Theory of Molecules and pairs in Degenerate Quantum Gases

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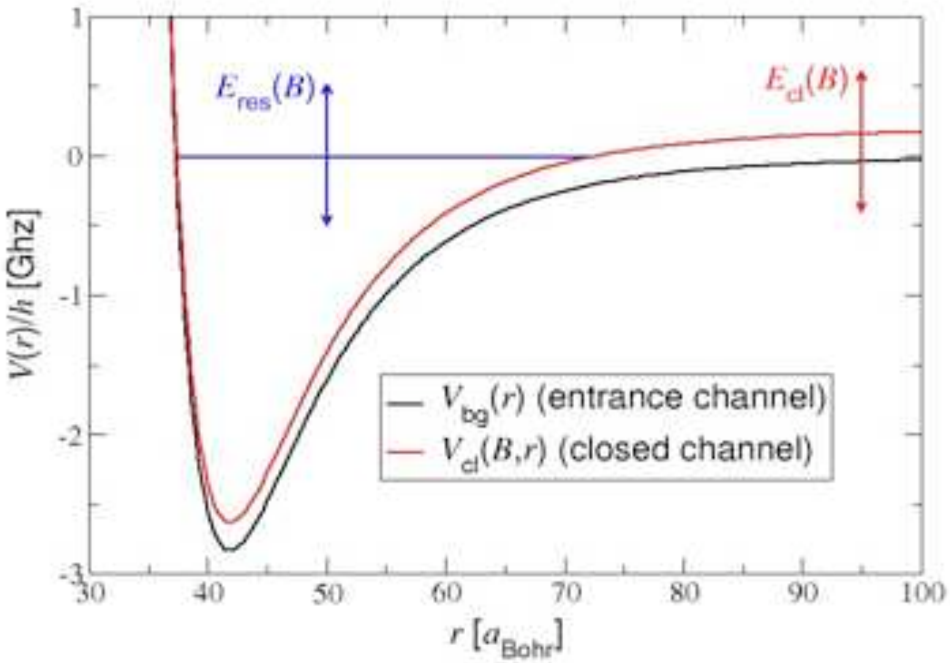
Outline

1. **Feshbach Resonances**
2. **Types of Resonance**
3. **Atom-Molecule Coherences**
4. **BEC Molecules to BCS Pairs**

Feshbach resonances



$$H_{2B}(B) = -\frac{\hbar^2}{m}\nabla^2 \left[|bg\rangle\langle bg| + |cl\rangle\langle cl|\right] + V_{2B}(B)$$

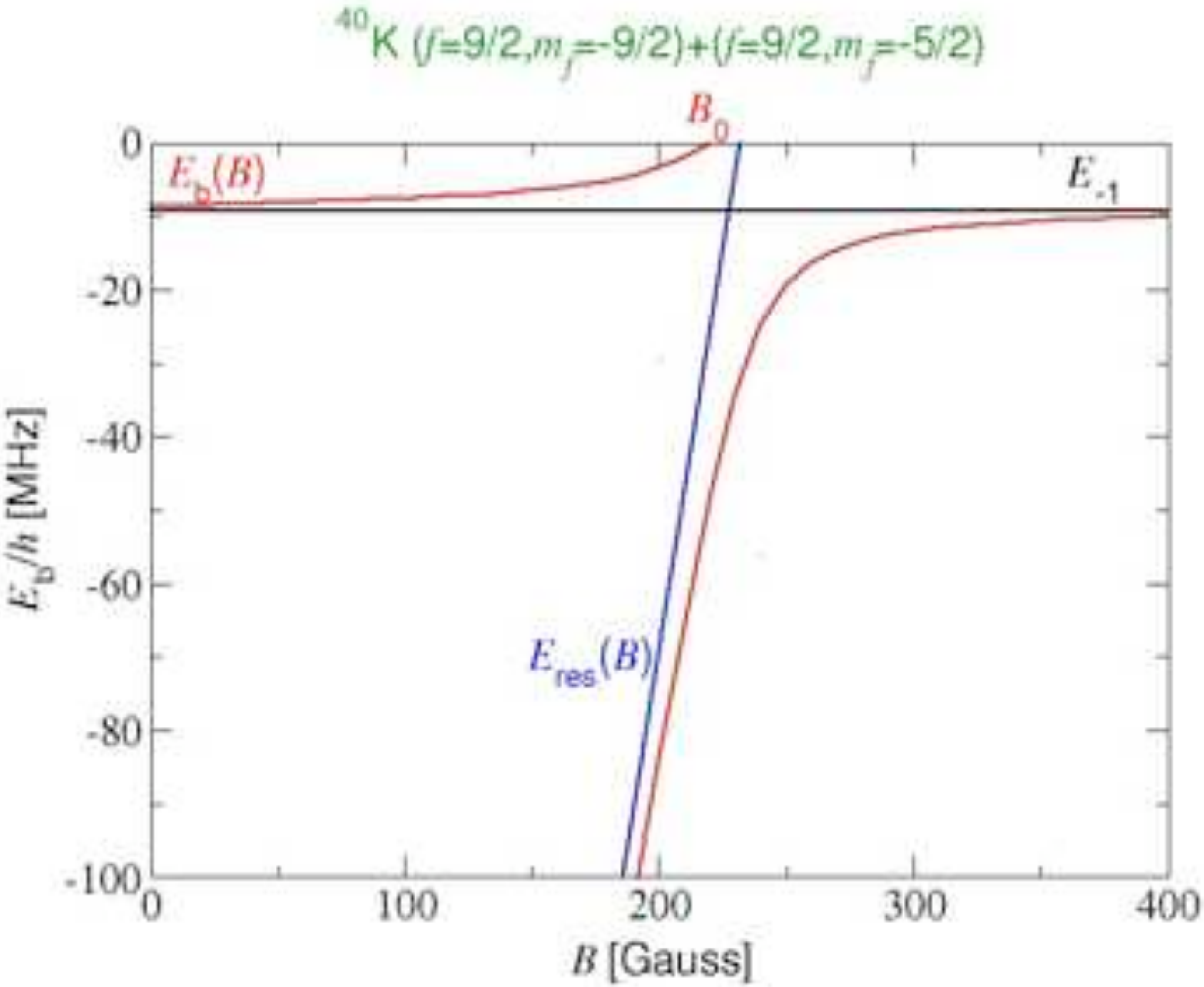
$$V_{2B}(B) = |bg\rangle V_{bg}(r)\langle bg| + |bg\rangle W(r)\langle cl|$$

$$+|cl\rangle W(r)\langle bg| + |cl\rangle V_{cl}(B,r)\langle cl|$$

$$-\frac{\hbar^2}{m}\nabla^2 + V_{cl}(B) \rightarrow |\phi_{res}\rangle E_{res}(B)\langle\phi_{res}|$$

Single resonance state approximation

Entrance channel dominated resonance



Parameters of Feshbach Resonances

The background scattering length a_{bg}

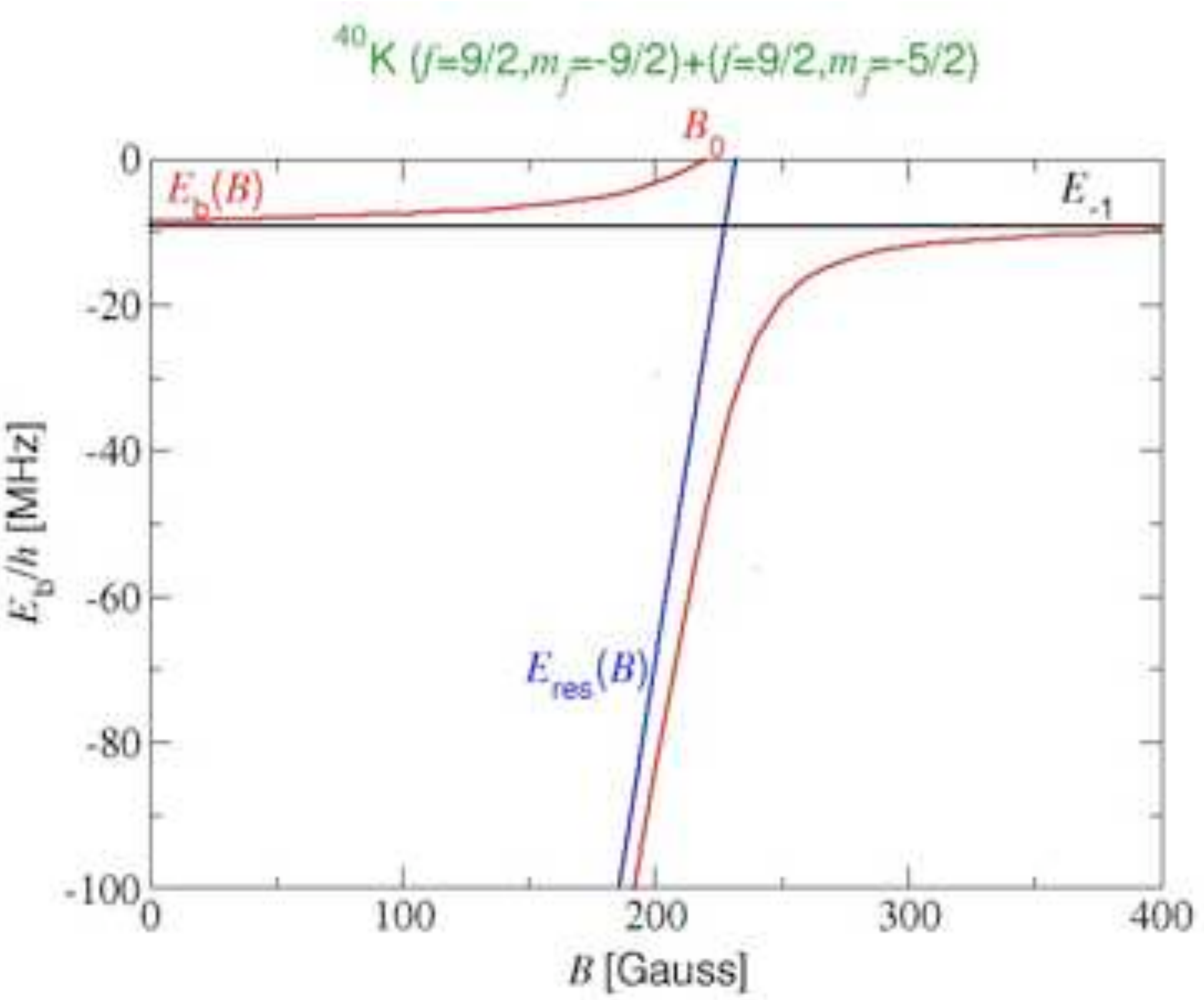
The resonance width ΔB

The magnetic moment of the resonance level dE_{res}/dB

The shift between the position of the zero energy resonance B_0 and the field where the resonance level crosses the dissociation threshold of the entrance channel B_{res}

