Quantum-Atom Optics: Present and Future

P. D. Drummond,
Australian Centre for Quantum Atom Optics

February 8, 2006
Quantum Optics: the 2005 Nobel Prize in Physics

The Royal Swedish Academy of Sciences awarded one half of the Nobel Prize in Physics for 2005 to

Roy J. Glauber

• for his contribution to the quantum theory of optical coherence
Roy at ETH

Postdoctoral Work:
Glauber followed
**EINSTEIN’S**
footsteps, working at:
Institute for Advanced Study (Princeton, USA)
Swiss Federal Polytechnic Institute (ETH, Zurich)
Caltech (USA)
Roy and Wolfgang Pauli

Wolfgang PAULI

inventor of the ‘exclusion principle’ - liked to tease his postdocs.
He is laughing at Roy who is trying to photograph him - while Pauli kicks a soccer ball at the camera.
Roy at Harvard

LYMAN/JEFFERSON LABS

where Glauber has worked for 53 years, since joining Harvard in 1952, with full tenure from 1955
Glauber’s correlation function

Define the $n$-th order correlation function:

$$G_{\mu_1 \ldots \mu_{2n}}^{(n)}(x_1, \ldots, x_{2n}) = \langle E_{\mu_1}^-(x_1) \ldots E_{\mu_n}^-(x_n) E_{\mu_{n+1}}^+(x_{n+1}) \ldots E_{\mu_{2n}}^+(x_{2n}) \rangle$$

For symmetric arguments, $G^{(n)}(x_1 \ldots x_n, x_n \ldots x_1)$ is:

• the rate of counting $n$ photons at locations $x_1 \ldots x_n$, where $x = (r, t)$. 
Coherence

Define the $n$-th order **normalized** correlation function:

$$g^{(n)}_{\mu_1...\mu_{2n}}(x_1 \ldots x_{2n}) = \frac{G^{(n)}_{\mu_1...\mu_{2n}}(x_1 \ldots x_{2n})}{\prod_{j=1}^{2n} \sqrt{G^{(1)}_{\mu_1,\mu_j}(x_j, x_j)}}$$

**First order coherence:** $$\left| g^{(1)}_{\mu_1\mu_2}(x_1, x_2) \right| = 1$$

**Second order coherence:** $$\left| g^{(2)}_{\mu_1...\mu_4}(x_1 \ldots x_4) \right| = 1$$
Lasers and Lightbulbs?

Roy’s coherence theory answered the question: is there a FUNDAMENTAL difference between lasers and lightbulbs??

• Is it because the laser has a narrow spectrum?

• NO - you can filter light to have a narrow spectrum!

• What is the difference?
PHOTON ARRIVAL TIMES

- Photons from a lightbulb BUNCH together.
- They are CORRELATED

- Photons from a laser arrive independently.
- They are UNCORRELATED
Lasers and Coherence

Lightbulbs:

✔️ Might have first order coherence -

❌ but NEVER second order

Lasers: can have coherence to ALL orders (if perfect)
Where did this lead?

**Photon antibunching**: Photons that never arrive together: suppressed intensity noise (Mandel, Walls)

**Bell inequalities**: Optical demonstrations of the Bell inequality (Bell, Aspect et al)

**Quantum Squeezing**: Reduced fluctuations in one quadrature, increased in another (Slusher, Gardiner)

**EPR correlations**: Optical demonstrations of the Einstein-Podolsky-Rosen paradox (Reid, Kimble)
What are coherent states?

These are idealized states which are coherent to all orders! If $\hat{a}$ is a field operator, then:

$$\hat{a}|\alpha\rangle = \alpha |\alpha\rangle$$

- Coherent states are a complete mathematical basis
- Also can have SU(N) coherent states for spins
Glauber-Sudarshan P-representation

• Coherent states can be used to construct quantum operator representations

\[ \hat{\rho} = \int P(\alpha) |\alpha\rangle \langle \alpha| d^2\alpha \]

• Glauber’s P-representation used to treat quantum noise in lasers

• Restricted to classical states \((g^{(2)}(0) \geq 1)\)
QUANTUM-ATOM OPTICS: PRESENT

Out of the last TEN Nobel prizes awarded to physicists

✔ THREE: low-temperature physics, many-body theory

✔ TWO: Ultra-cold atoms, BEC

✔ ONE: Computational physics/chemistry

✔ ONE: Quantum Optics/ Laser Spectroscopy

✔ SCIENCE (Top Ten breakthroughs in 2004): ultracold fermions
Atom laser experiments

