Superradiant light scattering from a Bose-Einstein condensate

Rob Ballagh & James Douglas

University of Otago
Dunedin
New Zealand

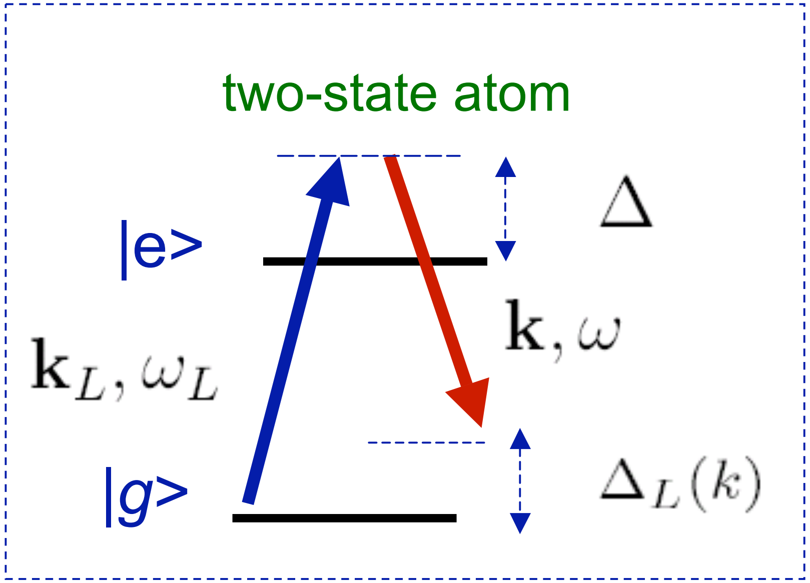
Background

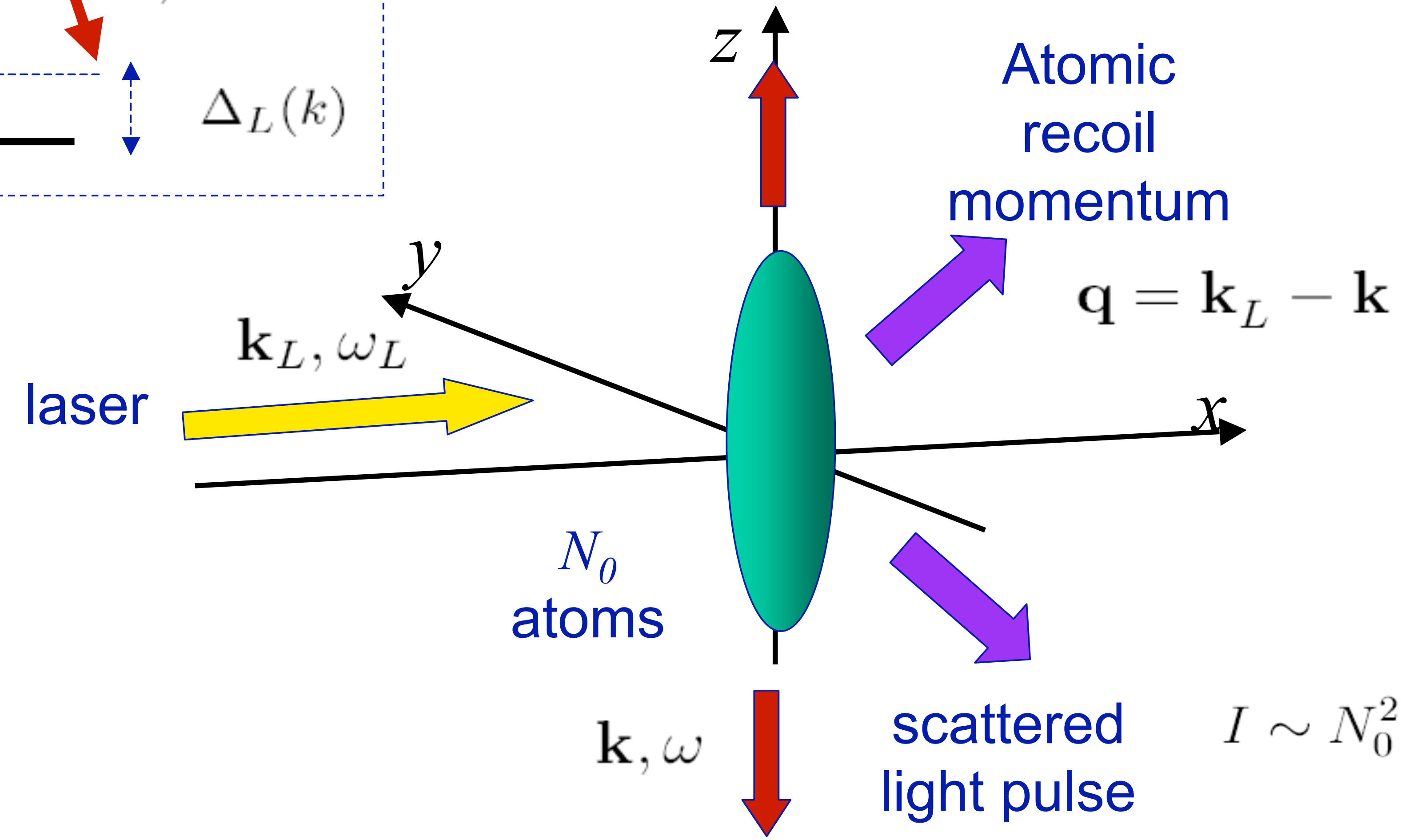
- An old topic of laser physics
  - Haake, Bonifacio, *PRA, 4, 302, (1971)*
  - Gross, Haroche, *Physics Repts, 93,302, (1982)*
- In the context of cold atoms
  - Bonafacio, CARL, *PRA, 50, 1776 (1994)*
  - Experiment ← Ketterle, *Science, 285, 571 (1999)* *[Nature 1999, PRL 2000, Science 2003]*
  - Experiment ← Kozuma et al , *Science 286,2309, (1999)*
  - Moore, Meystre, *PRL, 83, 5202 (1999)*

This Talk

- Obtain full spatio-temporal description of BEC superradiance
- Review experimental scenario
- Outline theoretical formalism
- Simulation results
    - solutions in 2D and 3D for condensate and light
    - survey of regimes
    - effects of condensate nonlinearity
    - Decoherence rates

Superradiant scattering

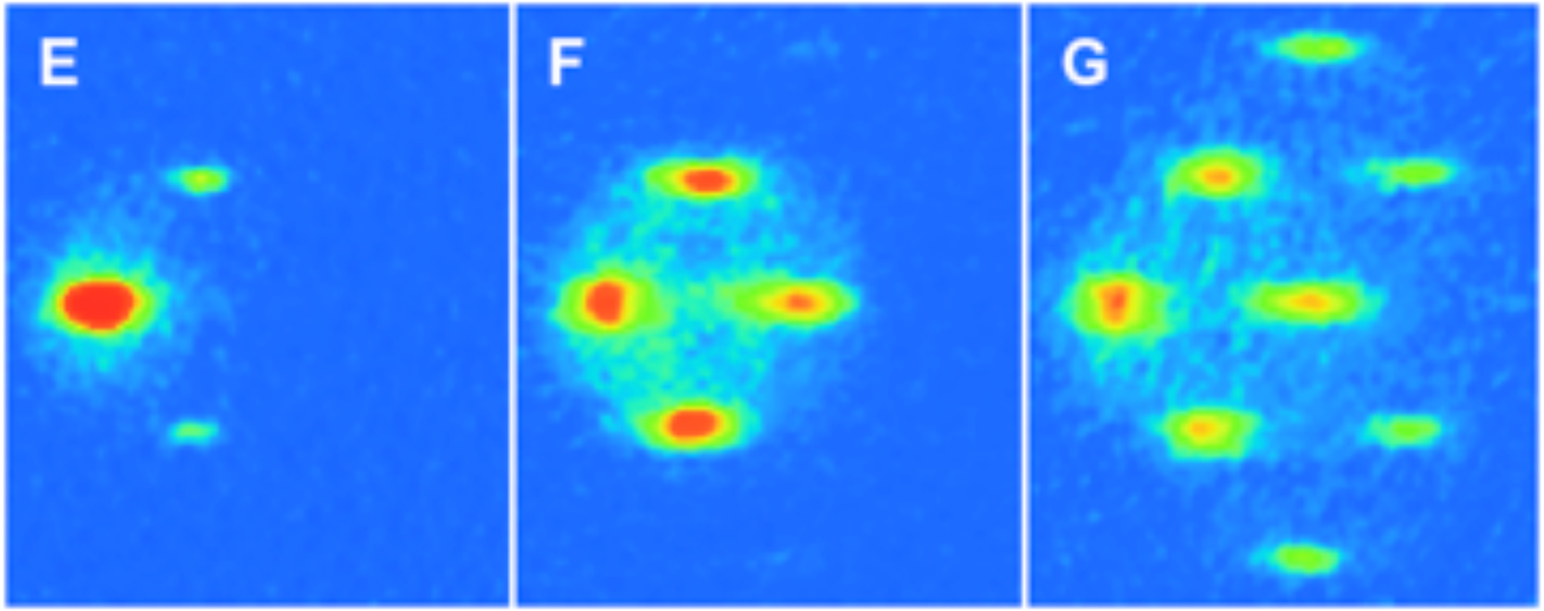




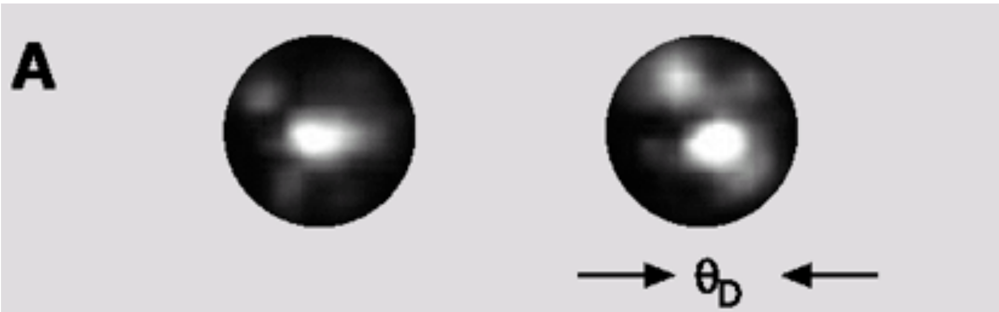
Experiment by Ketterle's group

Science, 285, 571, (1999)