The van der Waals dispersion coefficient C_6 (or binding energy of the highest excited bound state of the background scattering potential E)

Entrance channel dominated resonance



Formal Feshbach Result

$$a = a_{\mathrm{bg}} - \frac{\frac{m}{4\pi\hbar^2}(2\pi\hbar)^3 \left|\langle \phi_{\mathrm{res}}|W|\phi_0^{(+)}\rangle\right|^2}{E_{\mathrm{res}}(B) + \langle \phi_{\mathrm{res}}|W G_{\mathrm{bg}}(0) W|\phi_{\mathrm{res}}\rangle}.$$

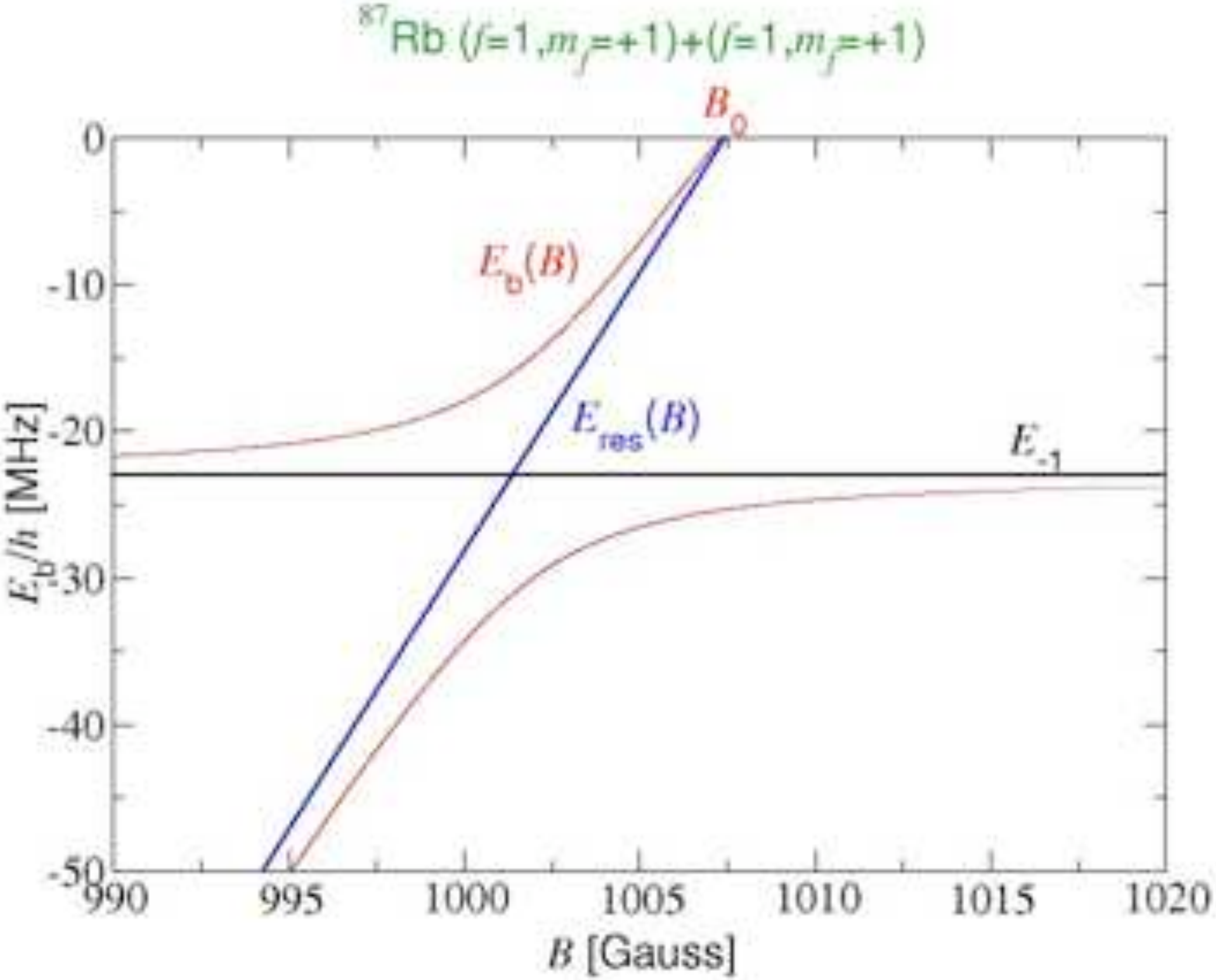
$$E_{\mathrm{res}}(B) = \left[\frac{dE_{\mathrm{res}}}{dB}(B_{\mathrm{res}})\right](B - B_{\mathrm{res}}).$$

$$(\Delta B) = \frac{m}{4\pi\hbar^2 a_{\mathrm{bg}}} \frac{(2\pi\hbar)^3 \left|\langle \phi_{\mathrm{res}}|W|\phi_0^{(+)}\rangle\right|^2}{\left[\frac{dE_{\mathrm{res}}}{dB}(B_{\mathrm{res}})\right]}$$

$$B_0 = B_{\mathrm{res}} - \frac{\langle \phi_{\mathrm{res}}|W G_{\mathrm{bg}}(0) W|\phi_{\mathrm{res}}\rangle}{\left[\frac{dE_{\mathrm{res}}}{dB}(B_{\mathrm{res}})\right]}$$

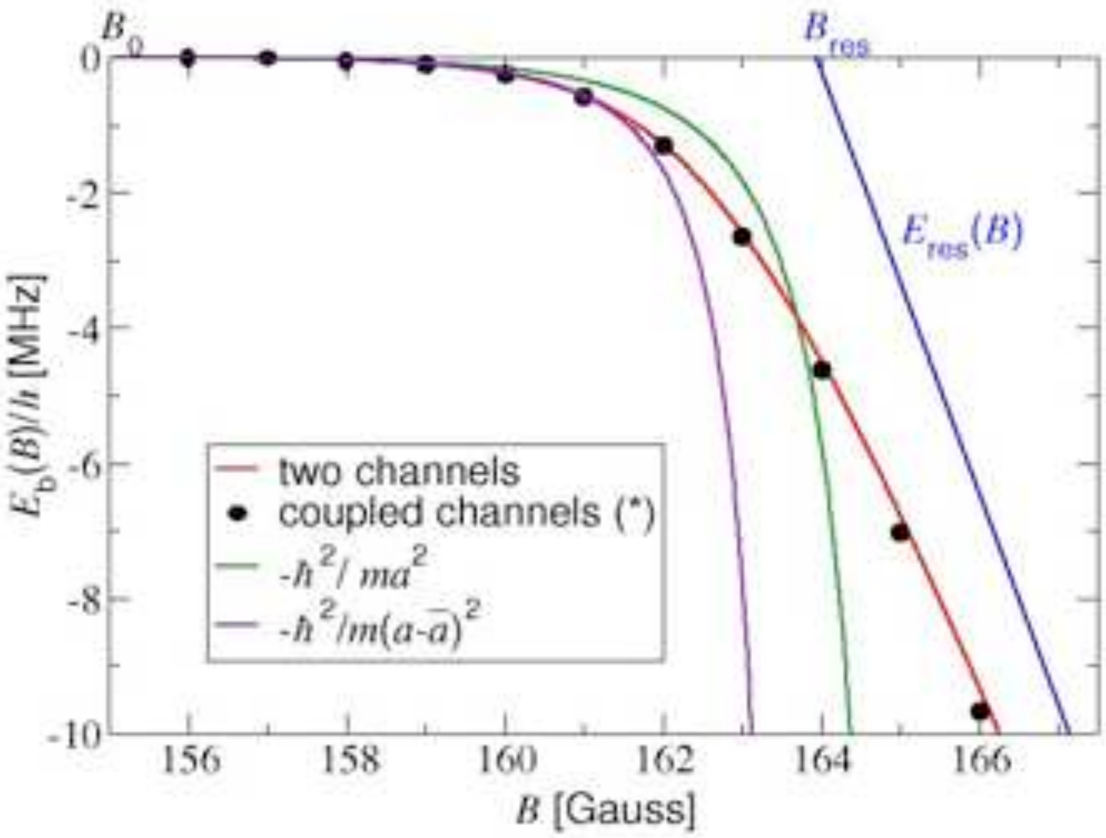
$$a(B) = a_{\mathrm{bg}}\left[1 - \frac{(\Delta B)}{B - B_0}\right],$$

