Inertial Sensors
Based on Cold Atoms
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Many disadvantages of inertia...
Atomic Sensors
-an alternative technique
Outline

1. Principle of Atom Interferometry
2. Accelerometers and Gyroscopes
3. Applications & Alternative Techniques
4. Outlook
Principle of Atom Interferometry
The atomic ruler

De-Broglie wavelength

Classical World

Atomic momentum

Planck’s constant

\[ |\vec{k}_{at}| = \frac{2\pi}{\lambda_{dB}} \]

\[ m_{at} \cdot \vec{v}_{at} = \vec{p}_{at} = \hbar \cdot \vec{k}_{at} \]

\[ m_{at} v_{at} = \frac{\hbar}{\lambda_{dB}} \]

Units in de Broglie wavelength

Louis Victor de Broglie
Nobel prize 1929
for the local measurement of tiny accelerations/forces and rotations with high resolution:

Measuring displacements with cold atoms
Comparing 2 rulers.

...with Atom Interferometers

using Light as coherent Beam Splitter:

Generating spatial modes
of matter waves
...based on

the mechanical effect of light

\[
|g, p_{at}\rangle \rightarrow |e, p_{at} + \hbar k\rangle \\

\vec{k}_L \\

\vec{v}_{rec} = \frac{1}{m_{\text{atom}}} \frac{\hbar \vec{k}_L}{m_{\text{atom}}}
\]
...transfer of two recoils by absorption & stimulated emission:

Bragg or Raman type beam splitter
... made out of Light

\[ \left| g_1, p + 0 \, \hbar \bar{k} \right\rangle \rightarrow \frac{1}{\sqrt{2}} \left[ \left| g_1, p + 0 \, \hbar \bar{k} \right\rangle + e^{i\phi} \left| g_2, p + 2 \, \hbar \bar{k} \right\rangle \right] \]

\[ \left| g_1, p + 0 \, \hbar \bar{k} \right\rangle \rightarrow e^{i\phi} \left| g_2, p + 2 \, \hbar \bar{k} \right\rangle \]

\[ \left| g_1, p + 0 \, \hbar \bar{k} \right\rangle + e^{i\phi} \left| g_2, p + 2 \, \hbar \bar{k} \right\rangle \rightarrow \frac{1}{\sqrt{2}} \left[ \left| g_1, p + 0 \, \hbar \bar{k} \right\rangle + e^{i\phi} \left| g_1, p + 0 \, \hbar \bar{k} \right\rangle \right] + e^{-i\phi} \left| g_2, p + 2 \, \hbar \bar{k} \right\rangle \]
Atomic Mach-Zehnder Interferometer

\[ S \sim \cos[(\phi_3 - \phi_2) - (\phi_2 - \phi_1)] \]

\[ \sim \cos (\phi_1 - 2\phi_2 + \phi_3) \]
Accelerometers & Gyroscopes
\[ \Delta \varphi = [\varphi_3(2T+t_0) - \varphi_2(T+t_0)] - [\varphi_2(T+t_0) - \varphi_1(t_0)] \]

Constant accelerations:

\[ \Delta \varphi_{\text{acc}} = T^2 \vec{k} \cdot \vec{a} \]

to be used as...
Accuracy of $\Delta g$ resp. $g$: 
1 part in $10^9$

$1 \text{ Gal} = 10^{-2} \text{ m/s}^2$

Gravimeter

A. Peters et al., Metrologia 38, 25 (2001)
\[ \Delta \varphi = [\varphi_3(2T + t_0) - \varphi_2(T + t_0)] - [\varphi_2(T + t_0) - \varphi_1(t_0)] \]

constant rotations:

\[ \Delta \varphi_{rot} = \frac{2 \, m_{\text{Atom}} \cdot \mathbf{A} \cdot \vec{\Omega}}{\hbar} \]

\( T \): drift time
\( v_1 \): atomic forward drift
Sagnac-Effect

Rotational induced Phase shift:

for Light: \[ \Delta \varphi_{rot} = \frac{4\pi}{\lambda c} \vec{A} \cdot \vec{\Omega} \]

for Atoms: \[ \Delta \varphi_{rot} = \frac{4\pi}{\hbar} m_{at} \vec{A} \cdot \vec{\Omega} \]

Gain by de Broglie-Waves: \(~ 10^{11}~\)
Differential Interferometry

\[ S_1 \sim \cos(\varphi_{\text{rot}} + \varphi_{\text{acc}}) \]

\[ S_2 \sim \cos(-\varphi_{\text{rot}} + \varphi_{\text{acc}}) \]

Substraction $\rightarrow \varphi_{\text{rot}}$

Addition $\rightarrow \varphi_{\text{acc}}$

Distinction between rotations and accelerations
- 2 atom sources
- thermal Cs-beams
- transverse laser cooling
- Sensitivity:
  close to shot noise
  $5 \times 10^{-10} \text{ rad/s}$
  
  $4.8 \mu\text{rad} = 1 \text{ arcsec}$

Earth's rotation: 72 $\mu\text{rad/s}$

Definition of the Area

Area enclosed by interferometer:

\[ A = L^2 \cdot \frac{V_T}{V_1} \ll 1 \]

Constant rotations:

\[ \vec{A} = \vec{V}_{rec} \frac{L}{\vec{V}_1} \times \vec{V}_1 \frac{L}{\vec{V}_1} \]

\[ \Delta \varphi_{rct} = \frac{2 m_{\text{Atom}}}{\hbar} \vec{A} \cdot \vec{\Omega} \]
Noise Sources

\[
(\Delta \varphi)^2_{ges} = \frac{1}{N_J} + \frac{1}{N_J n_{Ph}} + \frac{2\sigma_{\delta N}^2}{N_J^2} + \gamma + \ldots
\]