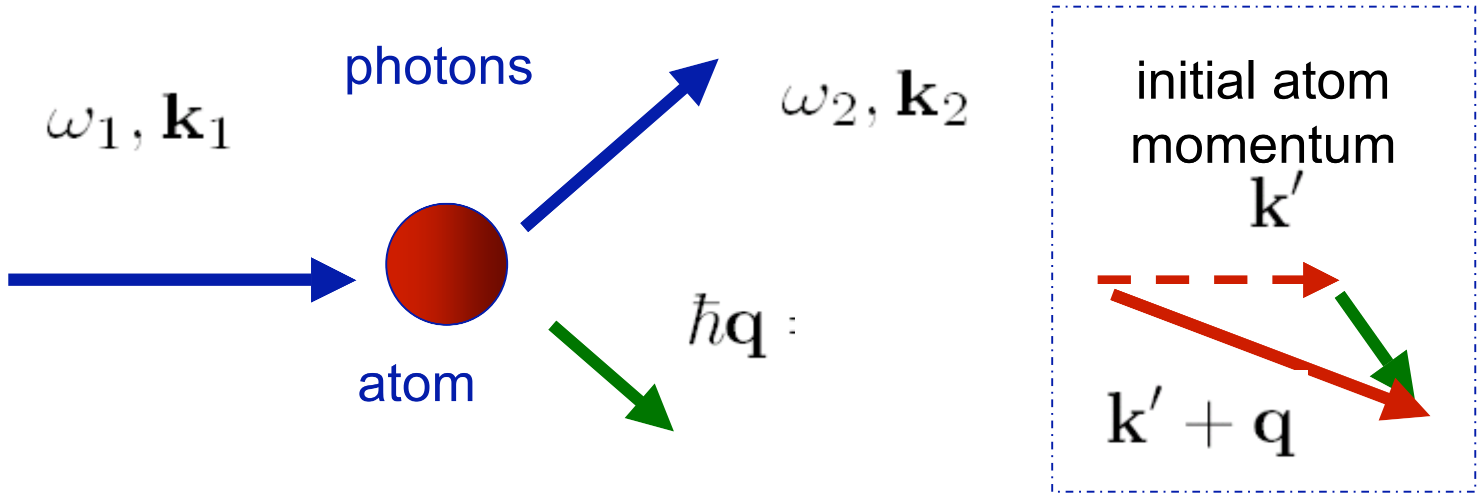
Time development of condensate momentum distribution



Corresponding scattered light



A Basic Mechanism



Each scattering event transfers to C.O.M of each atom

net energy :	$\hbar\delta = \hbar(\omega_1 - \omega_2)$
net momentum :	$\hbar\mathbf{q} = \hbar(\mathbf{k}_1 - \mathbf{k}_2)$

associated recoil energy $\hbar\omega_{recoil} = \dfrac{\hbar^2(\mathbf{k}' + \mathbf{q})^2}{2m} - \dfrac{\hbar^2(\mathbf{k}')^2}{2m}$

Resonant process; require $\delta \approx \omega_{recoil}$

Formalism

$$\hat{H} = \hat{H}_{atom} + \hat{H}_{rad} + \hat{H}_{atom-pump} + \hat{H}_{atom-rad} + \hat{H}_{atom-atom}$$

$$\hat{H}_{atom} = \int d^3\mathbf{r} \left[\hat{\Psi}_g^\dagger(\mathbf{r}) \left(-\frac{\hbar^2}{2m}\nabla^2 + \hbar\omega_g + V_g(\mathbf{r}) \right) \hat{\Psi}_g(\mathbf{r}) + (g \to e) \right.$$

$$\hat{H}_{rad} = \hbar \sum_{\mathbf{k}} \omega_k \hat{a}^\dagger(\mathbf{k})\hat{a}(\mathbf{k})$$

Interactions

$$\hat{H}_{atom-rad} = -i\hbar \sum_{\mathbf{k}} \int d^3\mathbf{r}\, g(\mathbf{k}) \hat{\Psi}_e^\dagger(\mathbf{r}) \hat{a}(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{r}} \hat{\Psi}_g(\mathbf{r}) + h.c.$$

vacuum radiation

$$\hat{H}_{atom-pump} = \frac{\hbar\Omega_0}{2} e^{-i\omega_L t} \int d^3\mathbf{r}\, \hat{\Psi}_e^\dagger(\mathbf{r}) e^{i\mathbf{k}_L\cdot\mathbf{r}} \hat{\Psi}_g(\mathbf{r}) + h.c.$$

Classical laser field

$$\hat{H}_{atom-atom} = \frac{\hbar U_o}{2} \int d^3\mathbf{r}\, \hat{\Psi}_g^\dagger(\mathbf{r}) \hat{\Psi}_g^\dagger(\mathbf{r}) \hat{\Psi}_g(\mathbf{r}) \hat{\Psi}_g(\mathbf{r})$$

Ground state collisions

Fundamental Equations

$$\frac{\partial \hat{\Psi}_e(\mathbf{r})}{\partial t} = \frac{1}{i\hbar}[\hat{\Psi}_e(\mathbf{r}), \hat{H}] \;= i\left(\frac{\hbar}{2m}\nabla^2 - \frac{1}{\hbar}V_e(\mathbf{r}) - \omega_{eg}\right)\hat{\Psi}_e(\mathbf{r})$$

$$-\underbrace{\frac{i\Omega_0}{2}e^{-i\omega_L t}\hat{\Psi}_g(\mathbf{r})e^{i\mathbf{k}_L\cdot\mathbf{r}}}_{\text{absorb laser photon}} - \sum_{\mathbf{k}}\underbrace{g(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{r}}\hat{a}(\mathbf{k})\hat{\Psi}_g(\mathbf{r})}_{\text{absorb scattered photon}}$$

$$\frac{\partial \hat{\Psi}_g(\mathbf{r})}{\partial t} = \frac{1}{i\hbar}[\hat{\Psi}_g(\mathbf{r}), \hat{H}] = i\left(\frac{\hbar}{2m}\nabla^2 - \frac{1}{\hbar}V_g(\mathbf{r}) - U_0\hat{\Psi}_g^\dagger(\mathbf{r})\hat{\Psi}_g(\mathbf{r})\right)\hat{\Psi}_g(\mathbf{r})$$

$$-\frac{i\Omega_0^*}{2}e^{i\omega_L t}\hat{\Psi}_e(\mathbf{r})e^{-i\mathbf{k}_L\cdot\mathbf{r}} + \sum_{\mathbf{k}} g(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{r}}\hat{a}^\dagger(\mathbf{k})\hat{\Psi}_e(\mathbf{r})$$

$$\frac{\partial \hat{a}(\mathbf{k})}{\partial t} = \frac{1}{i\hbar}[\hat{a}(\mathbf{k}), \hat{H}] \;= -i\omega_k\hat{a}(\mathbf{k}) + \int d^3\mathbf{r}\, g(\mathbf{k})\hat{\Psi}_g^\dagger(\mathbf{r})e^{-i\mathbf{k}\cdot\mathbf{r}}\hat{\Psi}_e(\mathbf{r})$$

Assume $\Delta = \omega_L - \omega_{eg} \gg \gamma$ Einstein A coefficient

Adiabatically eliminate internal upper state

Gives for **scattered photon** (in slowly varying form $\hat{a}'(\mathbf{k}) = \hat{a}(\mathbf{k})e^{i\omega_L t}$)

$$\frac{\partial \hat{a}'(\mathbf{k})}{\partial t} = i\Delta_L(k)\,\hat{a}'(\mathbf{k}) + \frac{\Omega_0}{2\Delta} g(\mathbf{k}) \int d^3\mathbf{r}\, e^{i(\mathbf{k}_L - \mathbf{k})\cdot\mathbf{r}} \left(\hat{\Psi}_g^\dagger(\mathbf{r})\hat{\Psi}_g(\mathbf{r})\right) + O(g^2)$$

$\Delta_L(k) = \omega_L - \omega_k$

laser couples to ground state density grating

And for **ground state** field

$$\frac{\partial \hat{\Psi}_g(\mathbf{r})}{\partial t} = i\left[\frac{\hbar}{2m}\nabla^2 - \frac{1}{\hbar}V_g(\mathbf{r}) - \frac{|\Omega_0|^2}{4\Delta} - U_0 \hat{\Psi}_g^\dagger(\mathbf{r})\hat{\Psi}_g(\mathbf{k}) + \frac{1}{2\Delta}\sum_{\mathbf{k}} \left(i\Omega_0^* g(\mathbf{k}) e^{-i(\mathbf{k}_L - \mathbf{k})\cdot\mathbf{r}}\hat{a}'(\mathbf{k}) + h.c.\right)\right]\hat{\Psi}_g(\mathbf{r}) + O(g^2)$$

light shift

light scattering terms

scatter ***into*** laser mode

out of laser mode

second order scattering

Classical field representation

Wigner function treatment

$$\hat{\Psi}_g(\mathbf{r},t) \rightarrow \sqrt{N_0}\Psi(\mathbf{r},t) +$$

Half particle of noise on each mode initially

$$\hat{a}'(\mathbf{k},t) \rightarrow \alpha(\mathbf{k},t)$$

(neglect vacuum radiation noise)

Find expression for scattered photon amplitude

Take **matter** 'wavefunction' to momentum representation

$$\Phi(\mathbf{k}) = \frac{1}{\sqrt{V}} \int d\mathbf{r} \Psi(\mathbf{r}) e^{-i\mathbf{k}\cdot\mathbf{r}} \quad ; \quad \Psi(\mathbf{r}) = \frac{1}{\sqrt{V}} \sum_{\mathbf{k}} \Phi(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{r}}$$

Dominant behaviour of matter wave in momentum space…

$$\frac{\partial \Phi(\mathbf{k})}{\partial t} = -i \left[\frac{\hbar k^2}{2m} + \frac{|\Omega_0|^2}{4\Delta} \right] \Phi(\mathbf{k}) + \ldots$$

So introduce slowly varying wave function (in momentum space)

$$\Phi'(\mathbf{k},t) = \Phi(\mathbf{k},t) e^{i\left(\frac{\hbar k^2}{2m} + \frac{|\Omega_0|^2}{4\Delta}\right)t}$$

Equation for **scattered photon** becomes

$$\frac{\partial \alpha(\mathbf{k})}{\partial t} = i\Delta_L(k)\,\alpha(\mathbf{k}) + \frac{\Omega_0 g(\mathbf{k}) N_0}{2\Delta}\sum_{\mathbf{k}'} \Phi'(\mathbf{k}')\Phi'^*(\mathbf{k}'+\mathbf{k}_L-\mathbf{k})e^{i\Delta^R_{\mathbf{k}',\mathbf{k}_L-\mathbf{k}}t}$$

where $\hbar\Delta^R_{\mathbf{k}',\mathbf{k}_L-\mathbf{k}} = E(\mathbf{k}'+\mathbf{k}_L-\mathbf{k}) - E(\mathbf{k}')$ $\rightarrow$ $\frac{\hbar k'^2}{2m}$

Integrate **photon equation** (ignore radiation noise)

$$\alpha(\mathbf{k},t) = \frac{\Omega_0 g(\mathbf{k}) N_0}{2\Delta}\int_0^t ds e^{i(\Delta_L(k)-\Delta^R)s}\sum_{\mathbf{k}'} \Phi'(\mathbf{k}',t-s)\Phi'^*(\mathbf{k}'+\mathbf{k}_L-\mathbf{k},t-s)$$

$\Delta_L(k)$ very fast, photons stay in system for time L/c

Φ' condensate, slow

Make **Markov** approximation, to give **photon amplitude** at time t

$$\alpha(\mathbf{k},t) = \frac{\pi\Omega_0 g(\mathbf{k}) N_0}{\Delta}\sum_{\mathbf{k}'} \delta\left(\Delta^R - \Delta_L(k)\right)\Phi(\mathbf{k}',t)\Phi^*(\mathbf{k}'+\mathbf{k}_L-\mathbf{k},t)$$

