Self-consistent theory of atomic Fermi gases with a Feshbach resonance at the superfluid transition

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A self-consistent theory is derived to describe the BCS–Bose-Einstein-condensate crossover for a strongly interacting Fermi gas with a Feshbach resonance. In the theory the fluctuation of the dressed molecules, consisting of both preformed Cooper pairs and "bare" Feshbach molecules, has been included within a self-consistent T-matrix approximation, beyond the Nozières and Schmitt-Rink strategy considered by Ohashi and Griffin. The resulting self-consistent equations are solved numerically to investigate the normal-state properties of the crossover at various resonance widths. It is found that the superfluid transition temperature T_c increases monotonically at all widths as the effective interaction between atoms becomes more attractive. Furthermore, a residue factor Z_m of the molecule's Green function and a complex effective mass have been determined to characterize the fraction and lifetime of Feshbach molecules at T_c. Our many-body calculations of Z_m agree qualitatively well with recent measurments of the gas of ^{6}Li atoms near the broad resonance at 834 G. The crossover from narrow to broad resonances has also been studied.

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INTRODUCTION

The recently demonstrated strongly interacting Fermi gases of ^{40}K and ^{6}Li near a Feshbach resonance have attracted a lot of attention [1–8]. In these systems, because of the coupling between a pair of atoms in an open channel and a bound molecular state of a closed channel, it is possible to obtain arbitrarily strong attractions between atoms by tuning the energy of the molecular state across the resonance with a magnetic field. It therefore provides us a unique opportunity to experimentally examine the crossover from a Bardeen-Cooper-Schrieffer (BCS) superfluidity to a Bose-Einstein condensate (BEC).

The use of the Feshbach resonance, however, significantly complicates the theoretical analysis of current experimental results, due to the presence of the bound molecular state. It has been argued that a two-channel (resonance) model describing explicitly both the open-channel and closed-channel physics should be adopted as a minimum ingredient for the theory [9–11]. In the model, the molecular bound state in the closed channel is treated as a featureless bosonic particle and two essential parameters—the threshold energy of molecules and the atom-molecule coupling constant—are required to characterize the total system. This is in contrast to the traditional single-channel model, in which the closed channel or the "bare" Feshbach molecules are neglected. Thereby the model is characterized solely by a single parameter of s-wave scattering length a; i.e., the interatomic interaction is described by a contact potential with strength $4\pi\hbar^2 a/m$. The applicability of these models to current experiments, together with their equivalence in certain limits, is actively debated at present. A closely related subtle problem is the nature of Fermi superfluidity near the resonance—i.e., whether the superfluid transition is associated with small pairs that are molecular in character or large objects akin to Cooper pairs.

In this paper we present a many-body theory of the two-channel model for an interacting ultracold Fermi gas with a Feshbach resonance at and above the superfluid transition temperature T_c. In the theory the strong fluctuations of the preformed Cooper pairs and of the Feshbach molecules have been incorporated within a self-consistent T-matrix approximation, similar to that considered by Haussmann for a single-channel model [12]. We then apply the theory to explore the normal phase of the gas at the BCS-BEC crossover, emphasizing the importance of the resonance width. The purpose of this exploration is threefold. (i) First, a reliable T_c is predicted. Until now there has been strong evidence for the emergence of superfluidity at the crossover [1–4,6–8]. Precise experimental characterizations of the thermodynamic and dynamical properties of strongly interacting Fermi gases are also undergoing. In this respect, a reliable theoretical prediction of T_c of the crossover gas is, therefore, useful for guiding future experimental investigations. Previous calculations of T_c are based on the so-called Nozires–Schmitt-Rink (NSR) scheme [13,14], which is an analysis equivalent to the ladder approximation using free-fermionic Green's functions. In both weak- and strong-coupling limits this approximation leads to correct results [13]. However, at the crossover it becomes less accurate as the fermionic quasiparticles are far from being free particles. Our theory has the advantage of taking a self-consistently dressed Green function in the T matrix and is expected to give a more reasonable T_c than the NSR method, although there is no specific control of the precision of the predicted value for T_c in our self-consistent scheme. (ii) Second, to characterize the nature of pairs at the crossover, the fraction and lifetime of the "bare" Feshbach molecules have been studied by extracting the residue factor Z_m and the effective mass from the Green function of molecules. The value of Z_m can be directly determined in experiments using the Stern-Gerlach selection technique to measure the magnetic moment of the pairs [15] or using optical spectroscopy to project out the singlet component of the pairs [16]. Our calculations of Z_m emphasize many-body