Closed channel dominated resonance



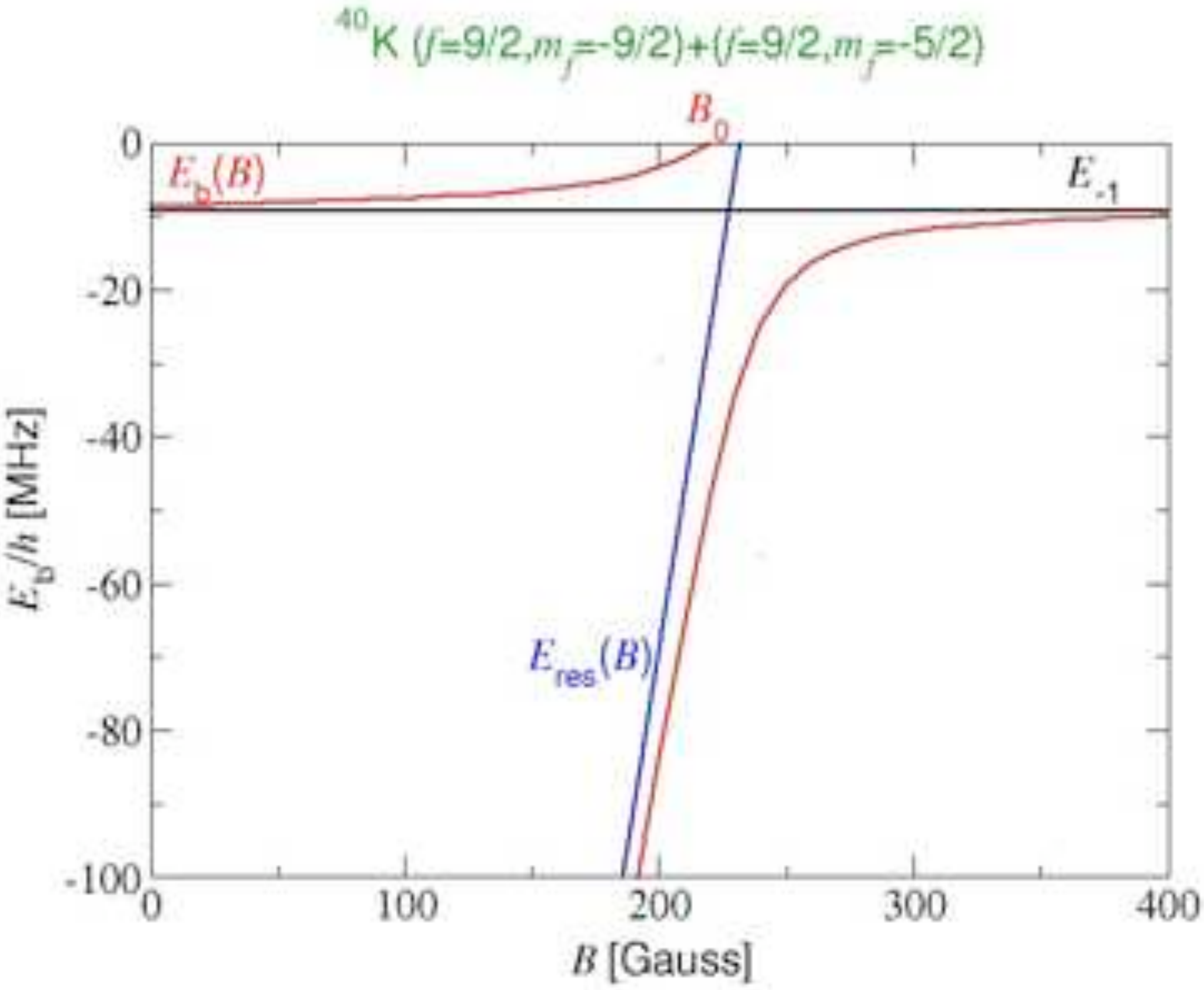
Molecular binding energies

^{85}Rb

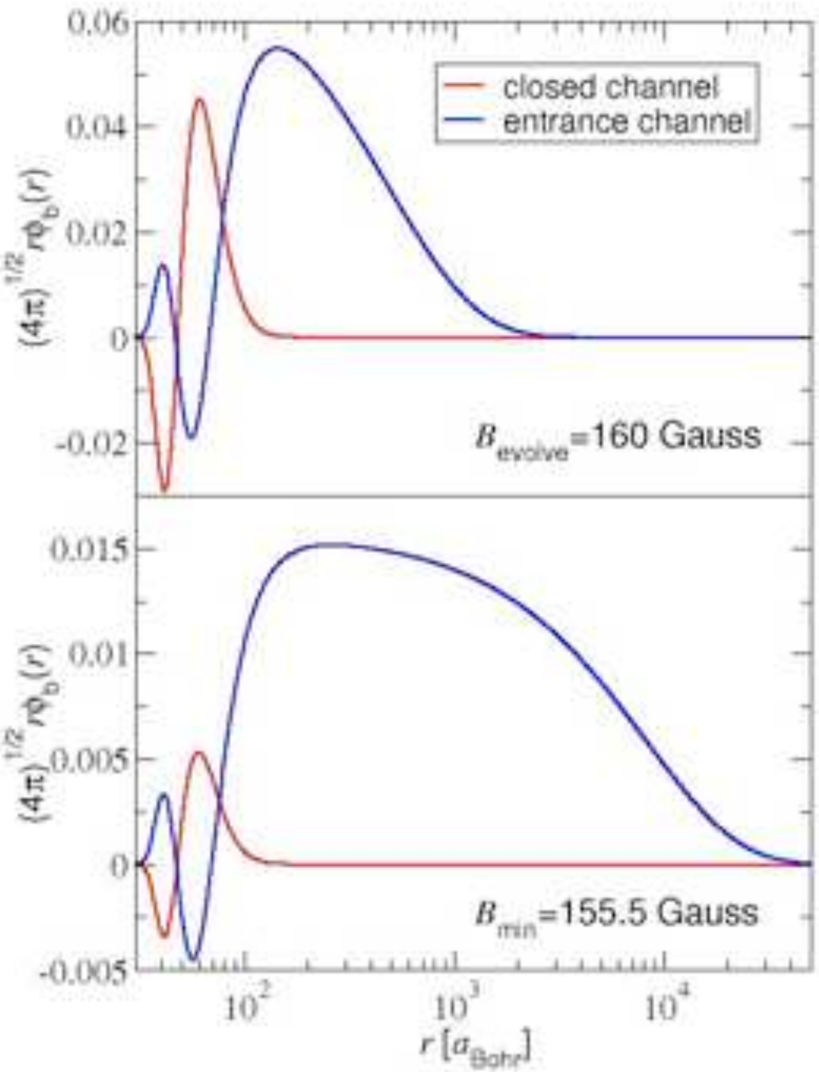


* N.R. Claussen et al., Phys. Rev. A **67**, 060701 (2003); S. Kokkelmans, private communication.

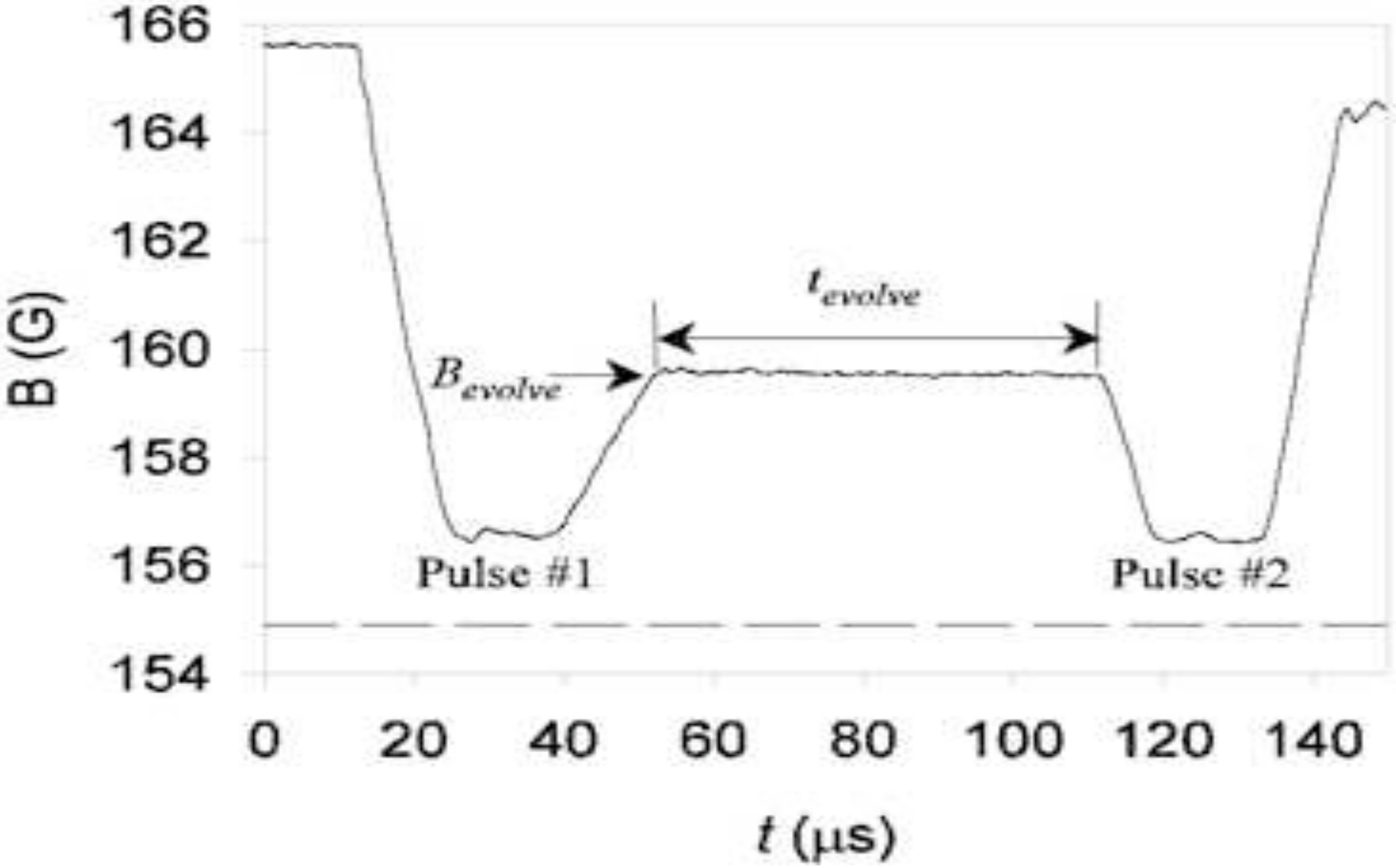
Entrance channel dominated resonance



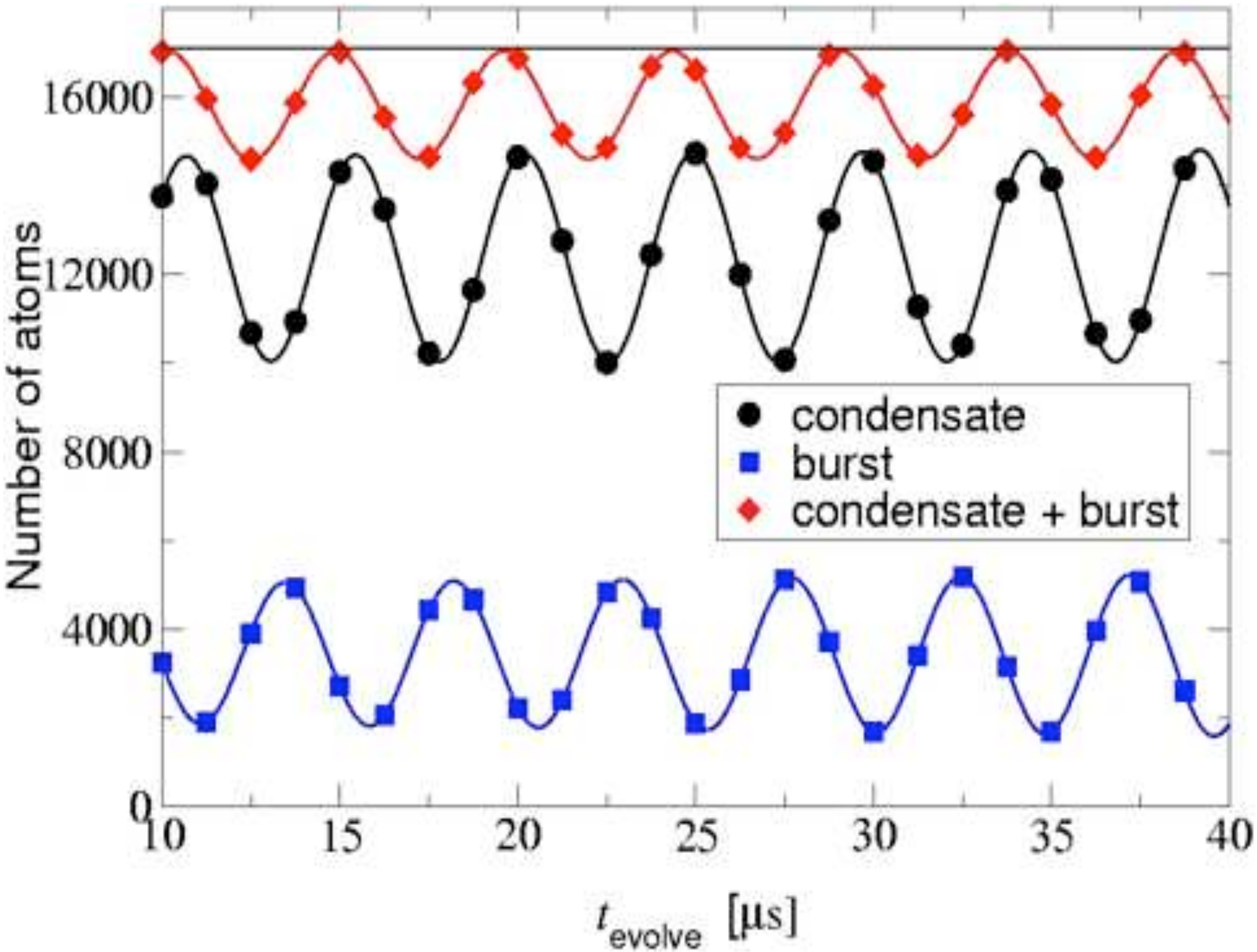
Feshbach molecules can be huge!