- Atomic projections noise
- Shot noise of photon detection
- Noise of electronics for detection
- Raman-Laser

„quantum limit“, all contributions negligible compared to \(1/N_J\)
for
(ultra-)cold atomic inertial sensors
Cold Atom Gyros

Gyromètre
d’ ondes matières
(A. Landragin, SYRTE Paris)

Cold Atom Sagnac Interferometer
(W.E., IQ Hanover)
MAGGIA
(G. Tino, Univ. Florence)

Paris Gravimeter
(F. Pereira d. Santos, SYRTE, Paris)
Mobile Atomic Gravity Gradiometer Prototype Instrument (MAGGPI)
Accelerometer Arrays
(M. Kasevich, Stanford Univ., US)
Airborne System,
140 dB common mode suppression

Space Gravigradiometer
(L. Maleki, JPL, US)
Applications of *inertial* sensors based on *cold atoms*
• Fundamental Physics

• Applied Physics and connected fields
Watt Balance: Replacement of the kg artifa

2 Steps:

1) Weighing by balancing with the magnetic force

   \[ F_g = m g = I \nabla \Phi \]

2) Measuring the flux gradient

AQS serves for measuring local gravity with a relative accuracy of 1 part in \(10^9\)
... the determination of the Gravitational constant „G“

G- the worst known constant
complementary method
Exp. Accuracy: 1 part in $10^4$
General Problem: The perfect Test mass

Pendulum of the „EotWash“ group
Earth Observation: The Geoid
global and high-resolution models of the static and the time variable components of the Earth's gravity:

- global mass distribution
- ocean heat flux,
- long term sea level change,
- upper oceanic heat content
- large scale evapo-transpiration and soil moisture changes,
- glaciology (Greenland ice sheet changes)
- Space Exploration (Mars!)
Earth and Planetary Observation

The Geoid

\[ U_s(r, \phi, \lambda; t) = \frac{GM_e}{r} + \frac{GM_e}{r} \sum_{l=2}^{N_{\text{max}}} \left( \frac{a_e}{r} \right)^l \sum_{m=0}^{l} \frac{P_{lm}(\sin \phi)}{r^l} \left[ \bar{C}_{lm}(t) \cos m\lambda + \bar{S}_{lm}(t) \sin m\lambda \right] \]
Correction of the non-gravitational accelerations

STAR & SUPERSTAR Sensors

P. Touboul, ONERA Paris

\[ 3 \cdot 10^{-10} \frac{g}{\sqrt{\text{Hz}}} \quad \text{at } 10^{-2} - 10^{-1} \text{Hz} \]

\[ 3 \cdot 10^{-11} \frac{g}{\sqrt{\text{Hz}}} \quad \text{at } 10^{-4} - 10^{-1} \text{Hz} \]
Earth Observation: The Spin
High resolution rotation sensors

The Earth’s rotation:
\[ \Omega_E \approx 7.2 \times 10^{-5} \text{ rad/s} \]

Applications:
- Investigation of the Earth’s rotation
- Geology
- Star motion
- Satellite navigation
- Variation of the Earth’s rotation
- Relativistic effects
- Relativistic Effects

Resolution:
- 10^{-8} – 10^{-9} rad in 24 h
- 10^{-9} rad in 1 year
- 10^{-10} – 10^{-11} \text{ rad/s } \sqrt{\text{Hz}^{-1}}
Outlook
are a promising alternative and complimentary technique for experiments in fundamental physics, like

- Absolute inertial references
- The measurement of relativistic effects
- Testing the Equivalence Principle
- Drag-free sensors perhaps in gravitational wave detectors?
Observing the Lense-Thirring effect <1%

(Results end of the year!)

Testing the Equivalence Principle
1 part in $10^{15}/10^{18}$

Detecting Gravitational Waves
HYPER: Precision Aperiodic Interferometry in Space

Aim: Spatial resolution of the Lense-Thirring-Effect: Schiff effect

http://sci.esa.int/home/hyper/index.cfm
Lense-Thirring effect measured by HYPER

Signature for HYPER

$\Omega$

$\theta$

$t/T_{\text{Orbit}}$

$10^{-14} \text{ rad/s}$
…performance

2 atomic MOTs
Launch of $10^8$ at @ 1µK
with 20 cm/s, $2T_{Drift} = 3$ s
Length: 60 cm

$\Omega_{SNL} = 2 \cdot 10^{-12}$ rad/s/$\sqrt{\text{Hz}}$
$A_{SNL} = 4 \cdot 10^{-14}$ g/$\sqrt{\text{Hz}}$
per shot, 0.3 Hz
Need for Femto-g

With cold atoms?
$\Omega_{SNL} = 2 \cdot 10^{-12}$ rad/s/$\sqrt{\text{Hz}}$

$A_{SNL} = 4 \cdot 10^{-14}$ g/$\sqrt{\text{Hz}}$

per shot, 0.3 Hz

**Time**: $3\text{s} \rightarrow ?$

+ **Resolution**: $\sim T_{\text{Drift}}^2$

- $T_{\text{at}} < 1\mu\text{K}$

- **Dynamic Range**

**Atoms**: $10^8 \rightarrow ?$

+ **Resolution**: $\sim \sqrt{N}$

- $T_{\text{at}} ?$

- **USO-Phase noise** ?

**Beam splitter**: Multiphoton? - New Beam splitters?

„**All-atomic“** Gravitational Wave detector?

**Thermal Atoms**: + High Flux

small de Broglie wavelength

Large Distances in short time

Beam splitter ?
Cold Atom Sagnac Interferometer

Michael Gilovski

Thijs Wendrich
Tobias Müller

Ernst M. Rasel
W.E.

Christian Jentsch
(now at SYRTE)
THANK YOU