effects at the crossover, in particular on the BCS side of the resonance. For an ultracold gas of ^{6}Li atoms at the broad resonance $B_0=834$ G, we find a reasonable agreement between our theoretical predictions and the recent spectroscopy measurements by Partridge *et al.* [16]. (iii) Finally, the crossover from a narrow to a broad resonance with increasing atom-molecule coupling strengths has been investigated.

SELF-CONSISTENT *T*-MATRIX APPROXIMATION

We consider a homogeneous gas of fermionic atoms in two different hyperfine states in the vicinity of a Feshbach resonance. In the grand canonical ensemble the gas of N atoms is described by an atom-molecule Hamiltonian [9–11]

$$\mathcal{H}=\sum_{\mathbf{k}\sigma}(\epsilon_{\mathbf{k}}-\mu)c^{\dagger}_{\mathbf{k}\sigma}c_{\mathbf{k}\sigma}+U_{bare}\sum_{\mathbf{k}\mathbf{k}'\mathbf{q}}c^{\dagger}_{\mathbf{k}+\mathbf{q}\uparrow}c^{\dagger}_{\mathbf{k}'-\mathbf{q}\downarrow}c_{\mathbf{k}'\downarrow}c_{\mathbf{k}\uparrow}+\sum_{\mathbf{q}}(\epsilon_{\mathbf{q}}/2+\nu_{bare}-2\mu)b^{\dagger}_{\mathbf{q}}b_{\mathbf{q}}+g_{bare}\times\sum_{\mathbf{k}\mathbf{q}}(b^{\dagger}_{\mathbf{q}}c_{\mathbf{k}+\mathbf{q}/2\uparrow}c_{-\mathbf{k}+\mathbf{q}/2\downarrow}+\mathrm{H.c.}). \quad (1)$$

Here the two hyperfine states are denoted as pseudospins $\sigma=\uparrow,\downarrow$, with $N_{\uparrow}=N_{\downarrow}=N/2$. $c^{\dagger}_{\mathbf{k}\sigma}$ and $b^{\dagger}_{\mathbf{q}}$ represent the creation operators of an atom with kinetic energy $\epsilon_{\mathbf{k}}=\hbar^2\mathbf{k}^2/2m$ and of a Feshbach molecule with dispersion $\epsilon_{\mathbf{q}}/2+\nu_{bare}=\hbar^2\mathbf{q}^2/4m+\nu_{bare}$, respectively. The magnetic detuning or the threshold energy ν_{bare} is a key parameter in this model and can be conveniently adjusted by an external magnetic field B. It changes sign across the resonance from above and enables the formation of stable molecules. As the molecule is made of two atoms, a chemical potential μ is introduced to impose conservation of the total number of bare Fermi atoms—i.e., $\langle\mathcal{N}\rangle=\langle\Sigma_{\mathbf{k}\sigma}c^{\dagger}_{\mathbf{k}\sigma}c_{\mathbf{k}\sigma}\rangle+\langle\Sigma_{\mathbf{q}}b^{\dagger}_{\mathbf{q}}b_{\mathbf{q}}\rangle=N$. Moreover, two contact interactions have been adopted for nonresonant (or background) atom-atom scatterings and the atom-molecule coupling, as parametrized by U_{bare} and g_{bare}, respectively.