Ramsey interferometry



Ramsey fringes



Microscopic quantum dynamics method

$$i\hbar \frac{\partial}{\partial t}\Psi(\boldsymbol{x},t) = \left[-\frac{\hbar^2}{2m}\nabla^2 + V_{trap}(\boldsymbol{x})\right]\Psi(\boldsymbol{x},t)$$

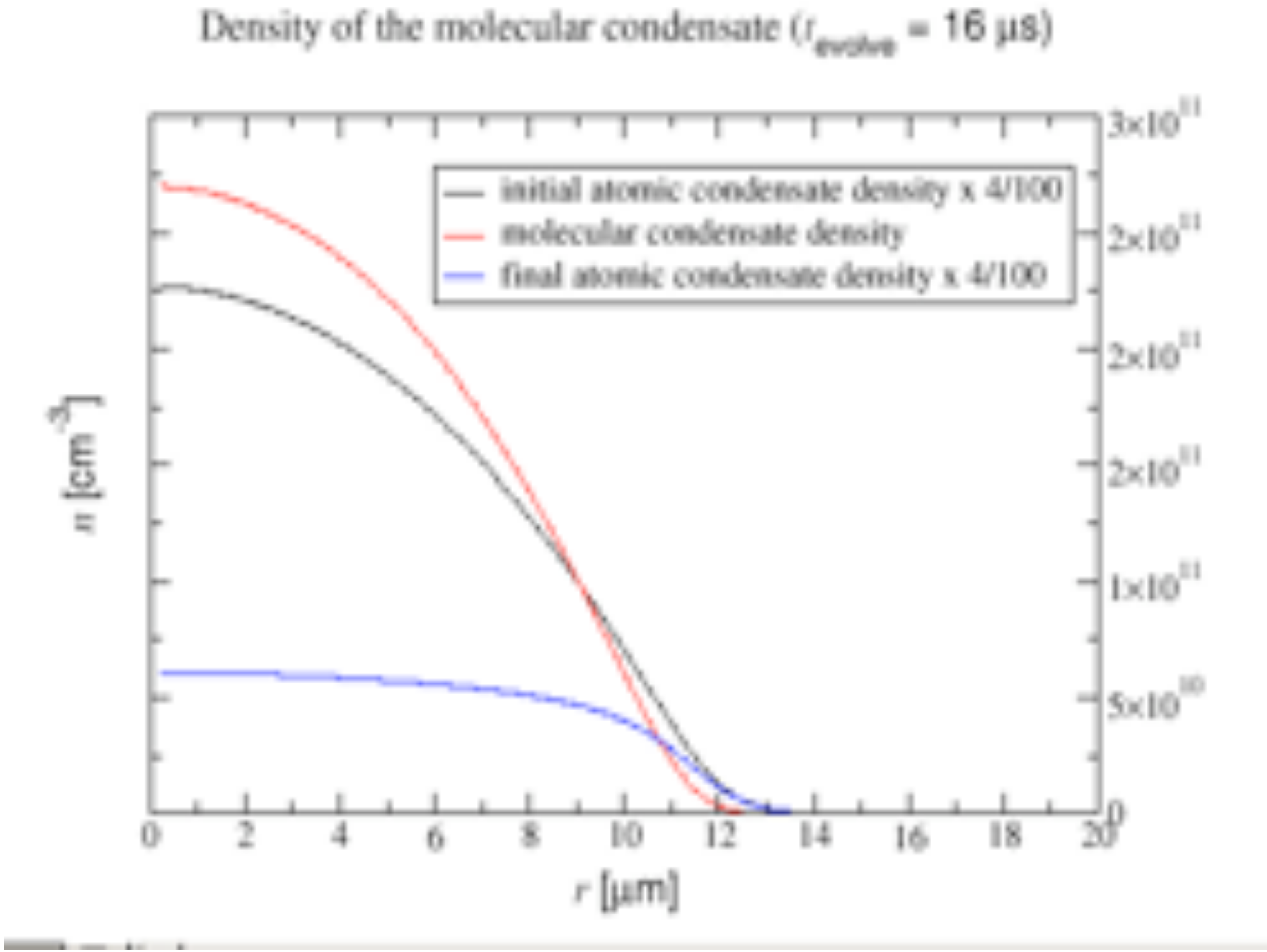
$$-\Psi^*(\boldsymbol{x},t)\int_{t_0}^{\infty} d\boldsymbol{\tau}\ \Psi^2(\boldsymbol{x},\boldsymbol{\tau})\frac{\partial}{\partial \boldsymbol{\tau}} h(t,\boldsymbol{\tau})$$

Non-Markovian non-linear Schrödinger equation

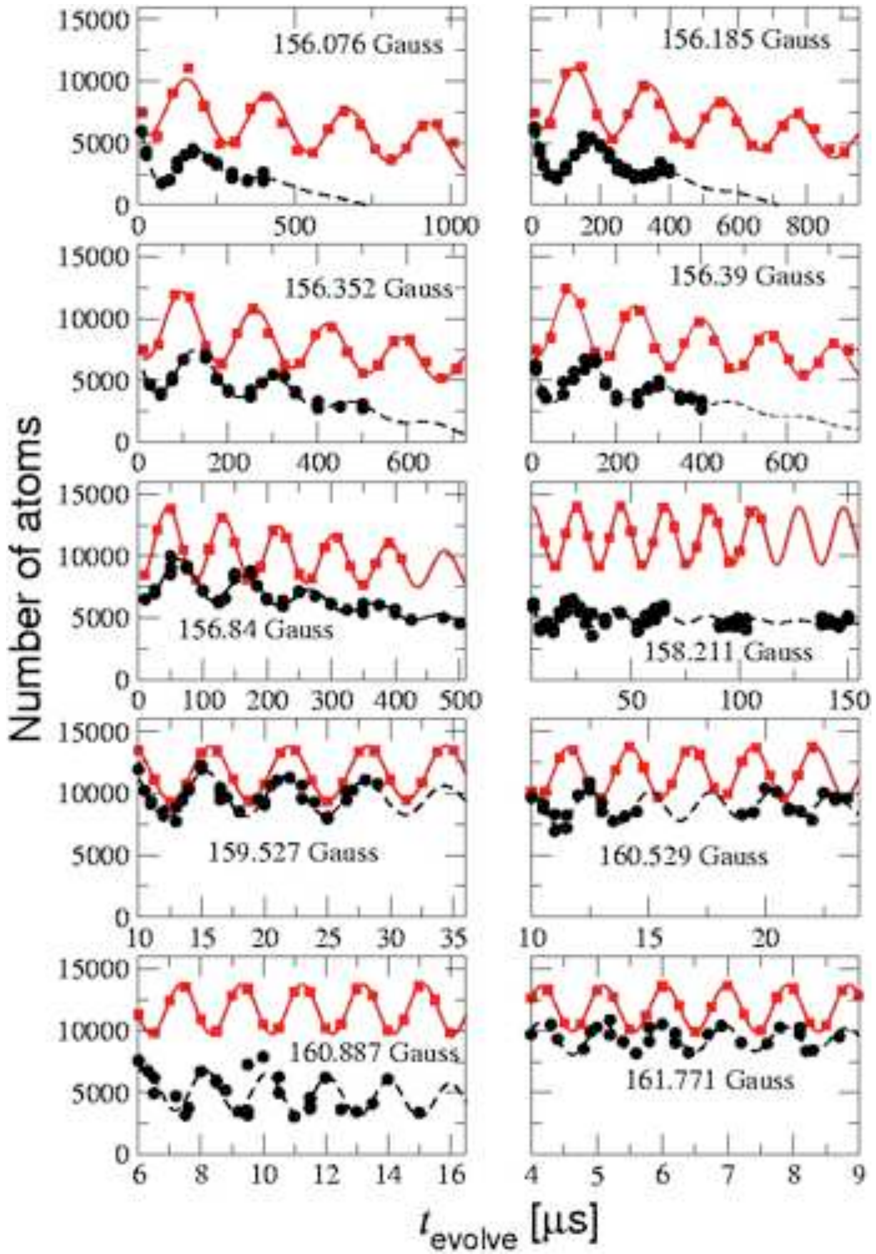
$$h(t,\boldsymbol{\tau}) = (2\pi\hbar)^3\theta(t-\boldsymbol{\tau})\left\langle 0,bg\left|V_{2B}(t)U_{2B}(t,\boldsymbol{\tau})\right|0,bg\right\rangle$$

$\left|0,bg\right\rangle$ Zero momentum plane wave of the relative motion of two atoms in the entrance channel