Finally, scattered photon amplitude

$$\alpha(\mathbf{k},t) = \frac{\pi\Omega_0 g(\mathbf{k}) N_0}{cdk\Delta}\mathcal{F}(k=k_L)\tilde{\rho}(\mathbf{k}-\mathbf{k}_L,t)$$

where $\tilde{\rho}(\mathbf{k}-\mathbf{k}_L,t) = \int d^3\mathbf{r}|\Psi(\mathbf{r},\mathbf{t})|^2 e^{-i(\mathbf{k}-\mathbf{k}_L)\cdot\mathbf{r}}$

and $\mathcal{F}(k=k_L)$ is the discrete version of the delta function

Then we get Gross-Pitaevskii Superadiance equation

$$\frac{\partial\Psi(\mathbf{r},t)}{\partial t} = i\left[\frac{\hbar}{2m}\nabla^2 - \frac{|\Omega_0|^2}{4\Delta} - U_0 N_0|\Psi(\mathbf{r},t)|^2 + i\sum_{\mathbf{k}} G(\mathbf{k})\,\mathcal{F}(k=k_L)\left(e^{-i(\mathbf{k}_L-\mathbf{k})\cdot\mathbf{r}}\tilde{\rho}(\mathbf{k}-\mathbf{k}_L,t) - c.c.\right)\right]\Psi(\mathbf{r},t)$$

$$G(\mathbf{k}) = \frac{|\Omega_0|^2\gamma}{4\Delta^2} N_0\left(\frac{dk^2}{4\pi k_L^2}\right)\frac{3}{2}|\hat{\mathbf{e}}_d\cdot\hat{\mathbf{e}}_{\mathbf{k}}^*|^2$$

Scattering rate (per mode) below threshold

Total photon scattering rate from one atom

Results

- Release condensate from trap, then apply laser
- In our simulations

units of time $t_0 = \frac{1}{\omega_{trap}}$; *length* $x_0 = \sqrt{\frac{\hbar}{2m\omega_{trap}}}$

- Most of our simulations in 2D (phenomena is primarily 2D)

- Require three dimensionless parameters

Condensate nonlinearity $C = \frac{U_0 N_0}{\hbar\omega_{trap} x_0^3}$

N atom gain $G = \frac{G(\mathbf{k})}{\omega_{trap}}$

Laser wavenumber $\mathbf{k}_L$

Main Themes

- **Low laser power**

  Linear condensate

  Gain and decoherence rates
  Effect of condensate shape

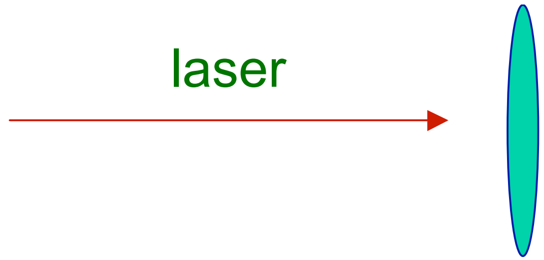
  Nonlinear condensate

- **High laser power**

- **Coherent matter wave amplifier**

Low laser power

$G \lesssim \omega_{recoil}$



$C=0$

$k_L = 6$

Superradiant Rayleigh Scattering

Momentum Space Evolution

$k_0 = 6, \; G = 36, \; \lambda = 0.1$

J. S. Douglas and R. J. Ballagh

University of Otago

Dunedin, New Zealand

June 2004

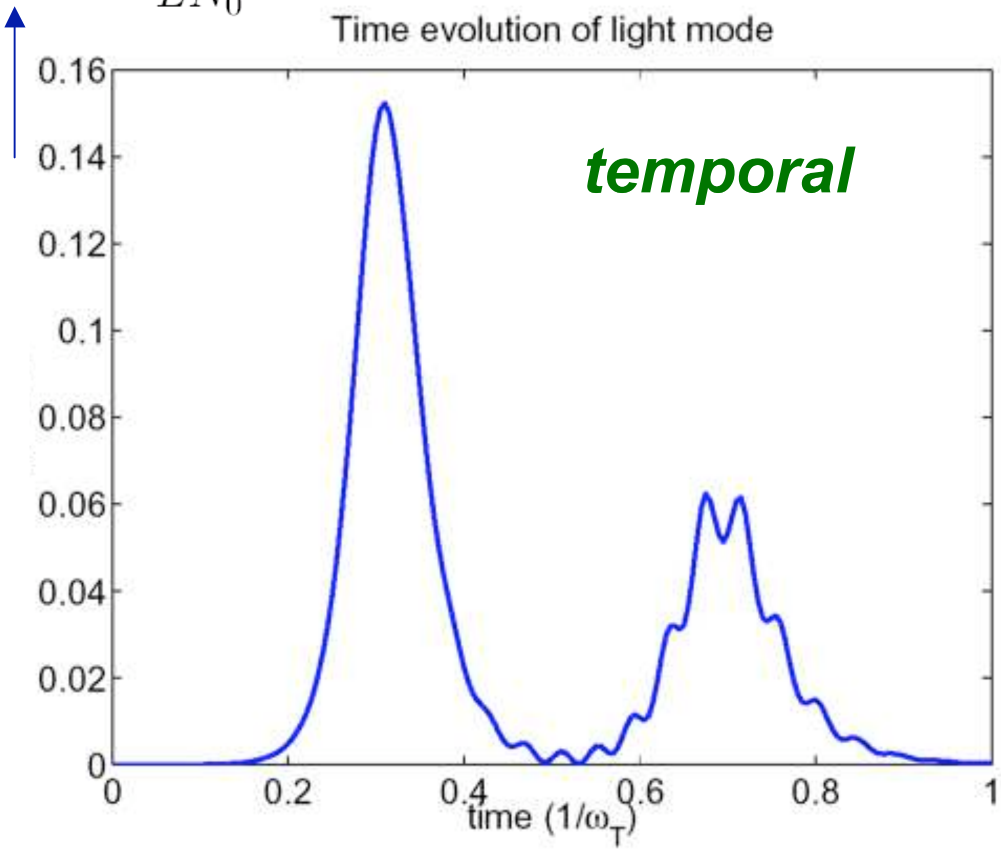
Behaviour of scattered radiation

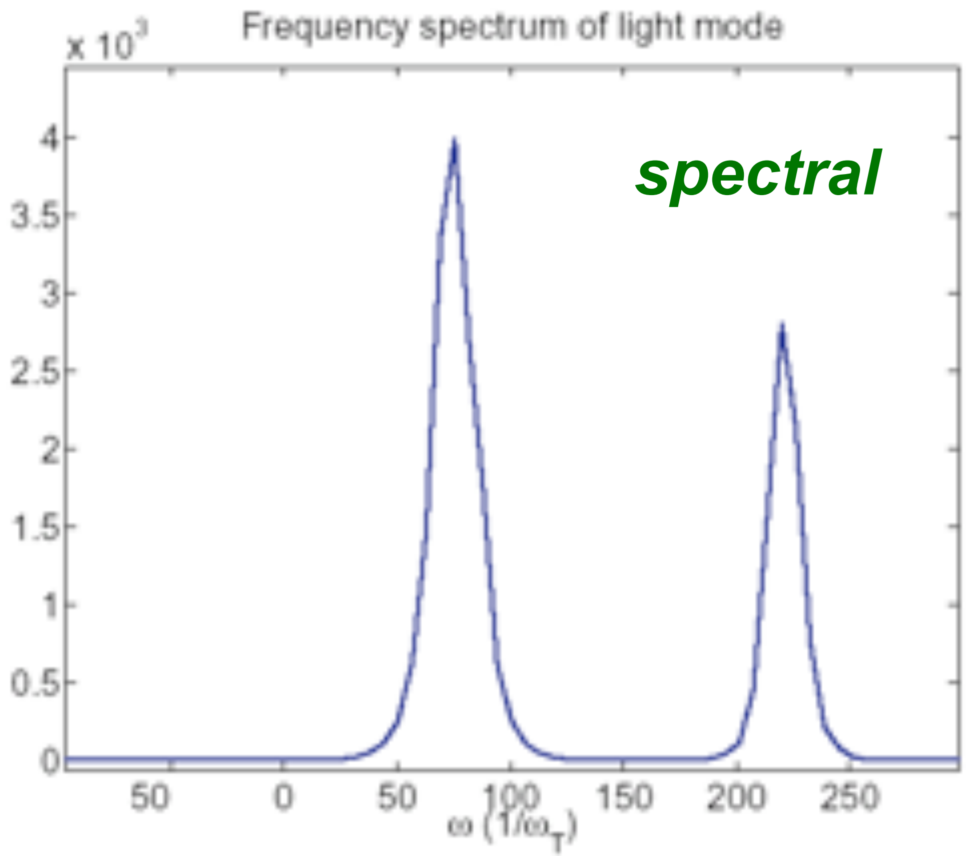
Number of photons scattered into mode $\mathbf{k}$ per unit time is

$$|a(\mathbf{k},t)|^2 \frac{c}{L} = G(\mathbf{k})\, N_0 \left[\tilde{\rho}(\mathbf{k}-\mathbf{k}_L, t)\right]^2$$