As usual, the use of contact interactions leads to an (unphysical) ultraviolet divergence and requires a renormalization that expresses the bare parameters U_{bare}, g_{bare}, and ν_{bare} in terms of the observed or renormalized values $U_0=4\pi\hbar^2a_{bg}/m$, g_0, and $\nu_0=\Delta\mu(B-B_0)$, where a_{bg} is the background s-wave scattering length for the atoms, g_0 is associated with the width of resonance, and $\Delta\mu$ is the magnetic moment difference between the atomic and bound molecular states. The renormalization can be done by solving either the Lippmann-Schwinger scattering equations or the vertex functions in the two-body limit. The outcome is very transparent in physics: one can just replace in everywhere the bare parameters by their corresponding observable values. In addition, the divergent Cooper pair or two-particle propagator (defined later) could be regularized as $\Pi(\mathbf{q},i\nu_n)\to\Pi(\mathbf{q},i\nu_n)-\Sigma_{\mathbf{k}}1/(2\epsilon_{\mathbf{k}})$. In Ref. [17], it is stated that this procedure yields the correct expression for the molecular binding energy and therefore incorporates the exact two-body scattering process. A similar conclusion has also been arrived by Falco and Stoof [18].

To relate the value of g_0 to experimentally known parameters, we compare the expression for the effective interactions between atoms in the low-density limit, $U_{eff}=U_0+g_0^2/(-\nu_0)$, with the s-wave scattering length of the atoms as a function of magnetic field, $a(B)=a_{bg}[1+\Delta B/(B-B_0)]$, where ΔB is the width of the resonance. Using the relation $U_{eff}=4\pi\hbar^2a(B)/m$ and eliminating U_0 and ν_0, we have that $g_0=2\hbar[\pi(-a_{bg}\Delta B\Delta\mu)/m]^{1/2}$. For the ^{6}Li atoms at the broad resonance $B_0=834$ G of interest here, $\Delta B\approx 300$ G, $\Delta\mu\approx 2\mu_B$,and $a_{bg}\approx -1405a_0$ [19], where μ_B is the Bohr magneton and $a_0=0.529$ is the Bohr radius.

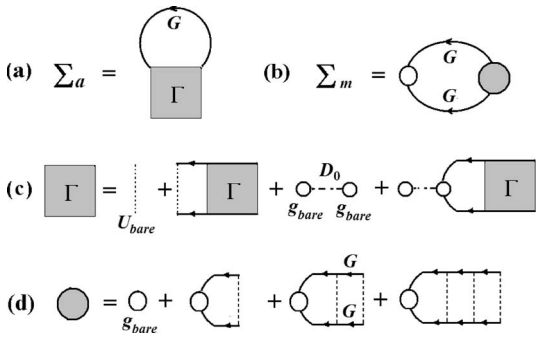

FIG. 1. Diagrammatic representations of the atomic self-energy (a), the molecular self-energy (b), the Bethe-Salpeter equation for the four-point vertex function (c), and the atom-molecule vertex function (d). The solid (dot-dashed) line represents G (D_0), the dotted line describes U_{bare}, and the open circle is the bare atom-molecule coupling g_{bare}.

We solve the atom-molecule Hamiltonian (1) in the self-consistent T-matrix approximation by summing up infinite ladder diagrams involving U_{bare} and particle-particle bubble diagrams involving g_{bare}. We consider here only the normal phase of the atomic gas. The generalization to the superfluid phase is more interesting (and, of course, more complicated) and will be addressed elsewhere. As shown in Fig. 1(a), diagrammatically the dressed Green function of atoms, $G(\mathbf{k},i\omega_m)$, and its self-energy, $\Sigma_a(\mathbf{k},i\omega_m)$, read, respectively,

$$G^{-1}(\mathbf{k},i\omega_m)=G_0^{-1}(\mathbf{k},i\omega_m)-\Sigma_a(\mathbf{k},i\omega_m), \quad (2)$$

$$\Sigma_a(\mathbf{k},i\omega_m)=T\sum_{\mathbf{q},i\nu_n}\Gamma(\mathbf{q},i\nu_n)G(\mathbf{q}-\mathbf{k},i\nu_n-i\omega_m), \quad (3)$$

where $G_0(\mathbf{k},i\omega_m)=[i\omega_m-(\epsilon_{\mathbf{k}}-\mu)]^{-1}$ is the free-atomic Green function and $i\omega_m$ ($i\nu_n$) is the fermionic (bosonic) Matsubara frequency. The approximate four-point vertex function $\Gamma(\mathbf{q},i\nu_n)$ is solved readily from the Bethe-Salpeter equation [see Fig. 1(c)]