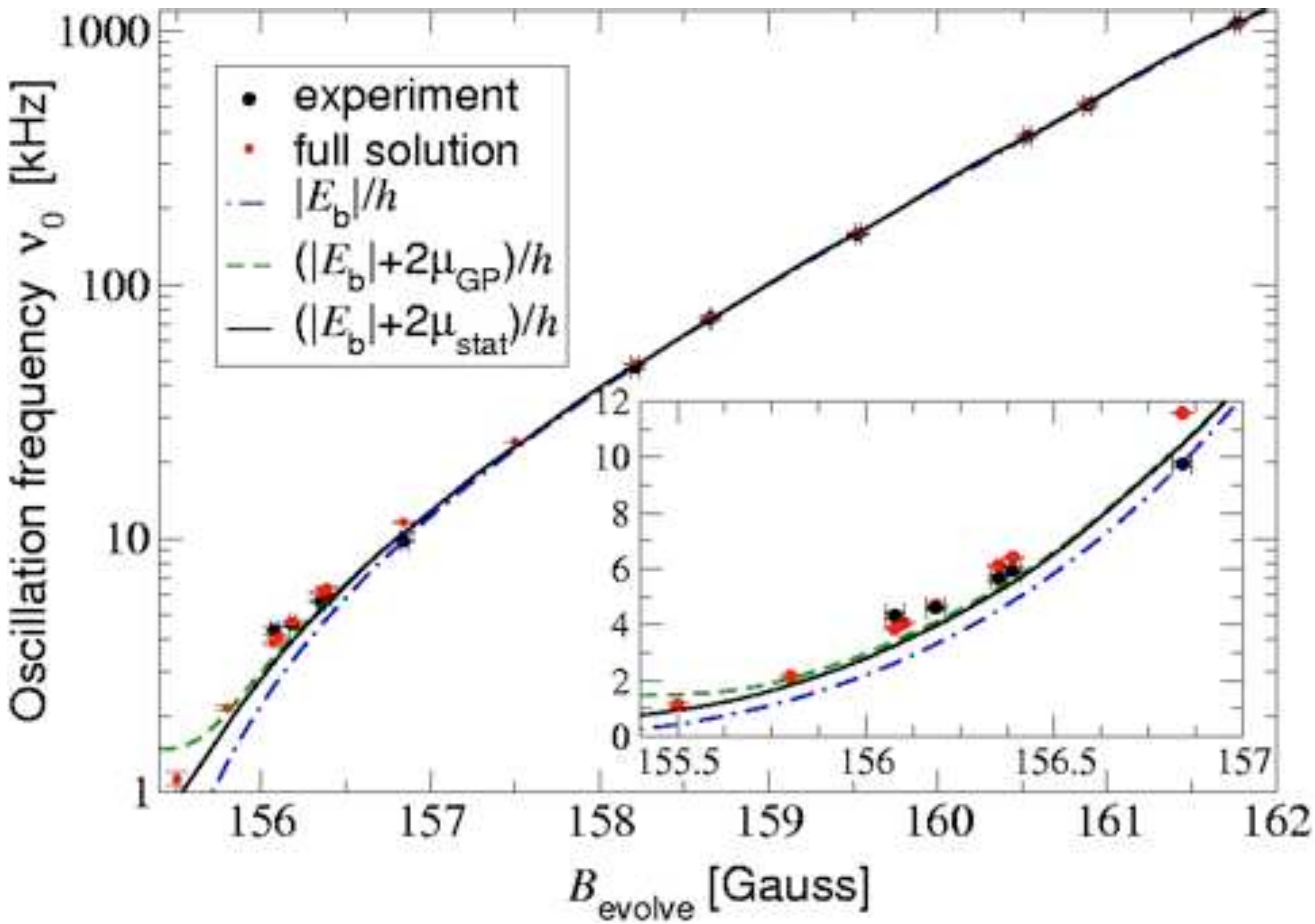
Simulation Results For molecular Condensate

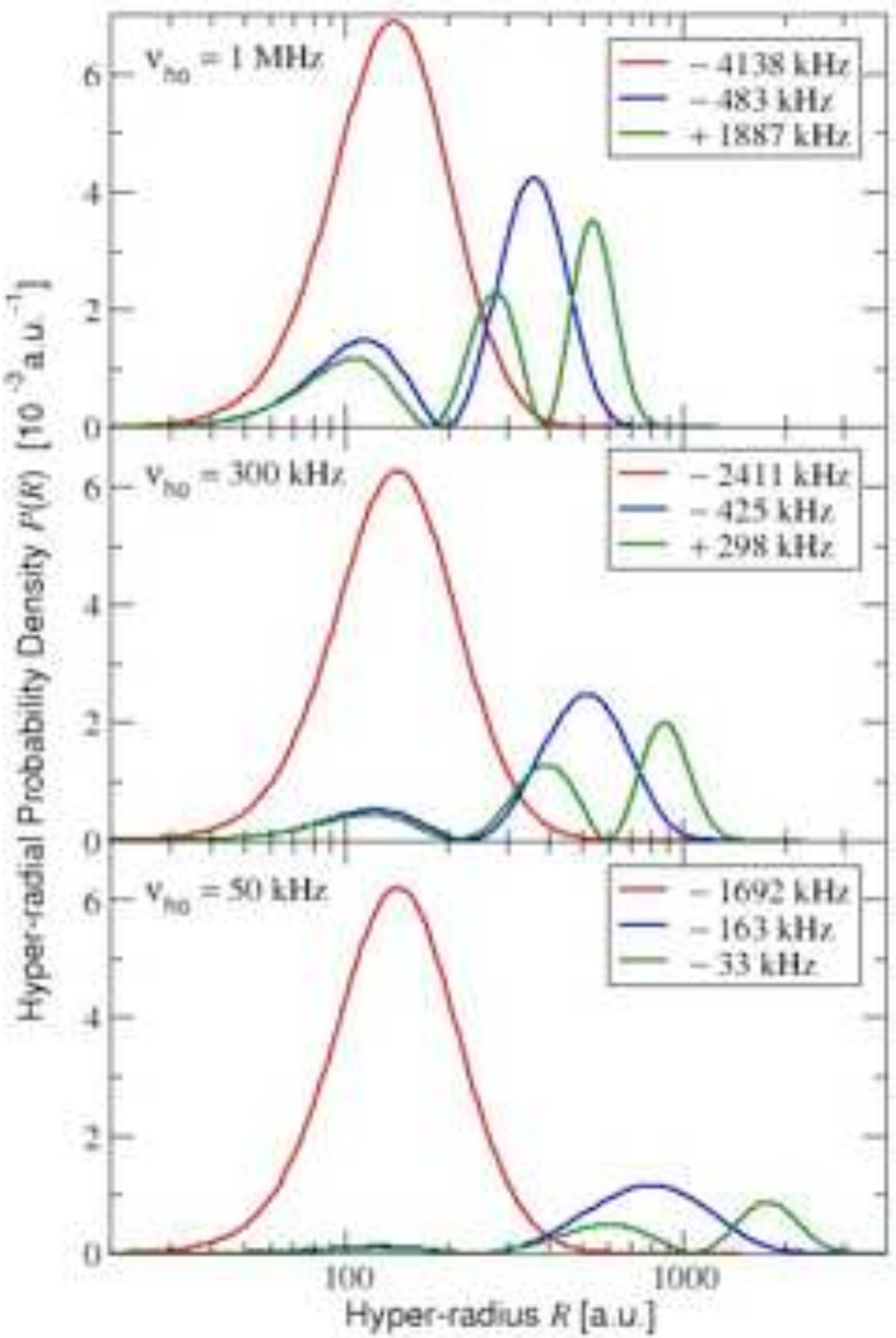


Decaying Oscillations



Oscillation frequency





Efimov States Koehler and Stoll

Hamiltonian for BCS-BEC in K

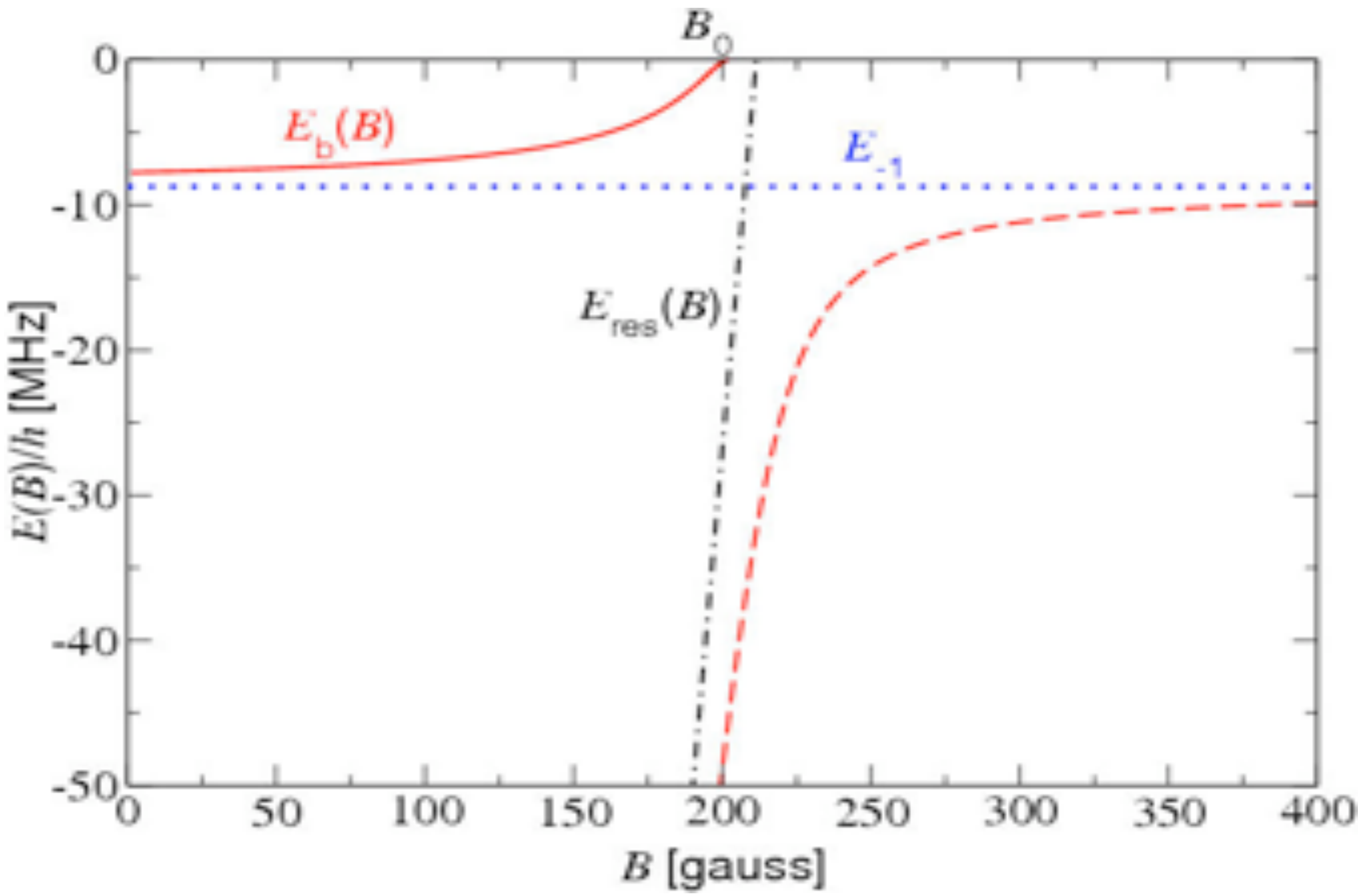
$$H_{\mathrm{MB}} = \sum_{s,\mathbf{k}} \epsilon_{\mathbf{k}} a^{\dagger}_{\mathbf{k},s} a_{\mathbf{k},s} +$$

$$\sum_{s,s'} \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} V(\mathbf{k},\mathbf{k}',B) a^{\dagger}_{\mathbf{k}+\mathbf{q},s} a^{\dagger}_{-\mathbf{k},s'} a_{-\mathbf{k}',s'} a_{\mathbf{k}'+\mathbf{q},s},$$