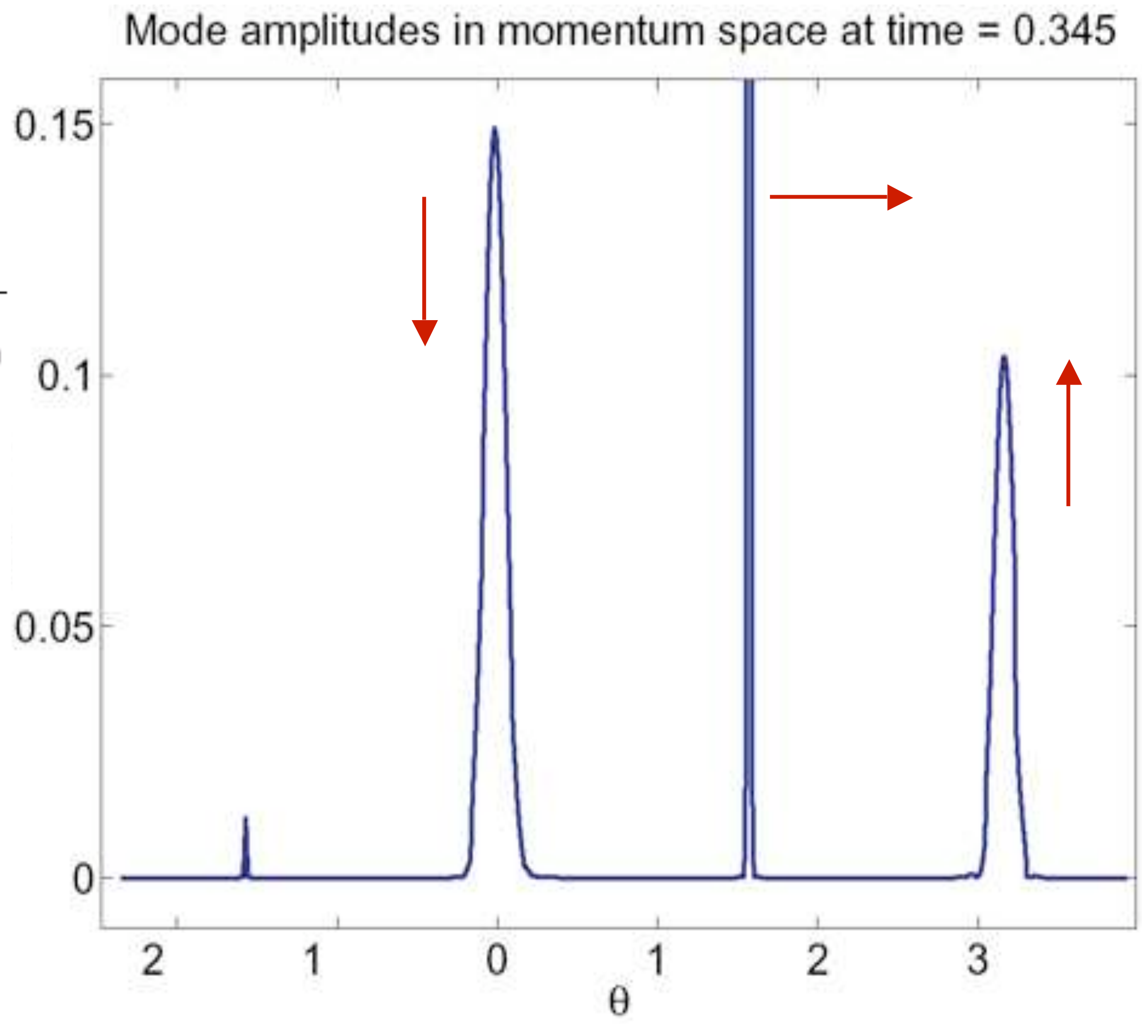
Mode along z direction

$|a(\mathbf{k},t)|^2 \frac{c}{LN_0}$





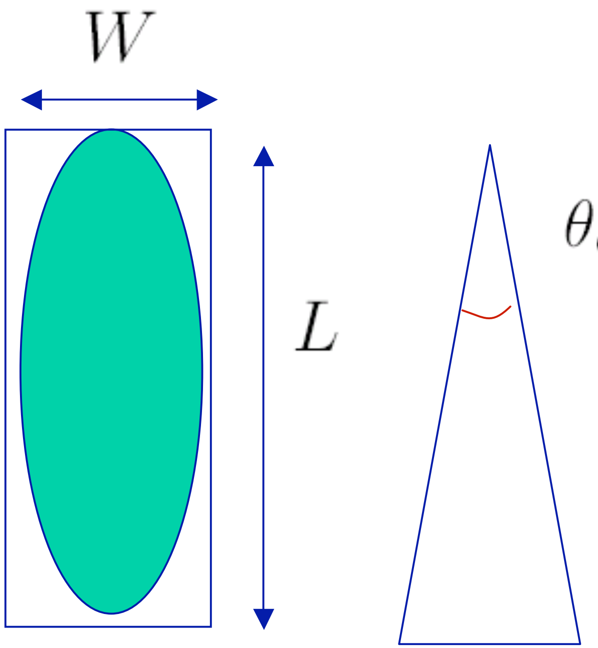
Angular distribution of scattered light



$$|a(\mathbf{k},t)|^2 \frac{c}{LN_0}$$

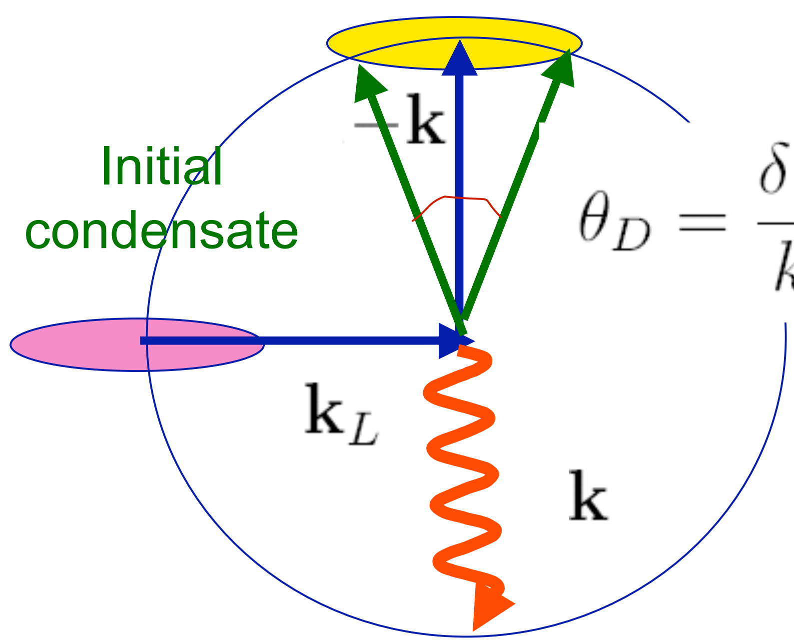
Angular quantities determining radiation angular width

Coordinate space



$$\theta_G = \frac{W}{L}$$

Momentum space



$$\theta_D = \frac{\delta k_x}{k_L} \approx \frac{\lambda_L}{R}$$

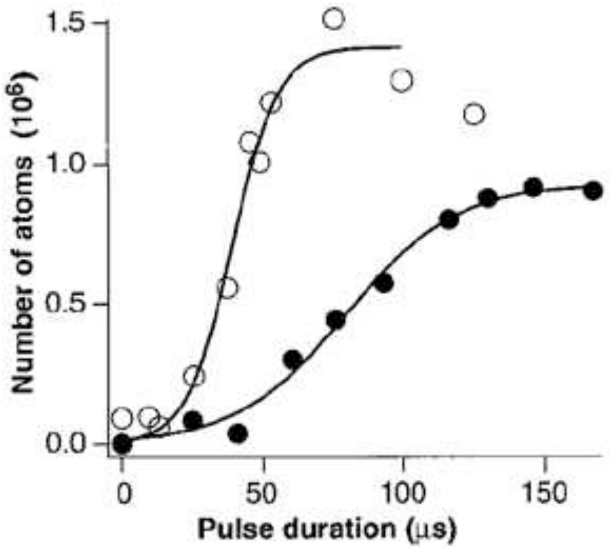
$\frac{\theta_G}{\theta_D} < 1$ Angular spread of radiation is θ_G

$\frac{\theta_G}{\theta_D} > 1$ Angular spread of radiation is determined by θ_D

(but may split into several modes)

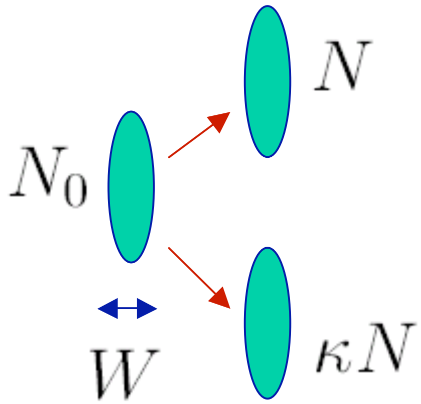
Temporal behaviour of scattered atoms

Ketterle experimental result



Suggests gain/loss behaviour

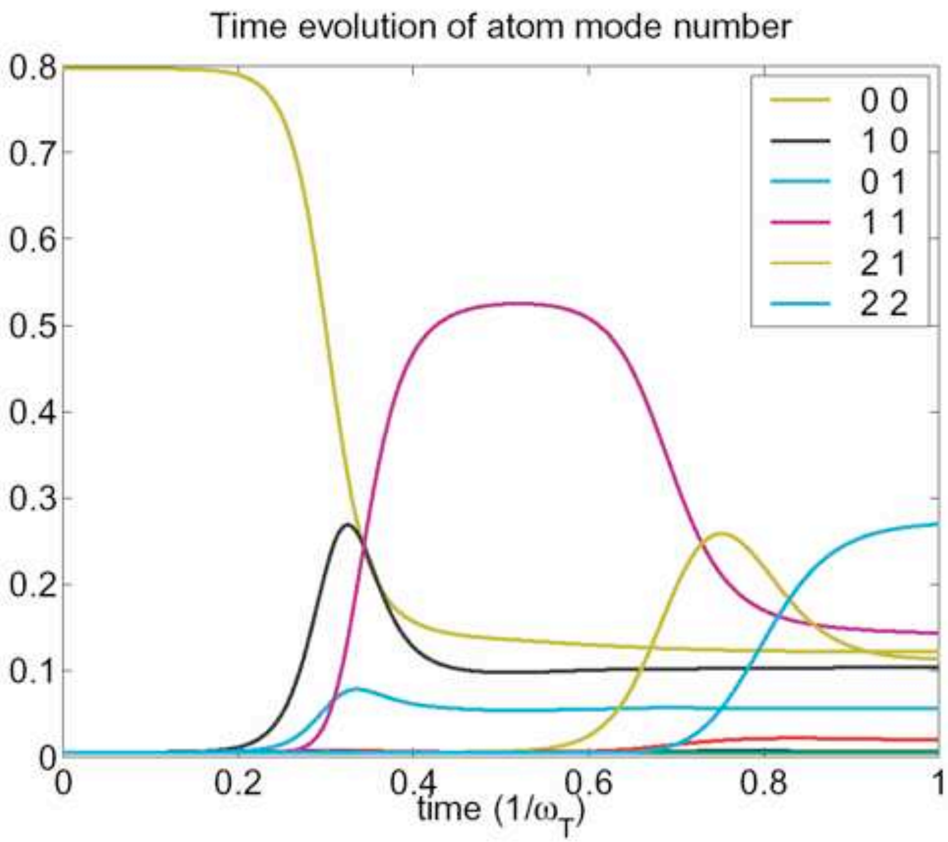
$$\frac{dN}{dt} = 2GN_0 - \Gamma N$$

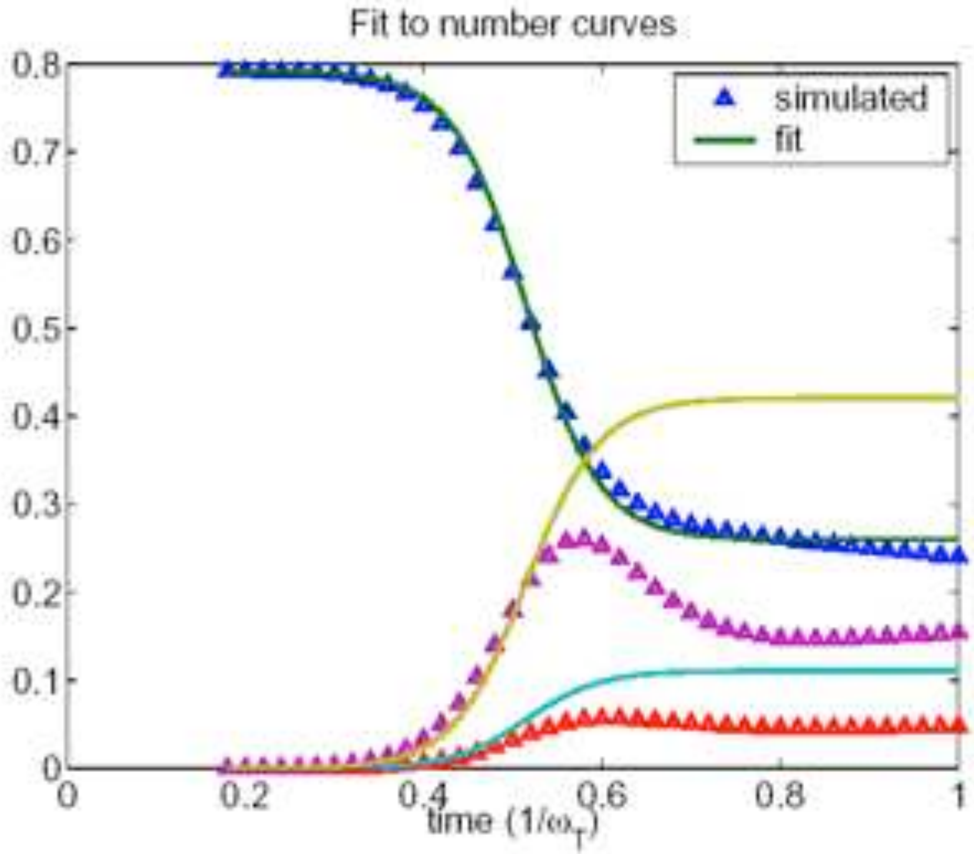


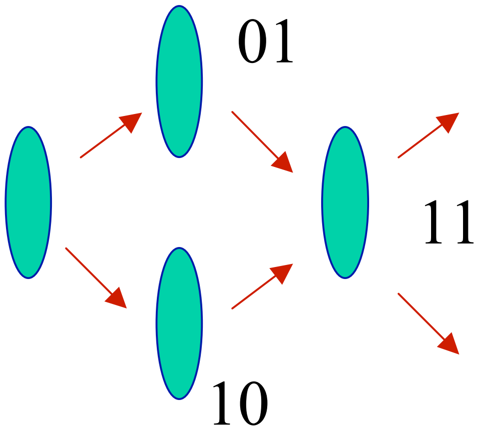
$$N_{tot} = N_0 + (1+\kappa)N$$

Suggested loss mechanism: *due to separation of packets*

implies $\Gamma \approx \frac{2v}{W}$







Fitted values	Estimated
$G = 37.8$	$G = 36$
$\Gamma = 9.9$	$\Gamma = 3$

Effect of condensate shape (1)