Atom laser gallery

MIT ‘97
Munich ‘99
Yale ‘98
NIST ‘99

Height:
5, 2, 0.5, 1 mm
Current experiments: Quantum Optics

- Quantum noise limited lasers to $1kHz$
- Sqeezed/entangled beams with up to $10dB$ squeezing
- Laser frequency stability to 1 part in $10^{15}$ Hz
- Demonstration of EPR (non-causal)
- Bell inequality tests (efficiency loopholes)
- Spin/light entanglement demonstration
Current experiments: Atom Optics

- Cold BEC and Fermi gases to $1nK$
- Dimensional control in optical lattices
- Nonlinear coupling, via four-wave mixing
- BEC-BCS crossover, via molecule formation
- Correlated atom *emission* measured by light scattering
- Correlated atom *detection* using MCP technology
Atoms on lattices
Atoms on chips
Current Theory: Quantum Optics

Many good techniques, weak interactions

- Direct calculations for small photon number
- Can linearize in some cases
- Truncated Wigner (semiclassical) OK for large photon number
- First-principles phase-space methods (positive-P) very successful
Positive P-representation

• Extends Glauber’s P-representation to **non-classical** states

\[ \hat{\rho} = \int P(\alpha, \beta) \frac{|\alpha\rangle \langle \beta|}{\langle \beta| \alpha\rangle} d^2\alpha d^2\beta \]

• Used for first principles quantum dynamical simulations

• Led to predictions of quantum squeezing in solitons
Current Theory: Atom Optics

More challenging, stronger interactions

- Mean-field and classical field approximations common
- Perturbation theory for ground states: but excited states difficult
- Approximate semiclassical has sampling error and other problems
- Monte-Carlo good in some cases at thermal equilibrium
- Positive-P useful, but only for short times
Largest Hilbert Space Ever Simulated

- BEC Four-wave mixing
- $10^5$ Rubidium atoms
- Total of $2 \times 10^6$ modes
- Experiment: Ketterle, MIT
- Theory: Drummond, Deuar, UQ (+P)
QUANTUM-ATOM OPTICS: FUTURE

- Lower temperatures: what is the current limit?
- More atomic/molecular species: can we cool every isotope?
- Light-atom entanglement: how strongly entangled?
- Spinor atoms on optical lattices
- Fermions in engineered environments
- Progress towards true ‘SCHROEDINGER CATS’
Theoretical Challenges

- Does the 2D Fermi-Hubbard model have superconductivity?
- Ground state of strongly interacting Fermi gas?
- How does a BEC interact with an optical cavity?
- Quantum ground state of spinor gas in a lattice?
- Excited states of Bose gases: are they bosonic/fermionic?
- First-principles time-domain quantum simulations?
Complexity Issues

- many-body problems become exponentially complex.

- consider $n$ atoms distributed among $m$ modes

- Each mode can have one or all atoms; take $n \approx m \approx 500,000$:

- Number $N_s$ of quantum states is ENORMOUS:

$$N_s = 2^{2n} = 10^{100,000}$$
Classical phase-space

Wigner and Glauber used a classical-like phase-space or quasi-probability description. Here, for $M = LD$ modes, and a maximum of $N$ particles/mode

**Usual** QM: $\rightarrow N^M$ (complex) coordinates

**Wigner, Glauber:** QM $\rightarrow M$ coordinates

Problem: the Wigner function has negative values and obeys a complicated differential equation. The Glauber-Sudarshan is negative or singular for non-classical states.
Quantum phase-space representations

Expand the density matrix $\hat{\rho}$, using operators $\hat{\Lambda}(\vec{\lambda})$:

$$\hat{\rho} = \int P(\vec{\lambda})\hat{\Lambda}(\vec{\lambda})d\vec{\lambda}$$

Quantum dynamics → Trajectories in $\vec{\lambda}$.

Different basis choice $\hat{\Lambda}(\vec{\lambda})$ → different representation

More than one $P(\vec{\lambda})$ is possible → different stochastic gauges
THE BIG QUESTIONS IN QUANTUM-ATOM OPTICS

✔ Is there a coldest temperature we can reach?

✔ Can we prove the existence of ‘Schroedinger Cat’ states?

✔ Are there fermionic excitations in 2D or 3D Bose gases?

✔ Does gravity play a role in quantum decoherence?

✔ Is there an ‘Infodynamics’ of quantum entanglement?

✔ Can we solve quantum complexity with digital computers?