Recent Applications of Classical Field Theory

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Overview

• 0. PGPE Review
• I. Application to (quasi)-2D trapped system
• II. Application to trapped critical system
0.
PGPE Review

(Projected Gross-Pitaevskii Equation)
Main Objective:
Dynamical Finite-T Theory

• Non-perturbative

• Flexible and computationally tractable as GPE

• Applications: Condensate growth, Vortex lattice formation, Atom lasers, Strongly fluctuating systems (near Tc or low dimensional).

• Quantitative comparison with experiment
Motivation: Blackbody Spectrum

Blackbody at 2500K

>3 photons per mode: classical field well-approximates quantum field
Classical Region for Atoms

Conservation of particles $\rightarrow \mu$

Classical regime $\varepsilon_h - \mu \lesssim kT$

Occurs only at extremely low temperatures
Ideal Bose Gas

2 million atoms, 100Hz harmonic trap

T_c

Ultra-Cool Region

BEC Mode

10th 35th 120th 1000th

3 atoms/Mode

Temperature [nK]
Classical Region

120th state

1000th state

Atom Number $N_T$ [$\times 10^6$]

Temperature [nK]

$T_c$

$N_{below}$

$\varepsilon_{cut}$

$T$ [nK]
PGPE Formalism

(EOM for the classical region)

The Projected Gross-Pitaevskii Equation:

\[ i\hbar \frac{\partial \Psi_C}{\partial t} = \hat{H}_{sp} \Psi_C + \mathcal{P} \left\{ U_0 |\Psi_C|^2 \Psi_C \right\} \]

restricted to classical region

single particle modes:

Eigenstates of \( \hat{H}_{sp} \)

\[ \Psi_C = \sum_{j=1}^{n} c_j \phi_j(x) \]

Harmonic trap: \( \phi_j(x) \) harmonic oscillator states

P. B. Blakie and M. J. Davis, PRA 72, 063608 (2005)
Above cut off atoms?

- Treat as weakly coupled system (thermal and diffusive equilibrium)
- Describe using semiclassical meanfield theory
Other Details

- Equilibrium: Ergodicity
- Techniques to extract temperature, chemical potential and condensate fraction
Condensation

Time averaged momentum col. density
Fluctuations

Density Fluctuations:

\[ g^{(2)}(r) = \frac{\langle \psi^+(r)\psi^+(r)\psi(r)\psi^+(r) \rangle}{\langle \psi^+(r)\psi(r) \rangle^2} \]
I. Application to 2D Trapped Systems
2D Bose Gas

Fundamental question:

What is the nature of the low temperature superfluid phase(s) of the 2D trapped Bose gas?
Observation of Phase Defects in Quasi-Two-Dimensional Bose-Einstein Condensates

Experimental Issues

- Temperature unclear $0.7T_c - 0.9T_c$ (?)
- Vortex pairs unobservable (?)
Classical field simulations 1/3

Classical field simulations 2/3

Temperature Movie

QuickTime™ and a decompressor are needed to see this picture.
Current Work

• Phase diagram - understanding the BKT window

• Looking at experimental diagnostics: scissor modes, 3-body loss
II. Application to Systems Near $T_c$
Interacting Bose Gas
Critical Temperature


\[ \frac{\Delta T_c}{T_c^0} = ca n^{1/3} \]

- Inhomogeneous gas?

\[ c = 1.4 \ldots \text{PGPE gets this right!!} \]

(Davis et al., 2003)
Experiments

- 50-to-1 cigar trap

Gerber et al., PRL 93, 030405 (2004)
summary

• Quantitative agreement between PGPE and experiment

• Quantitative calculations of critical effects in the trapped Bose gas

• currently: looking for regimes to enhance critical effects
Conclusions & Outlook

• Presented theoretical results for low dimensional and critical systems.

• Good agreement with experiment

• *future*: Correlations functions in the critical region and for quasi-low dimensional gases

• *future*: Non-equilibrium phenomenon
the end
References

• Gerber et al., PRL 93, 030405 (2005)
• P. B. Blakie and M. J. Davis, PRA 72, 063608 (2005)
• Stock et al., PRL 95, 190403 (2005)
• P. B. Blakie and M. J. Davis, PRL (To appear Feb. 06)