Low laser power

$C=0$

Superradiant Rayleigh Scattering

Momentum Space Evolution

$k_0 = 6, \; G = 36, \; \lambda = 1$

J. S. Douglas and R. J. Ballagh

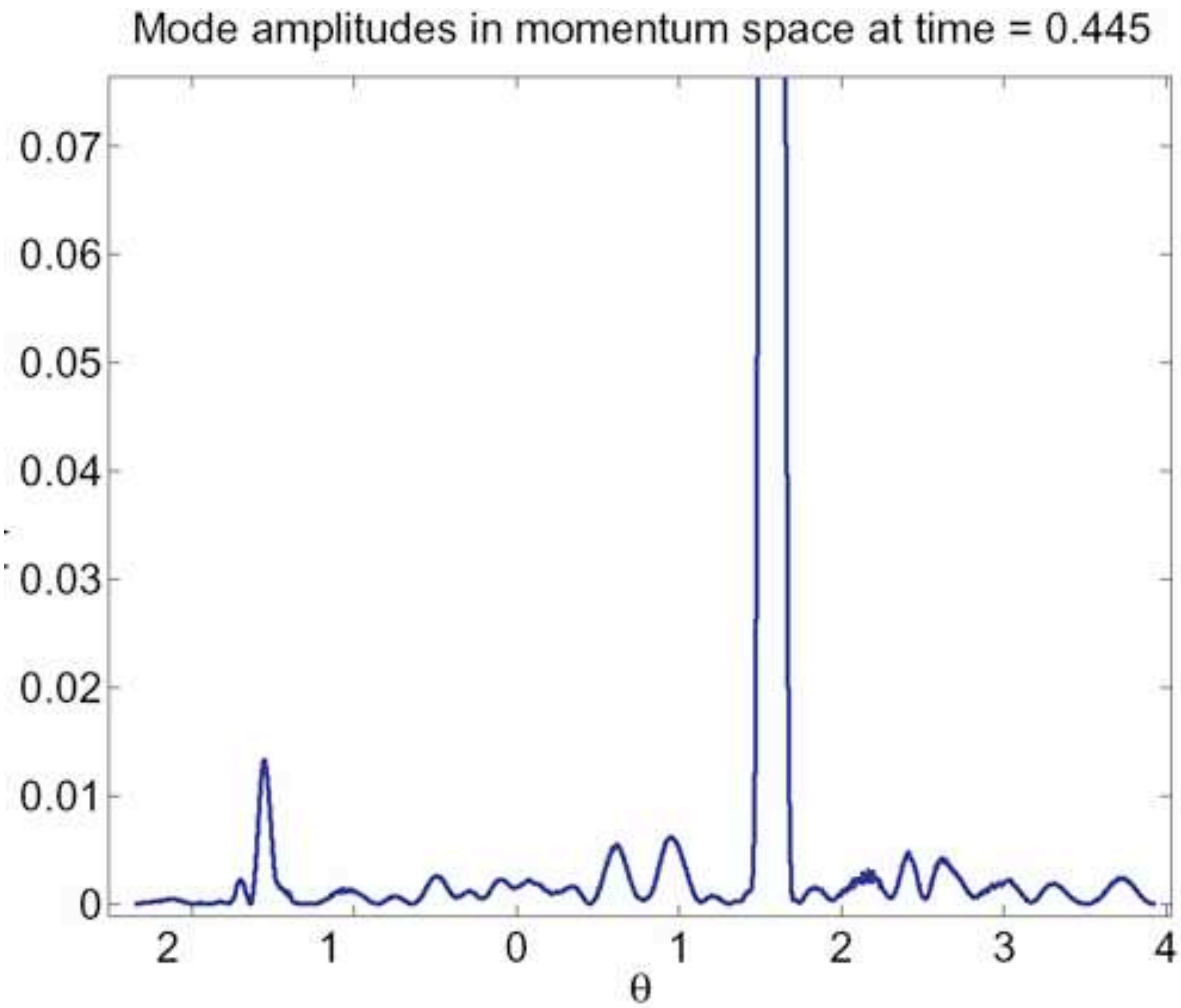
University of Otago

Dunedin, New Zealand

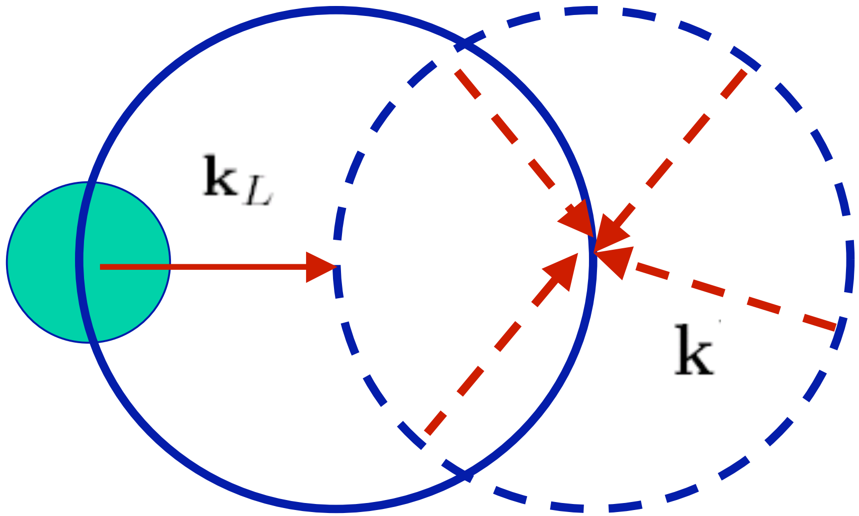
June 2004

Angular distribution of scattered radiation

$$|a(\mathbf{k},t)|^2 \frac{c}{LN_0}$$



Enhancement of forward recoil in spherical geometry



Effect of condensate shape (2)

