# Atom-Light Entanglement

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Objectives

Quantum information storage

- Quantify EIT-based Quantum memories
  - Theoretically → Stochastic Simulations of light storage
  - Experimentally → Quantum Information Delay/Storage in atomic ensemble.

- Generate non-classical light at atomic wavelengths
  - Self rotation → Too noisy (M.Hsu et al. To be published in PRA)
  - SHG/OPO @ 795 nm → In progress
Transfer of quantum states between optical and atomic fields

**Tool:** Electromagnetically induced transparency (EIT)

We want to consider the interaction of the two quadratures of the probe field when interacting with such a system in **single pass** and measure their **variances**.
Quantum information delay

\[
|3\rangle \xrightarrow{\hat{E}_p(z,t)} |1\rangle \xrightarrow{\gamma_0} |2\rangle \\
\Omega_c(t)
\]

Langevin treatment of the Lambda system in the weak probe approx.:

\[
V_{\text{out}}^\theta(\omega) = \eta(\omega)V_{\text{in}}^\theta(\omega) + (1 - \eta(\omega)) V_{\nu}(\omega)
\]

Input/Output quadrature variances

Vacuum field

Beam splitter like relation: No extra atomic noise in this regime.

For example: With 4 dB of squeezing input to the system: \(V_{\text{squ}}^{\text{out}}(\omega) = 3.8 dB\)

Parameters:

\[
\begin{aligned}
\Omega_c(t) &= 5\gamma \\
\gamma_0 &= 1 kHz \\
N &= 1.10^{12} at.cm^{-3} \\
\Delta\nu &= 2 MHz \\
\tau_d &= 10 \mu s
\end{aligned}
\]

A.Peng et al. PRA 71 033809 (2005)
Quantum Information Storage

- Switching off the coupling field: the probe field information is stored into the long lived coherence between the ground states when being delayed.
- Switching on the coupling field: the probe field is retrieved.

All the experiments in vapor cells present so far the same exponentially decaying output pulse.

Controversy on the interpretation.

Phillips et al.

Liu et al.

Smaller decoherence rate.
Longer storage time.
Quantum Information Storage

- Could the pulse shape be preserved?
- What is the conjugate observable of a pulse shape?
- How much quantum information can be stored?
- Is it a noisy process?

What we want to do:

\[ \hat{p}(z,t) \approx \mu s \]

\[ \approx \text{kHz} \]

\[ \hat{\varepsilon}_p(z,t) \]

EIT → Homodyne detection
Quantum Information Storage

Quantification of this process using quantum information tools:

- Average fidelity: \( \mathcal{F} = \langle \Psi_{in} | \rho | \Psi_{in} \rangle \)

  Bounded by 1/2 for a classical measure and prepare protocol. 
  \textit{Hammerer et al. (PRL 2005)}

- Signal to noise transfer and conditional variances (T and V). Useful when dealing with non-unity gain
  
  \textit{Ralph and Lam criteria (PRL 1998)}

Is it a Quantum Memory when taking into account realistic experimental parameters?
Modeling using Stochastic methods

Interaction Hamiltonian for an optically thick medium:

\[ \hat{H}_{\text{int}} = i \hbar n A_{\text{eff}} \int_0^l dz \left[ -g \left( \hat{E}_p(z,t) \sigma_{13}(z,t) + \hat{E}_p^*(z,t) \sigma_{31}(z,t) \right) 
+ \Omega_c(t) \sigma_{32}(z,t) + \Omega^*_c(t) \sigma_{32}(z,t) \right] \]

Maxwell equations in the rotating wave approximation:

\[
\begin{align*}
\frac{\partial}{\partial t} + c \frac{\partial}{\partial z} \bar{\alpha}(z,t) & = -gN \, \sigma_{13}(z,t) \\
\frac{\partial}{\partial t} + c \frac{\partial}{\partial z} \bar{\beta}(z,t) & = -gN \, \sigma_{31}(z,t)
\end{align*}
\]

Master equation:

\[ \frac{\partial}{\partial t} \hat{\rho} = i \hbar [\hat{H}_{\text{int}}, \hat{\rho}] + \mathcal{L}_{13}[\hat{\rho}] + \mathcal{L}_{23}[\hat{\rho}] + \mathcal{L}_{[1,2]}[\hat{\rho}] \]

We use the Positive P representation, define normal ordering and obtain a Fokker Planck equation. We finally convert the FPE into a set of SDE (18 noise terms...)}
Numerical results

**Time domain**

- Input
- Output

![Graph showing time domain results](image)

**Fourier domain**

\[ \langle X^+(\omega) X^-(\omega) \rangle \]

![Graph showing Fourier domain results](image)

Similar for the phase quadrature...
Realistically...

**Next step:** Characterize the Quantum state transfer including decoherence factors:

\[
\mathcal{L}_{[1,2]}[\hat{\rho}] = \gamma_{\text{coll}} \sum_{z_k \in \delta z} (\hat{\sigma}_{12}^k \hat{\rho} \hat{\sigma}_{21}^k - \frac{1}{2} \hat{\sigma}_{12}^k \hat{\sigma}_{21}^k \hat{\rho} - \frac{1}{2} \hat{\rho} \hat{\sigma}_{21}^k \hat{\sigma}_{12}^k) \\
+ \gamma_{\text{coll}} \sum_{z_k \in \delta z} (\hat{\sigma}_{21}^k \hat{\rho} \hat{\sigma}_{12}^k - \frac{1}{2} \hat{\sigma}_{21}^k \hat{\sigma}_{12}^k \hat{\rho} - \frac{1}{2} \hat{\rho} \hat{\sigma}_{12}^k \hat{\sigma}_{21}^k) \quad \text{Collisions}
\]

\[
\mathcal{L}_{[1,2]}[\hat{\rho}] = \gamma_{\text{deph}} \sum_{z_k \in \delta z} \left( (\hat{\sigma}_{11}^k - \hat{\sigma}_{22}^k) \hat{\rho} (\hat{\sigma}_{11}^k - \hat{\sigma}_{22}^k) - \frac{1}{2} (\hat{\sigma}_{11}^k + \hat{\sigma}_{22}^k) \hat{\rho} - \frac{1}{2} \hat{\rho} (\hat{\sigma}_{11}^k + \hat{\sigma}_{22}^k) \right) \\
+ \gamma_{\text{deph}} \sum_{z_k \in \delta z} \left( (\hat{\sigma}_{22}^k - \hat{\sigma}_{11}^k) \hat{\rho} (\hat{\sigma}_{22}^k - \hat{\sigma}_{11}^k) - \frac{1}{2} (\hat{\sigma}_{22}^k + \hat{\sigma}_{11}^k) \hat{\rho} - \frac{1}{2} \hat{\rho} (\hat{\sigma}_{22}^k + \hat{\sigma}_{11}^k) \right) \quad \text{Dephasing}
\]

**Main theoretical issue:** Need for computing time to calculate the noise. 

\[\rightarrow\rightarrow\rightarrow\] requires a lot more trajectories (≈10^6)

**Main experimental issue:**

Need for a good delay (to fit the whole pulse inside the cell)

\[\rightarrow\rightarrow\rightarrow\] narrow transparency window.

\[\rightarrow\rightarrow\rightarrow\] Low frequency information (Squeezing, AM/PM).