1. Feshbach means we can tune from +ve to -ve scattering lengths and see BEC to BCS

2. Two body scattering changed by surrounding particles.

BCS-BEC in K



B_0
$E_b(B)$
E_{-1}
$E_{res}(B)$
$E(B)/h$ [MHz]
0
-10
-20
-30
-40
-50
0
50
100
150
200
250
300
350
400
B [gauss]

Bose-Fermi Hamiltonian for BCS-BEC

$$H_{\text{MB-BF}} = \sum_{s,\mathbf{k}} \epsilon_{\mathbf{k}} a^{\dagger}_{\mathbf{k},s} a_{\mathbf{k},s} + \sum_{\mathbf{q}} (E_{\mathbf{q}} + E \quad (B)) b^{\dagger}_{\mathbf{q}} b_{\mathbf{q}}$$

$$+ \sum_{s,s'} \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} V \quad (\mathbf{k},\mathbf{k}') a^{\dagger}_{\mathbf{k}+\mathbf{q},s} a^{\dagger}_{-\mathbf{k},s'} a_{-\mathbf{k}',s'} a_{\mathbf{k}'+\mathbf{q},s}$$

$$+ \sum_{s,s'} \sum_{\mathbf{k},\mathbf{q}} g(\mathbf{k}) (b^{\dagger}_{\mathbf{q}} a_{-\mathbf{k},s} a_{\mathbf{k}+\mathbf{q},s'} + h.c.) .$$

1) Can be mapped onto the two body problem but don't think of b as representing molecules in the open channel dominated case.

2) Spin composition complicates composite boson picture.

Gap Equations for BCS-BEC

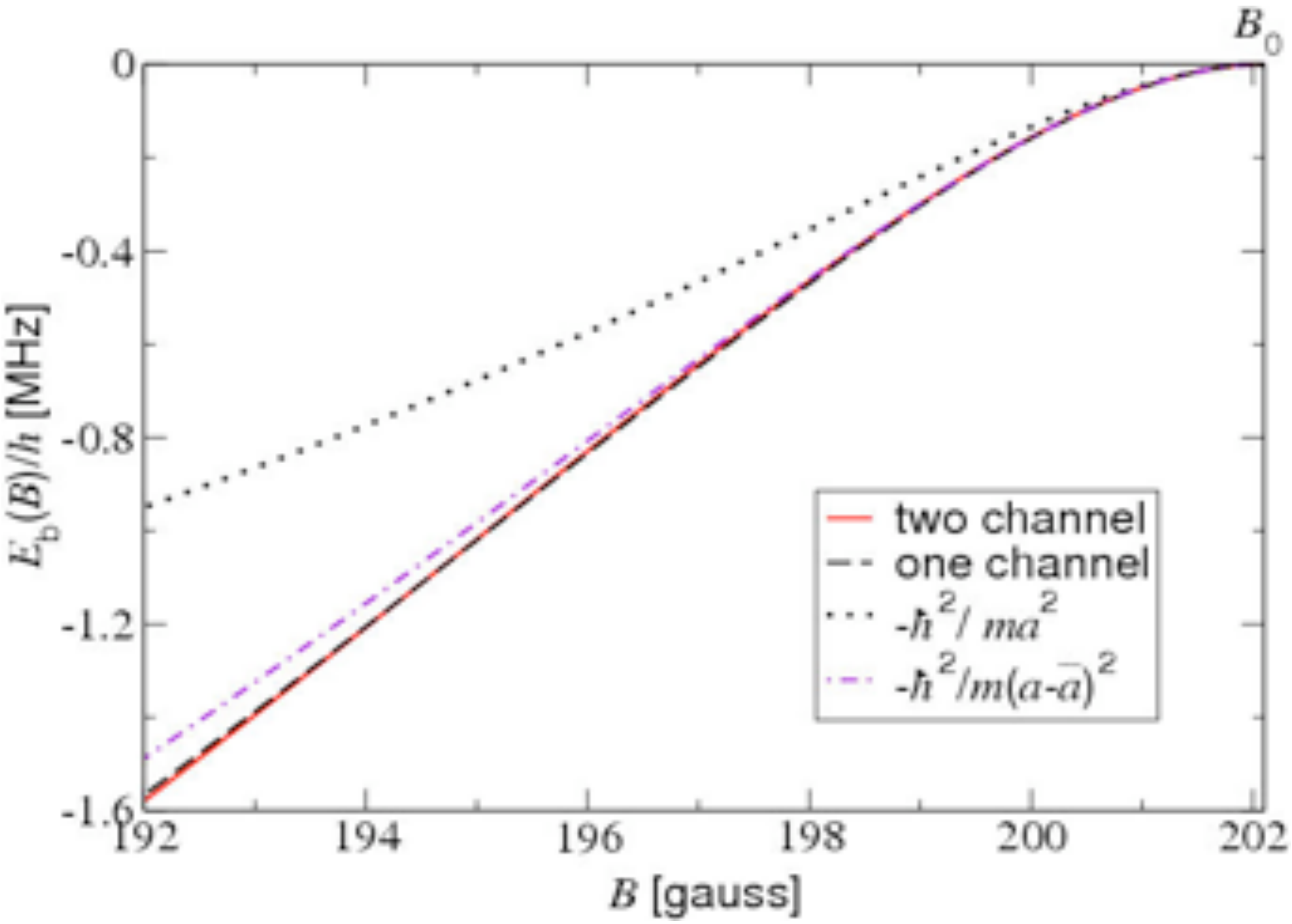
$$\Delta = \int d^3p \frac{\chi_{\text{bg}}(p)\Sigma(p)}{2\sqrt{(\epsilon_p - \mu)^2 + |\Sigma(p)|^2}} \tanh\left(\frac{\beta E_p}{2}\right),$$

$$[E_{\text{res}}(B) - 2\mu]b_0 =$$

$$\int d^3p \frac{\zeta\chi(p)\Sigma(p)}{2\sqrt{(\epsilon_p - \mu)^2 + |\Sigma(p)|^2}} \tanh\left(\frac{\beta E_p}{2}\right),$$

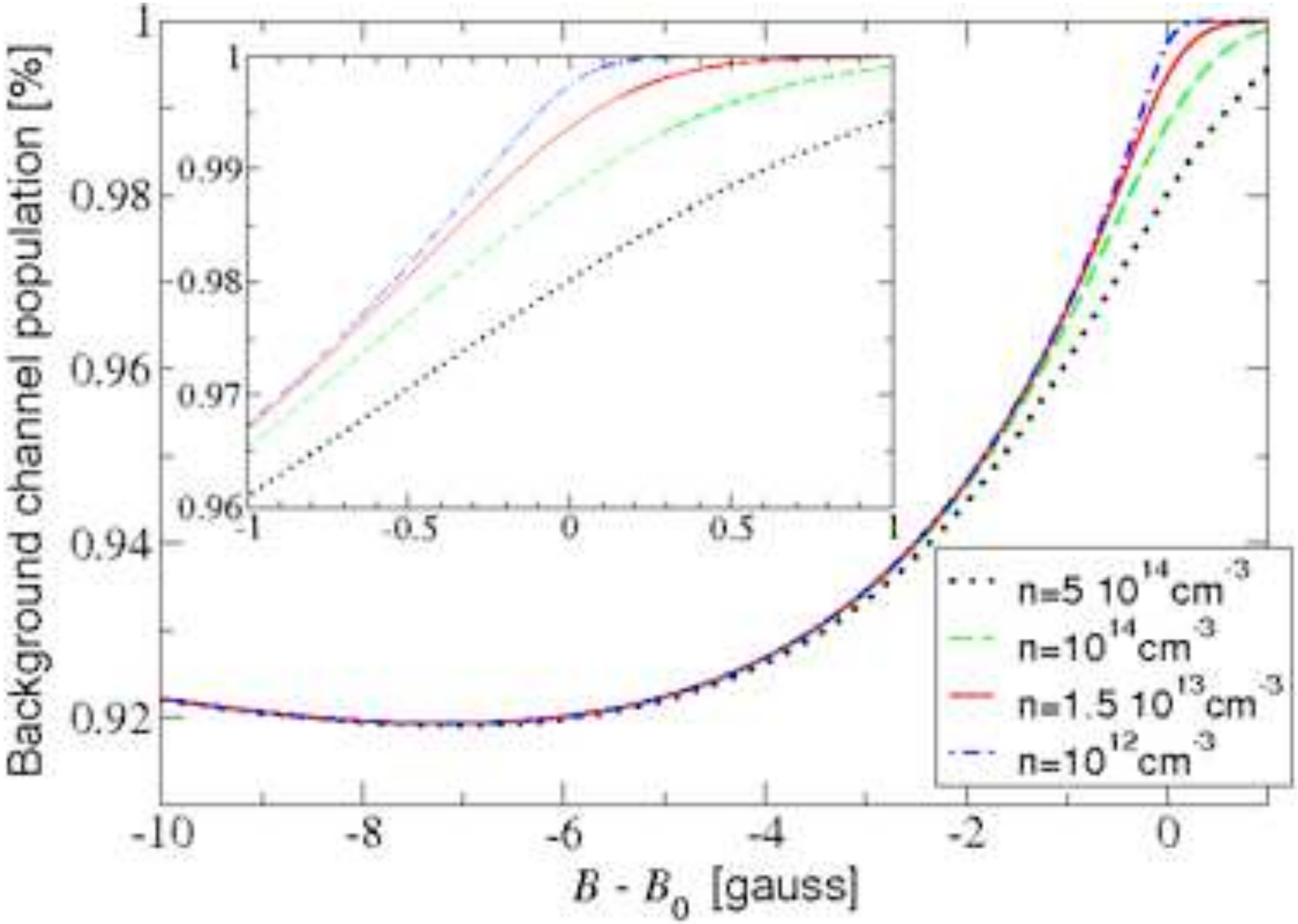
1) Single channel system works fine close to resonance.
Two channel works over wider range.
2) Mean-Field Theory Solution