Low laser power

$C=0$

Superradiant Rayleigh Scattering

Momentum Space Evolution

$k_0 = 6,\ G = 36,\ \lambda = 10$

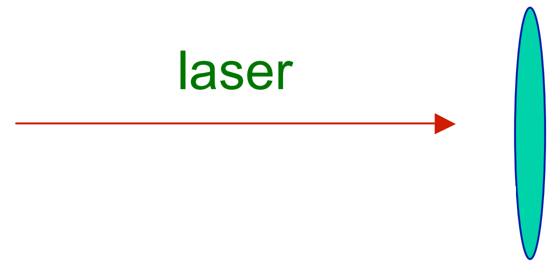
J. S. Douglas and R. J. Ballagh

UNIVERSITY of OTAGO

Dunedin, New Zealand

June 2004

Effect of condensate nonlinearity



Low laser power

C=5000

Superradiant Rayleigh Scattering

Momentum Space Evolution

$k_0 = 6,\ G = 36,\ \lambda = 0.1,\ C = 5000$

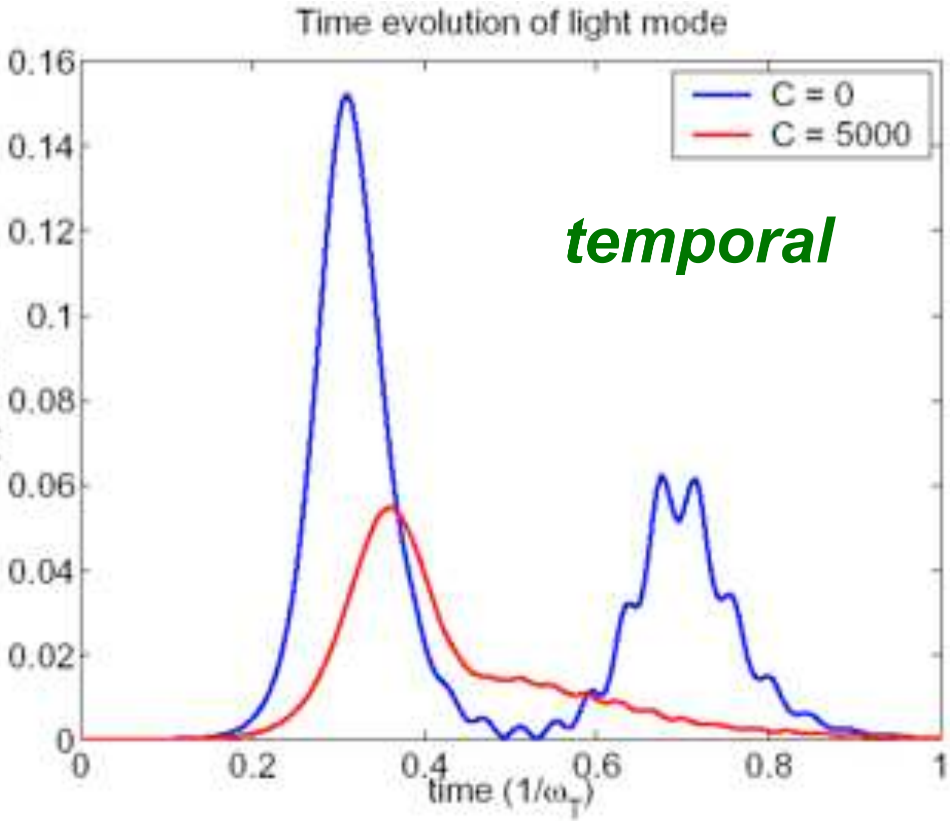
J. S. Douglas and R. J. Ballagh

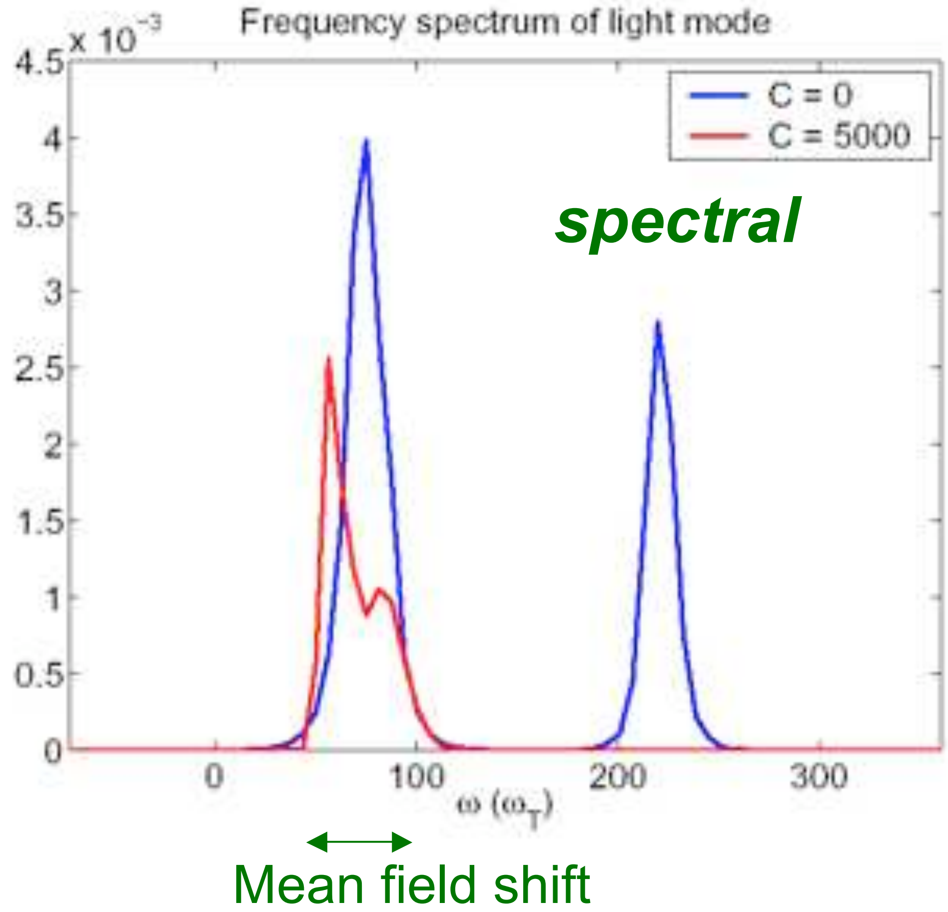
UNIVERSITY of OTAGO

Dunedin, New Zealand

July 2004

Behaviour of scattered radiation

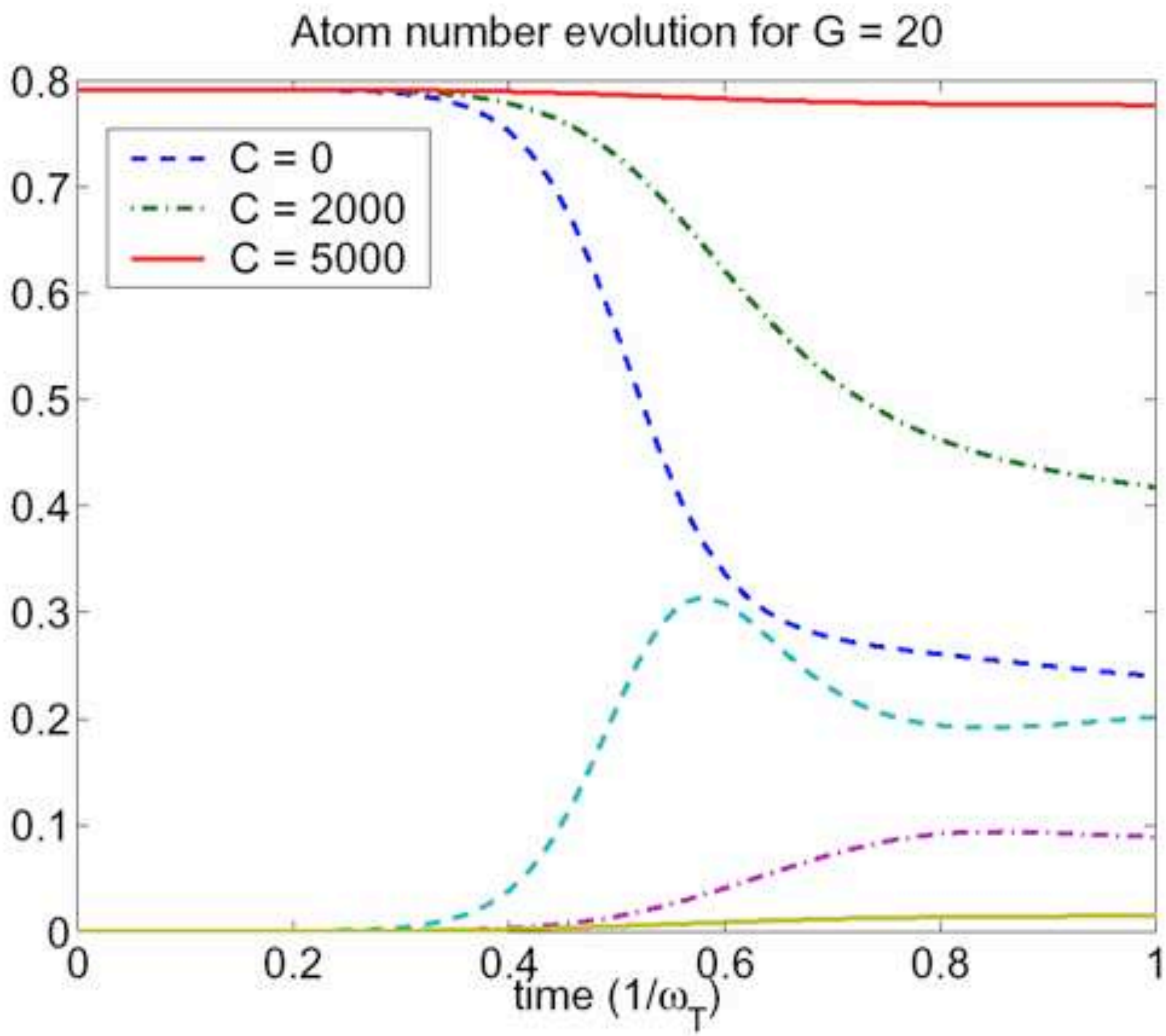




Recall Bragg resonance condition

$$\omega_k = \omega_L - \left(\omega_R + \frac{4\mu}{7\hbar} \right)$$

Superradiance suppressed by condensate nonlinearity

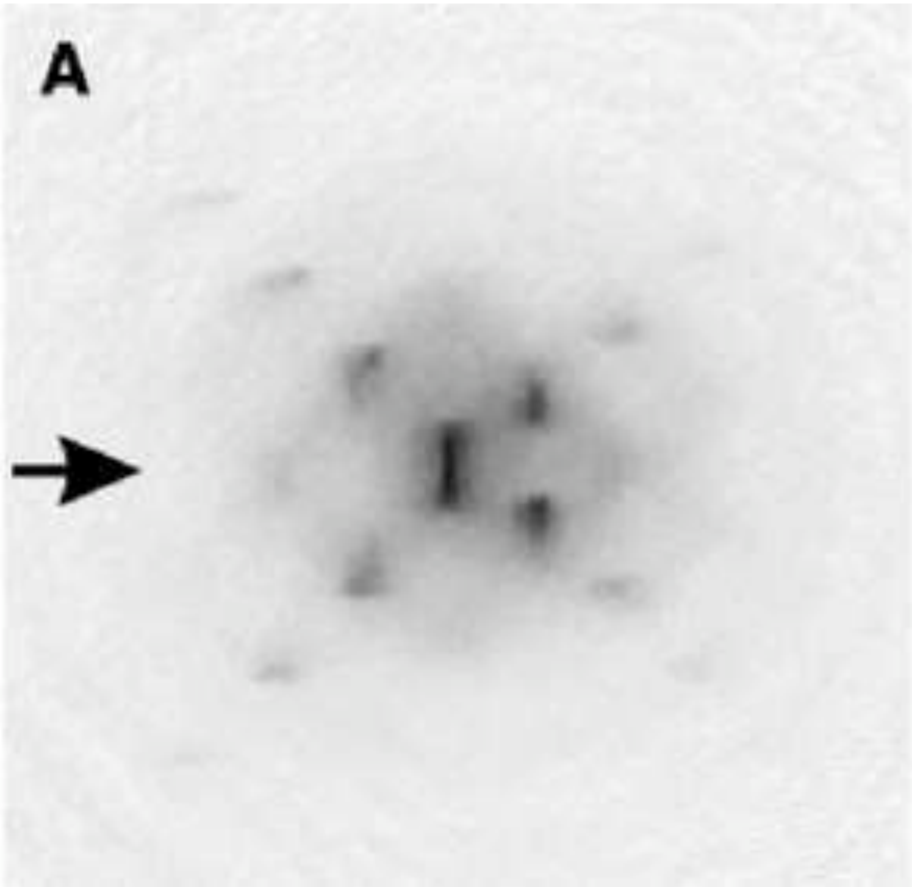


Decoherence rate increases with nonlinearity

C	Γ
0	*12*
2000	*21.5*

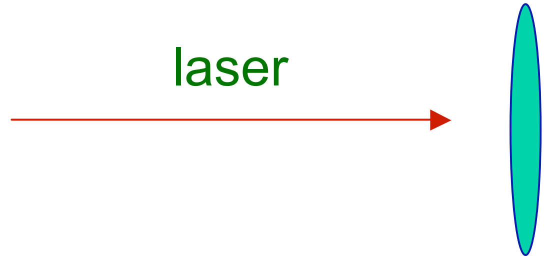
High laser power

Ketterle experiment (Science, 300,475,2003)



