Information is physical »: P. Shor)

Quantum information
- Quantum cryptography
- Quantum processing

Quantum cryptography with entangled pairs (A. Ekert)

The goal: distribute to two partners (Alice and Bob) two identical random sequences (encoding keys), while being sure that no eavesdropper (Eve) has got a copy of the key.



Alice and Bob randomly select an orientation out of ↕ or ⤢ and make a measurement on a photon belonging to an EPR pair. Repeat.

Then they communicate on a public channel :
- All the chosen orientations
- A sample of results of measurements

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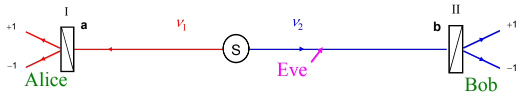


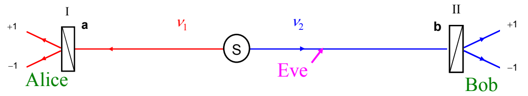
When the orientations are the same the outcome are identical: they have two identical keys.

With the transmitted results, they can make tests (for instance of Bell's inequalities) to be sure that there was no eavesdropper on the path.

The basic idea behind quantum cryptography

It is impossible to make a measurement on a quantum system without leaving a footprint (no cloning theorem). Also true for the Bennet Brassard protocole.

An appealing feature of quantum cryptography with entangled pairs



The key does not exist until the moment when the measurements by Alice and Bob are done (otherwise there would be hidden variables).

There seems to be nothing to spy in between!

L'ordinateur quantique?

On a pu montrer (P. Shore) qu'un ordinateur quantique permettrait de mettre en œuvre des algorithmes spécifiques permettant de réaliser dans des temps raisonnables des calculs défiant la puissance des ordinateurs classiques (décomposition en facteurs premiers)

Un matériel conceptuellement différent permet l'émergence de logiciels conceptuellement différents (cf. cryptographie)

Que serait un ordinateur quantique ?

Un ensemble de portes logiques quantiques interconnectées, travaillant sur des bits quantiques (Q bits)

Système travaillant avec des états intriqués: ⇒ Parallélisme massif

En effet, l'espace des états produit tensoriel de N qbits a une dimension 2^N ! L'essentiel de cet espace est constitué d'états intriqués.

Entanglement as a resource for quantum information:

Quantum cryptography

Quantum processing

Quantum processing

A system of N qbits (two level systems) is described in a enormous Hilbert space → dimension 2^N

Massively parallel calculations possible → 2^N values calculated simultaneously

The complexity of a problem can be drastically reduced.

Example: factorization (P. Shor). Time of calculation grows polynomially rather than exponentially with the size of the number.

One must use quantum gates to combine qbits:

entanglement on demand requested (immune of decoherence!)

Entanglement and quantum gates (2 qbits)

Example of a CNOT gate, combining q_1 and q_2

Definition

$$\boxed{\begin{array}{l} |0_1\rangle|0_2\rangle \xrightarrow{\text{CNOT}} |0_1\rangle|0_2\rangle \\ |0_1\rangle|1_2\rangle \xrightarrow{\text{CNOT}} |0_1\rangle|1_2\rangle \\ |1_1\rangle|0_2\rangle \xrightarrow{\text{CNOT}} |0_1\rangle|1_2\rangle \\ |1_1\rangle|1_2\rangle \xrightarrow{\text{CNOT}} |0_1\rangle|0_2\rangle \end{array}}$$

Apply it to a « rotated » qbit

$$\left(\frac{|0\rangle_1+|1\rangle_1}{\sqrt{2}}\right)|0_2\rangle \xrightarrow{\text{CNOT}} \frac{1}{\sqrt{2}}\left(|0_1\rangle|0_2\rangle+|1_1\rangle|1_2\rangle\right)$$

Bell (maximally entangled) state

To develop quantum gates, one must be able to implement entanglement on demand

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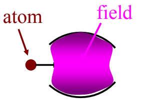
Entanglement on demand in Cavity QED



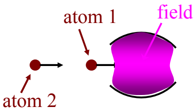
2 level atom (qbit q_1) interacting with a single mode of the field with 0 or 1 photon (qbit q_2)
Example (Haroche et al.): Rydberg atom interacting with a RF field in a supraconducting cavity

Controlled interaction time: π / 2 or π evolution time for the interacting system

$$\boxed{\begin{array}{l} |e\rangle|0\rangle \xrightarrow{\pi} |g\rangle|1\rangle \\ |g\rangle|1\rangle \xrightarrow{\pi} -|e\rangle|0\rangle \\ |g\rangle|0\rangle \xrightarrow{\pi} |g\rangle|0\rangle \end{array}}$$

$$\boxed{|e\rangle|0\rangle \xrightarrow{\pi/2} \frac{1}{\sqrt{2}}\left(|e\rangle|0\rangle+|g\rangle|1\rangle\right)}$$

Entanglement on demand in Cavity QED



Two atoms successively interacting with the same mode of the cavity

First interaction: $|e_1\rangle|0\rangle \xrightarrow{\pi/2} \frac{1}{\sqrt{2}}\left(|e_1\rangle|0\rangle+|g_1\rangle|1\rangle\right)$

Second interaction:

$$\frac{1}{\sqrt{2}}\left(|e_1\rangle|0\rangle+|g_1\rangle|1\rangle\right)|g_2\rangle \xrightarrow{\pi} \frac{1}{\sqrt{2}}\left(|e_1\rangle|0\rangle|g_2\rangle-|g_1\rangle|0\rangle|e_2\rangle\right)$$

Two entangled atoms (spatially separated) on demand (atom timing controlled) $\frac{1}{\sqrt{2}}\left(|e_1\rangle|g_2\rangle-|g_1\rangle|e_2\rangle\right)\otimes|0\rangle$

Entanglement on demand with two trapped ions

Analogy to Cavity QED scheme:

When it changes its internal state (under the effect of Raman lasers) each 2 level atom is coupled to the collective center of mass motion restricted to fundamental or first excited vibrational levels

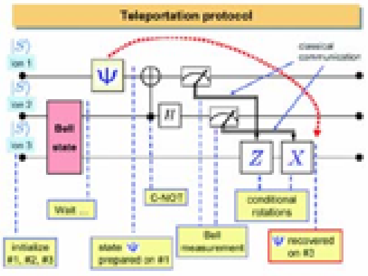
Ion qbit: $\{|g\rangle,|e\rangle\}$

Collective motion qbit: $\{|n_v=0\rangle,|n_v=1\rangle\}$

Theoretical proposal: Cirac and Zoller (1995); refined by Moelmer and Soerensen

Experimental implementation: Boulder, Innsbruck.

Example of quantum processing with trapped ions: teleportation



Innsbruck, Boulder

A genuine quantum processing algorithm, involving a series of controlled operations on three qbits, with a very limited decoherence

Not yet a quantum computer, but an elementary quantum processor!

The amazing properties of entanglement

Since the EPR paper (1935), it took more than 40 years (and the genius of John Bell) for a significant fraction of the physicists to recognise non locality of entangled pairs as a new amazing concept in quantum mechanics.

It took another decade to discover that entanglement can be a physical resource for new ways of handling information.

We have certainly not yet discovered all the amazing properties of entanglement, and the most efficient ways of using entanglement remain probably to be invented.

There are plenty of possibilities with photons, ions, atoms, supra conductor or semi conductor nanochips… to develop the applications of the new quantum revolution.