BCS-BEC in K



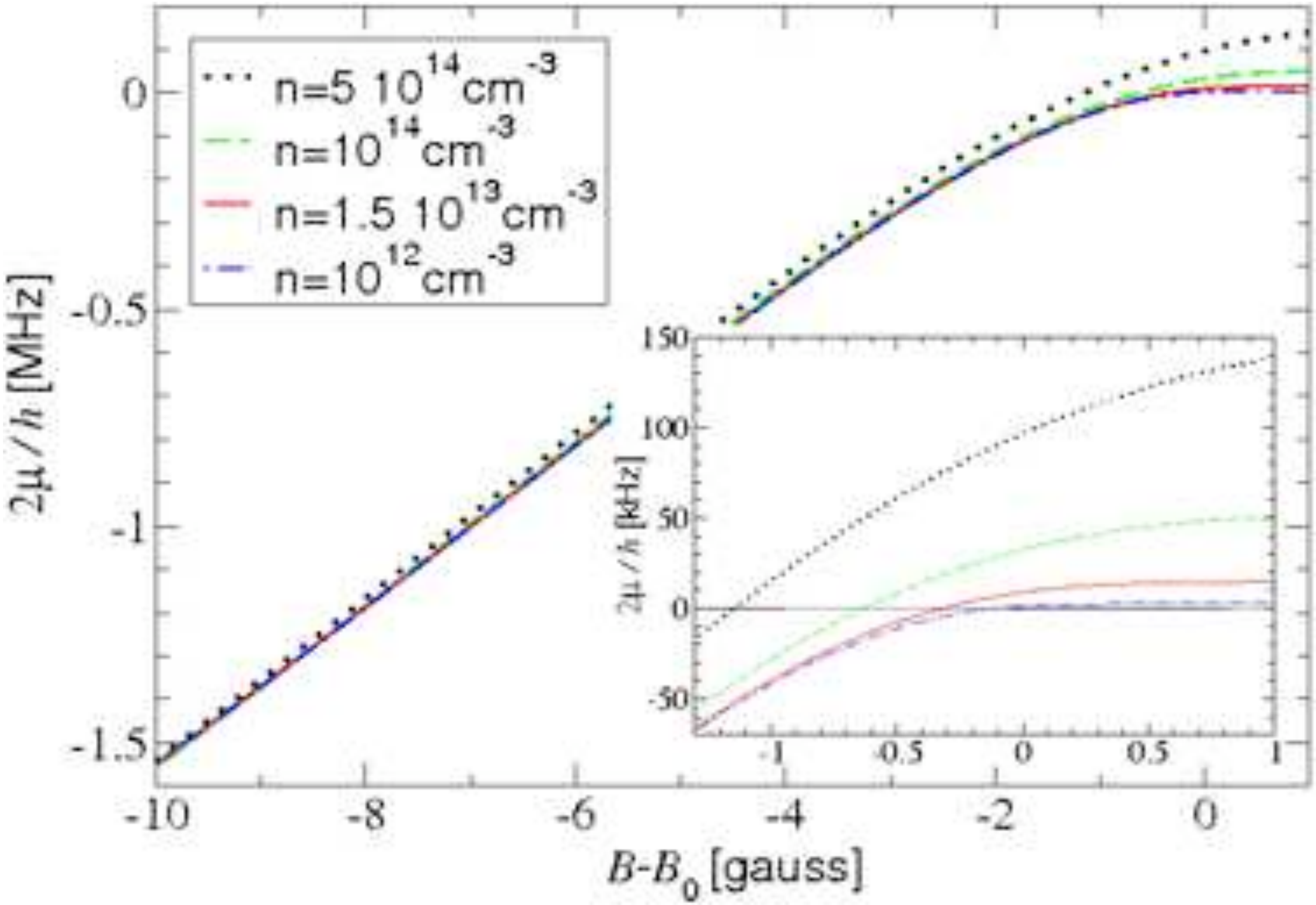
B_0
0
-0.4
-0.8
-1.2
-1.6
192
194
196
198
200
202
$E_b(B)/h$ [MHz]
B [gauss]
two channel
one channel
$-\hbar^2/ma^2$
$-\hbar^2/m(a-\bar{a})^2$

BCS-BEC in K



Background channel population [%]
B - B$_0$ [gauss]
n=5 10^{14}cm^{-3}
n=10^{14}cm^{-3}
n=1.5 10^{13}cm^{-3}
n=10^{12}cm^{-3}

BCS-BEC in K



n=5 10^14 cm^-3
n=10^14 cm^-3
n=1.5 10^13 cm^-3
n=10^12 cm^-3
2μ / h [MHz]
0
-0.5
-1
-1.5
-10
-8
-6
-4
-2
0
B-B0 [gauss]
2μ / h [kHz]
150
100
50
0
-50
-1
-0.5
0
0.5
1

Pair Wavefunctions in K

$$\kappa_{\mathbf{k}} = u_{\mathbf{k}} v_{\mathbf{k}} = \frac{\Sigma(\mathbf{k})}{2\sqrt{(\epsilon_{\mathbf{k}} - \mu)^2 + |\Sigma(\mathbf{k})|^2}},$$

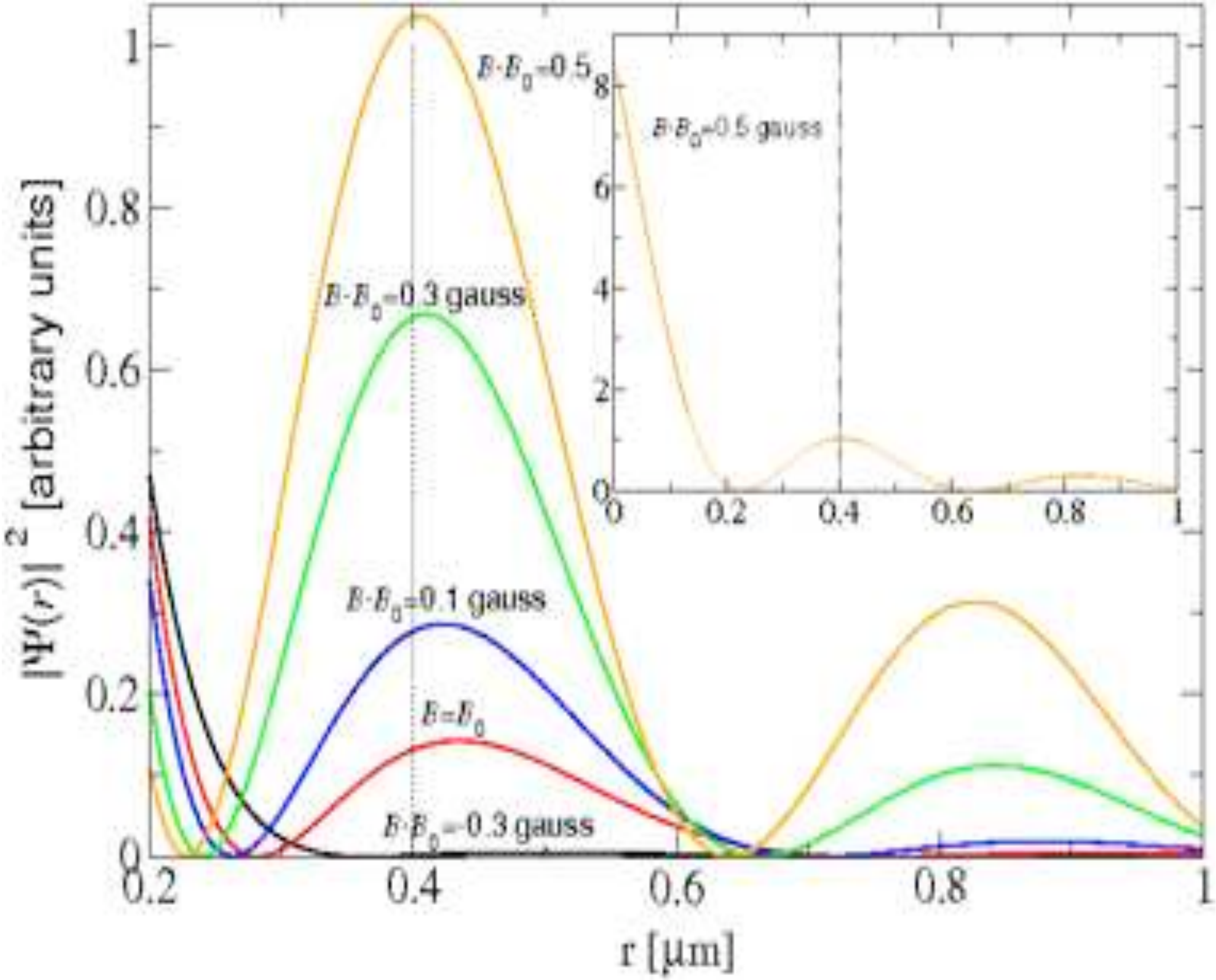
$$\Phi_{\mathbf{k}} = |v_{\mathbf{k}}|^2 = \frac{1}{2}\left[1 - \frac{\epsilon_{\mathbf{k}} - \mu}{\sqrt{(\epsilon_{\mathbf{k}} - \mu)^2 + |\Sigma(\mathbf{k})|^2}}\right],$$

$$E_{\mathbf{k}} = \sqrt{(\epsilon_{\mathbf{k}} - \mu)^2 + |\Sigma(\mathbf{k})|^2}\,.$$

$$\xi^2 = \frac{\int dr\, r^2 [\kappa(r)]^2}{\int dr [\kappa(r)]^2}.$$

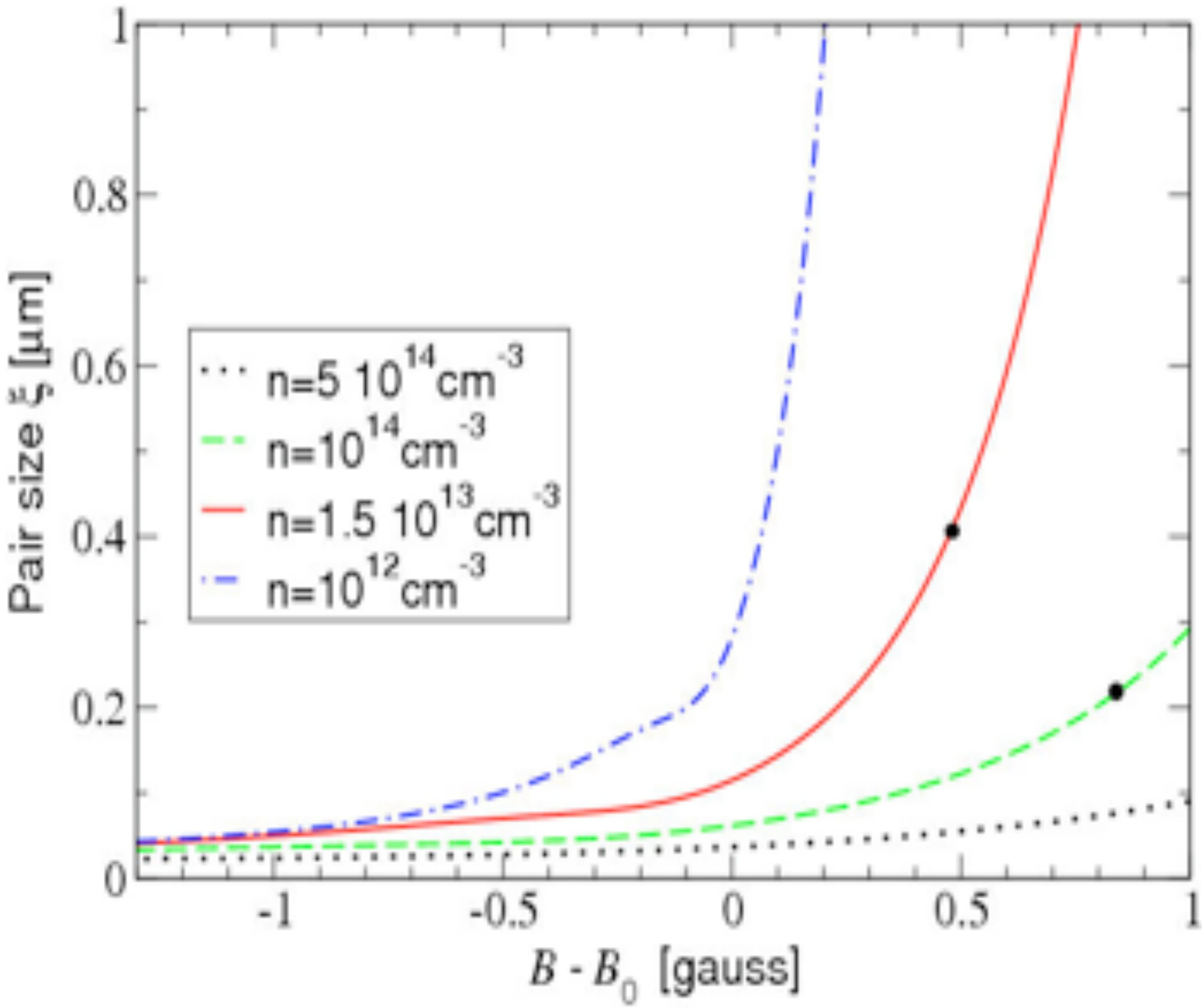
$$n_c = \frac{\int d^3p\, |\kappa_p|^2}{\frac{1}{2} n}.$$