High laser power

C=5000



$$G \gg \omega_{recoil}$$

Superradiant Rayleigh Scattering

Momentum Space Evolution

$k_0 = 6,\ G = 13000,\ \lambda = 0.1,\ C = 5000$

J. S. Douglas and R. J. Ballagh

University of Otago

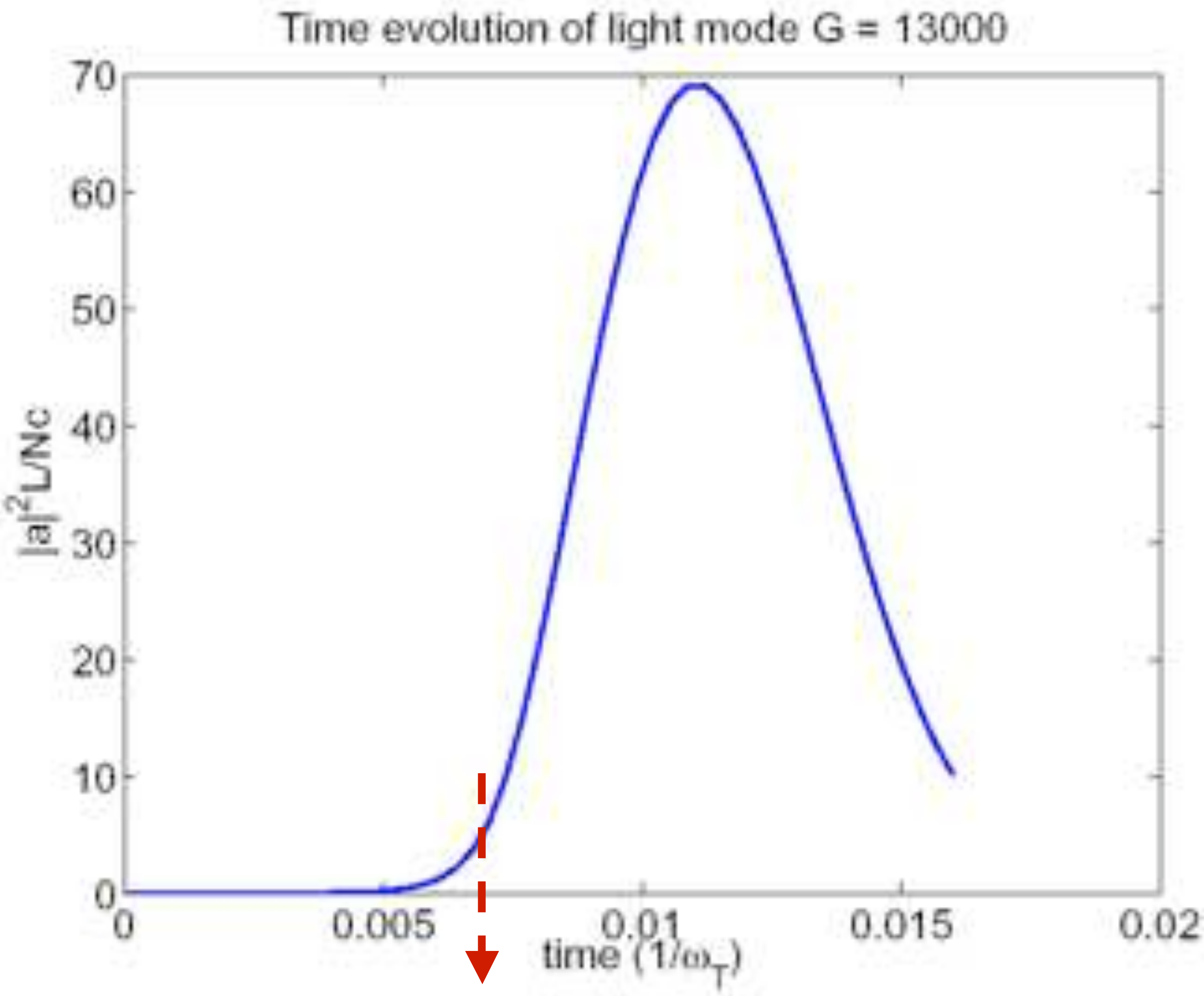
Dunedin, New Zealand

July 2004

Behaviour of scattered radiation

temporal

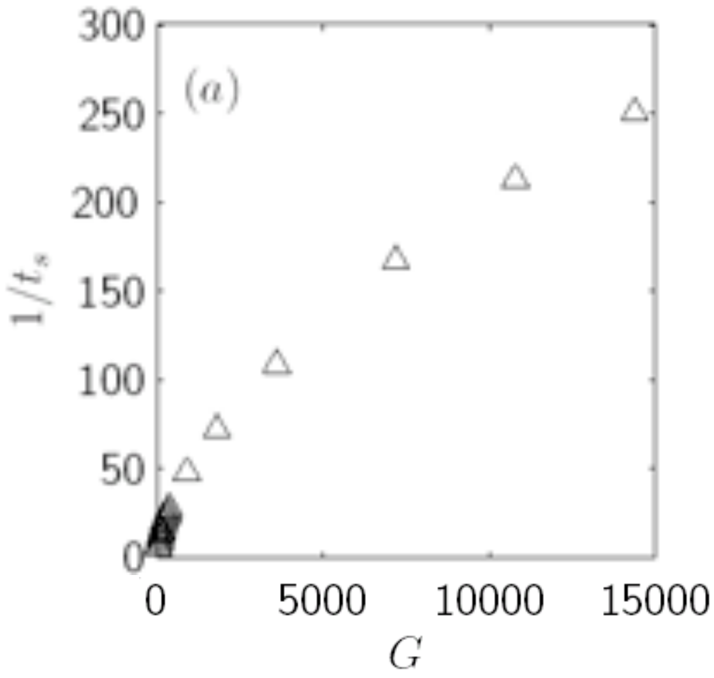
spectral



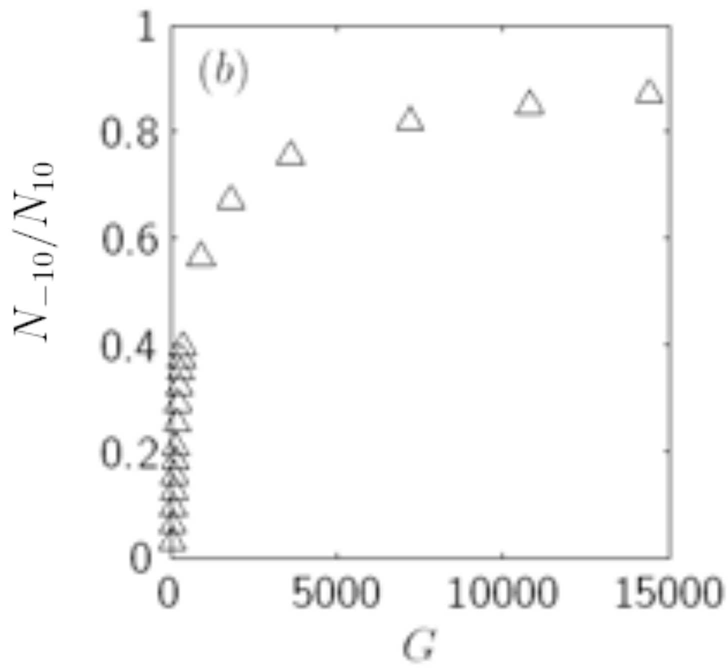
Laser turned off

Features of behaviour with high laser power

Inverse time to scatter 10% of condensate

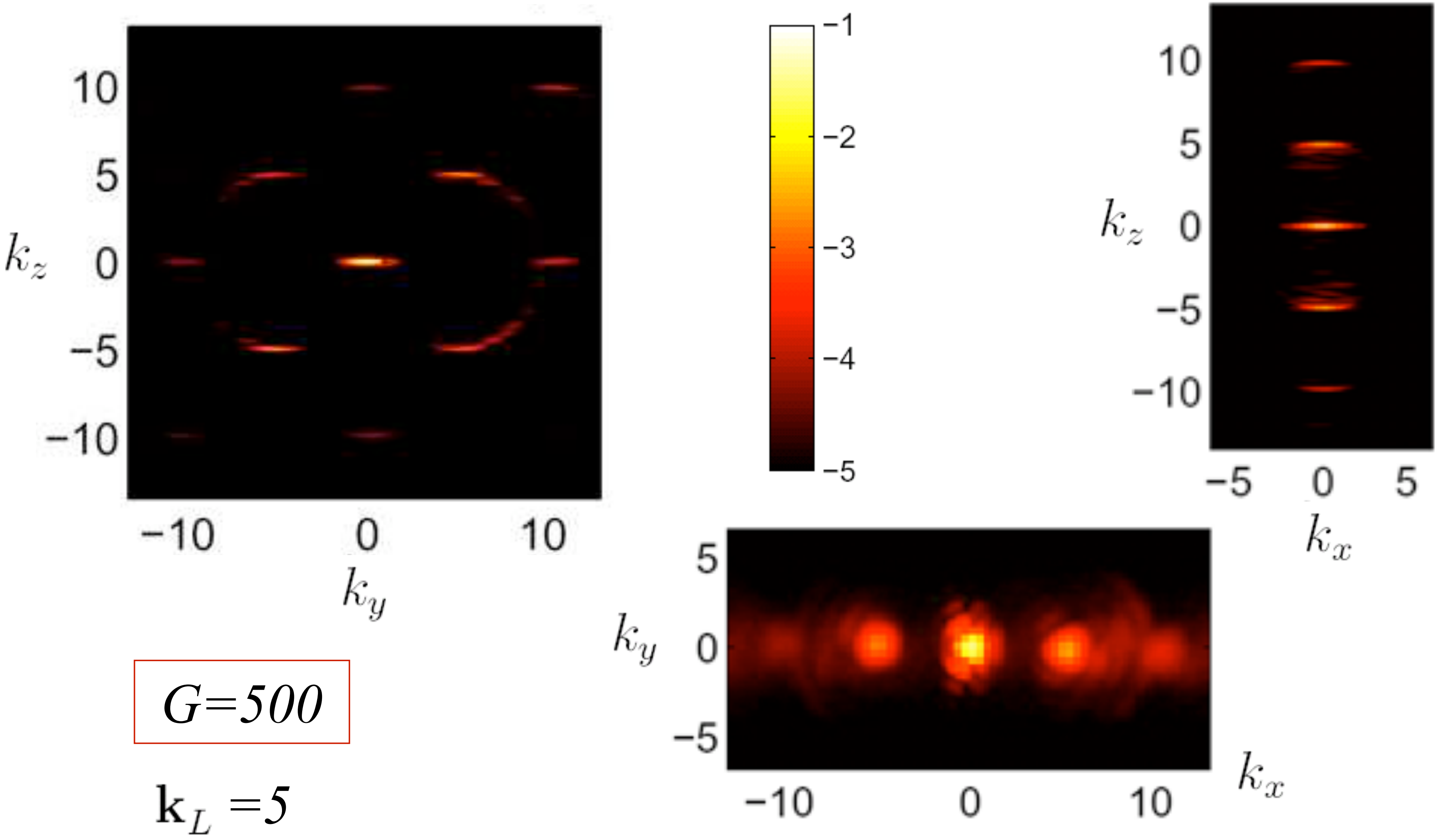


Early forward/backward asymmetry



Three dimensional simulations

Condensate: *summed momentum populations*

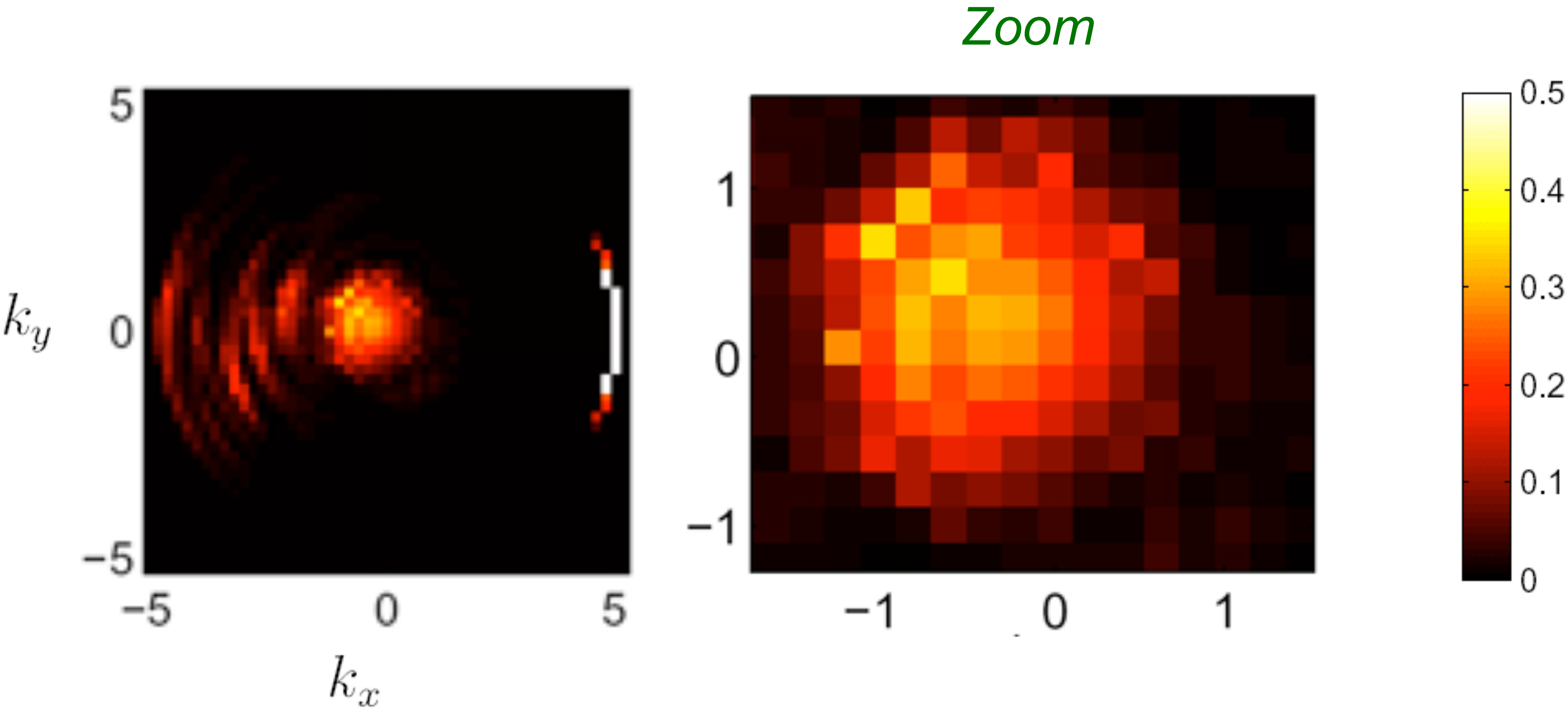


G=500

$\mathbf{k}_L$ =5

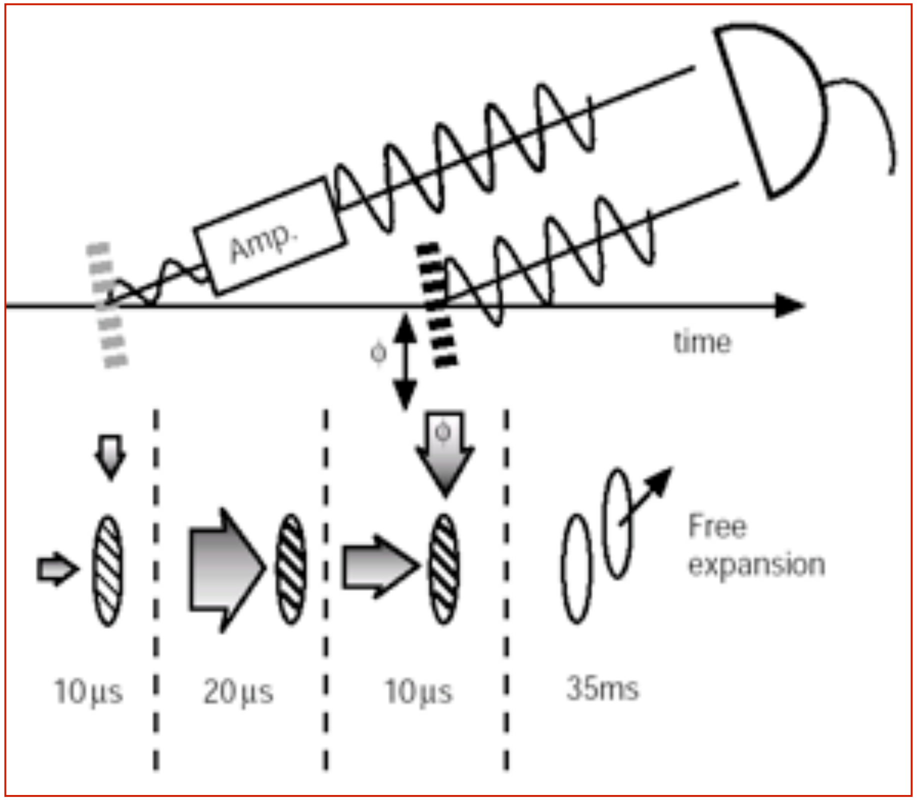
Three dimensional simulations

Scattered light

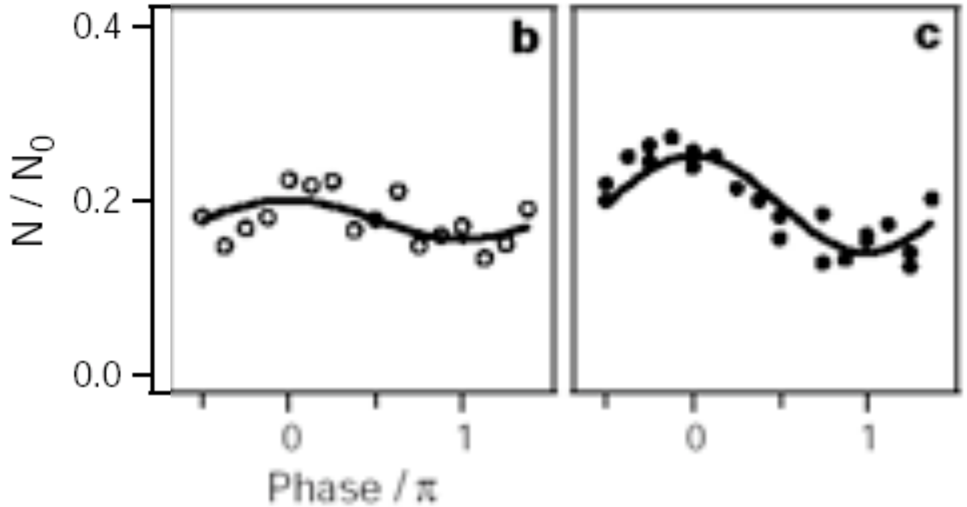


Coherent Matter wave amplifier

Ketterle: Nature,402,641,(1999);PRL,85,4225,(2000)
Kozuma et al: Science,286,17,(1999)



Number of output atoms



Small seed, NO amplifier

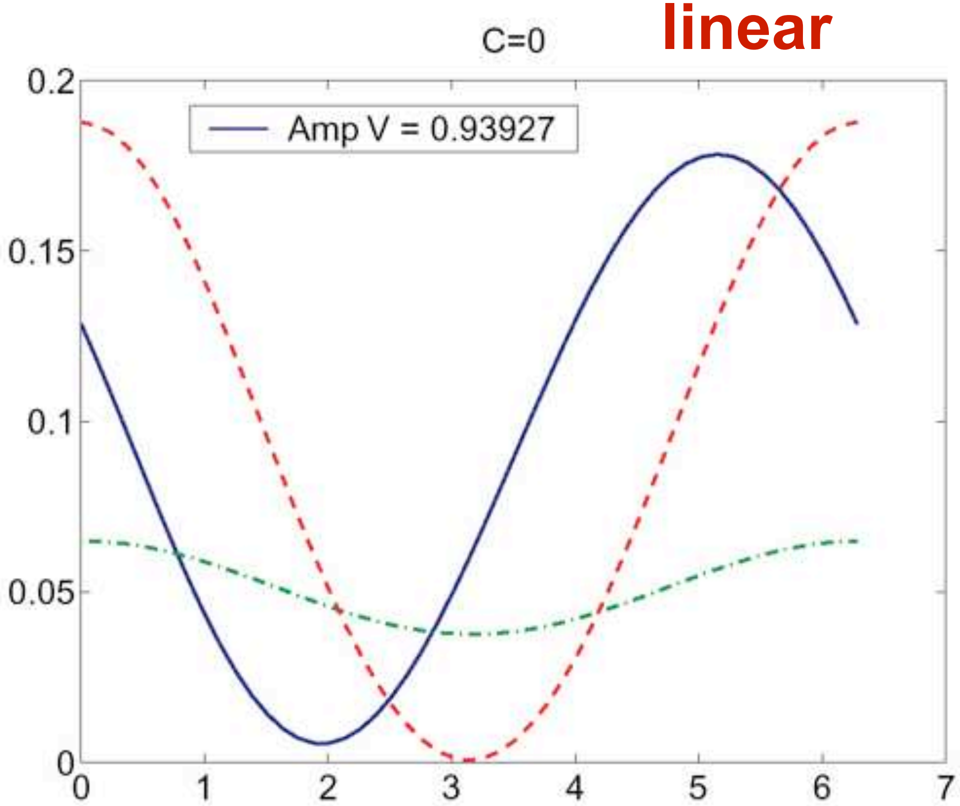
Small seed, PLUS amplifier

Numerical simulation

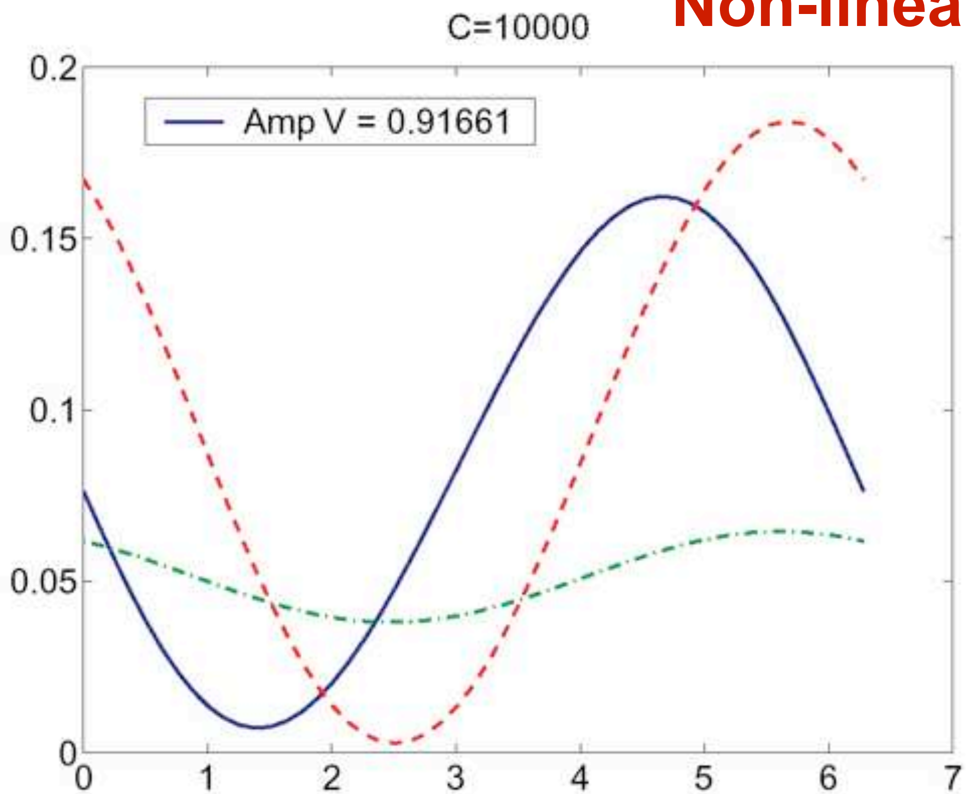
- seed 0.1%
- superradiant scatter 10%
- Final Bragg – scatter 10%
  (on original condensate)

Fraction of atoms output

linear



Non-linear



Phase

Kozuma experiment

(NIST)

Somewhat similar to Ketterle, **but better!**

- Mach Zehnder interferometer
- Longer superadiant pulse, 50% scattered
- Observe fringe visibility of 71%
- Two effects
  - (i) change of shape of scattered condensate
  - (ii) nonlinearity

Summary

- 2D and 3D spatio-temporal treatment of superadiance in BEC by classical field method

- Qualitative agreement with most experimental features

- Insight into effects of nonlinearity, including decoherence effects