BCS-BEC in K



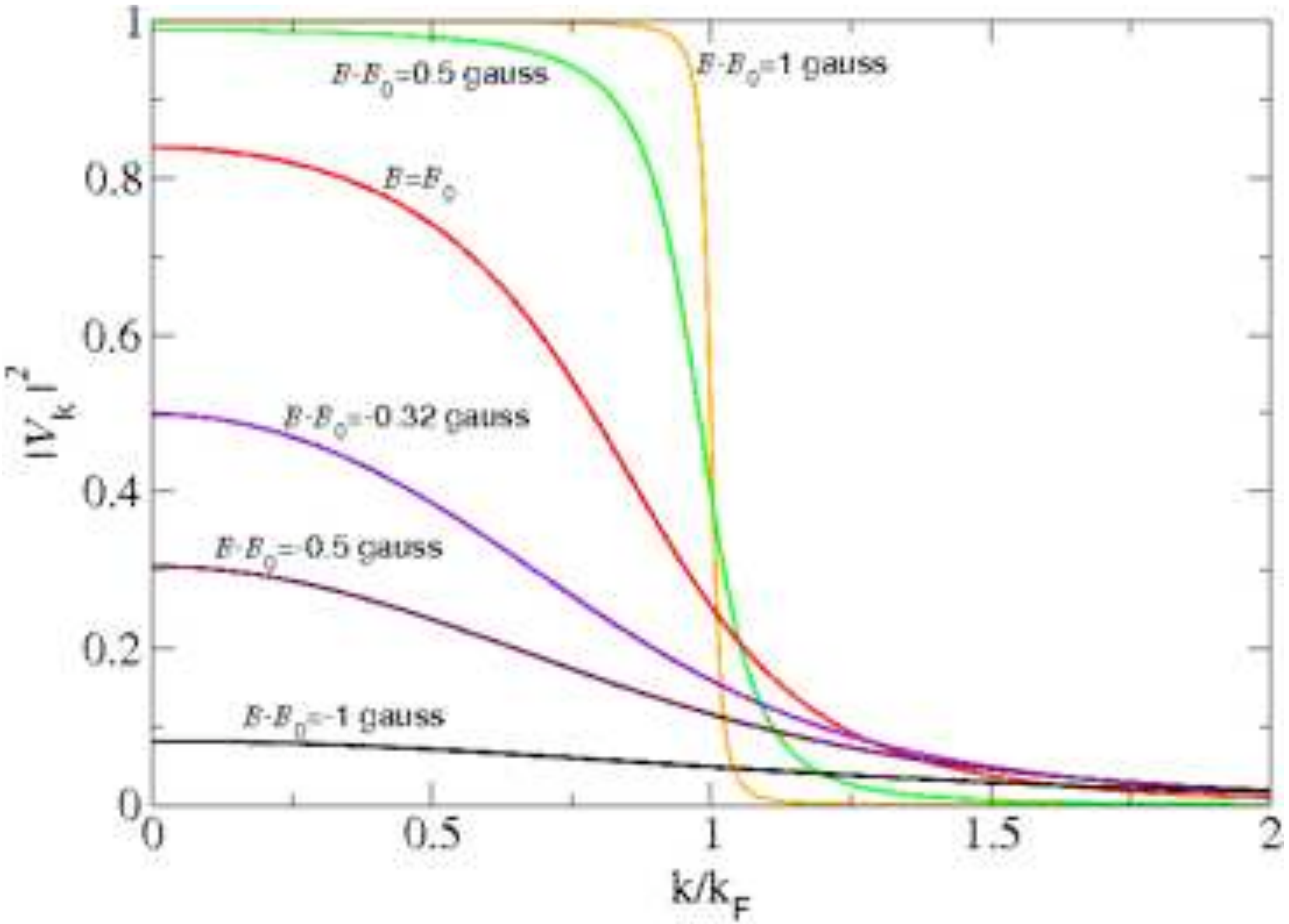
B-B0=0.5
B-B0=0.3 gauss
B-B0=0.1 gauss
B=B0
B-B0=-0.3 gauss
B-B0=0.5 gauss
|Ψ(r)|² [arbitrary units]
r [μm]

BCS-BEC in K



Pair size ξ [μm]
B - B0 [gauss]
n=5 10^14 cm^-3
n=10^14 cm^-3
n=1.5 10^13 cm^-3
n=10^12 cm^-3
0
0.2
0.4
0.6
0.8
1
-1
-0.5
0
0.5
1

BCS-BEC in K



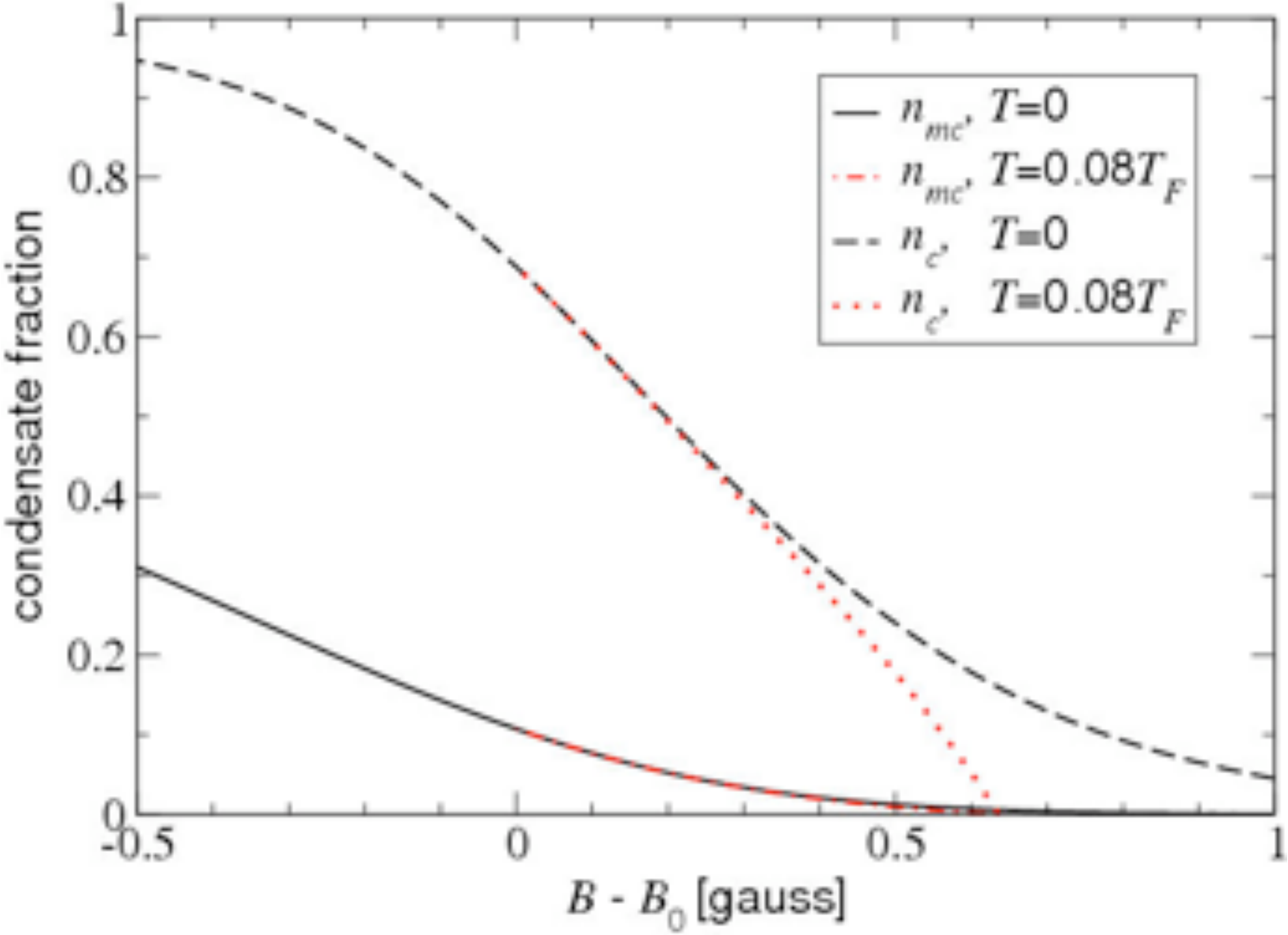
$|V_k|^2$
k/k_F
$B-B_0$=0.5 gauss
$B-B_0$=1 gauss
$B=B_0$
$B-B_0$=-0.32 gauss
$B-B_0$=-0.5 gauss
$B-B_0$=-1 gauss

Condensate BCS-BEC in K

$$n_{Overlap} = \frac{\left|\int d^3p\, \kappa_p\, \phi_{Bound}(p, B_{Initial})\right|^2}{\frac{1}{2}n}.$$

Overlap zero at around
half a Gauss above resonance

BCS-BEC in K



n_{mc}, T=0
n_{mc}, T=0.08T_F
n_c, T=0
n_c, T=0.08T_F
condensate fraction
B - B_0 [gauss]
0
0.2
0.4
0.6
0.8
1
-0.5
0
0.5
1

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