$$\Gamma^{-1}(\mathbf{q},i\nu_n)=U_{eff}^{-1}(\mathbf{q},i\nu_n)+\Pi(\mathbf{q},i\nu_n), \quad (4)$$

where $\Pi=T\Sigma_{\mathbf{k},i\omega_m}G(\mathbf{q}-\mathbf{k},i\nu_n-i\omega_m)G(\mathbf{k},i\omega_m)-\Sigma_{\mathbf{k}}1/(2\epsilon_{\mathbf{k}})$ and the effective pairing interaction $U_{eff}=U_0+g_0^2D_0(\mathbf{q},i\nu_n)$ consists of the background interaction U_0 and the induced interaction mediated by Feshbach molecules $g_0^2D_0(\mathbf{q},i\nu_n)$, where $D_0(\mathbf{q},i\nu_n)=1/[i\nu_n-\epsilon_{\mathbf{q}}/2-\nu_0+2\mu]$ is the noninteracting molecular Green function. Note that the induced interaction dominates near the Feshbach resonance and acquires a dependence on the relative energy of the colliding atoms (i.e., retarded effects) that becomes stronger for a narrow

resonance. Analogously, it is straightforward to write down the atom-molecule vertex function

$$g^{-1}(\mathbf{q},i\nu_n) = g_0^{-1} + g_0^{-1}U_0\Pi(\mathbf{q},i\nu_n), \tag{5}$$

whose diagrammatic equivalent is shown in Fig. 1(d). This leads to the Dyson equation for molecules [see Fig. 1(b)],

$$D^{-1}(\mathbf{q},i\nu_n) = D_0^{-1}(\mathbf{q},i\nu_n) - \Sigma_m(\mathbf{q},i\nu_n), \tag{6}$$

$$\Sigma_m(\mathbf{q},i\nu_n) = -g_0\Pi(\mathbf{q},i\nu_n)g(\mathbf{q},i\nu_n), \tag{7}$$

where $D(\mathbf{q},i\nu_n)$ and $\Sigma_m(\mathbf{q},i\nu_n)$ are, respectively, the molecular Green function and self-energy.

Equations (2)–(7) form a close set of equations, which we refer to as a self-consistent theory for the two-channel model. We emphasize that in these equations, we have used a fully dressed fermionic Green function that has to be determined self-consistently. This is contrasted to the NSR scheme considered by Ohashi and Griffin, where a free fermionic Green function is adopted [14]. Actually, with replacing G by G_0 in Π and Σ_a and writing $G=G_0+G_0\Sigma_a G_0$, we reproduce the NSR formalism. Furthermore, if we set $U_{eff}\equiv U_0$ in the four-point vertex function (4) and neglect the molecular part, we recover the self-consistent theory for the single-channel model, as proposed earlier by Haussmann [12].

We have solved Eqs. (2)–(7) in a self-consistent manner by an iteration procedure, which is repeated successively to obtain G and D in higher orders until convergence is achieved. At a finite temperature $T\geqslant T_c$, the only quantity to be determined is the chemical potential μ, which has to be adjusted to satisfy the constraint for the total density $n_{tot}=n_F+2n_m$, where $n_F=2G(\mathbf{x}=\mathbf{0},\ \tau=0^-)$ and $n_m=-D(\mathbf{x}=\mathbf{0},\ \tau=0^-)$ are the density of atoms and molecules, respectively. In the calculation, the energy and length are taken in units of $\epsilon_F=\hbar^2k_F^2/2m$ and k_F^{-1}, where $k_F=(3\pi^2n_{tot})^{1/3}$ is the characteristic Fermi wavelength. Without further clarity, in the calculation we take a weak background atom-atom interaction $k_Fa_{bg}=-0.1$ in accordance with the real experimental situation [16].

It is worth noticing that the properties of Feshbach resonances are naturally characterized by a dimensionless atom-molecule coupling constant $g=g_0\hbar^{-2}m(2\pi k_F)^{-1/2}$, quoted later as an *intrinsic* width of resonances. To illustrate the physical implication of this dimensionless parameter, we note that the important energy scale of resonances can be defined as $\epsilon_R=\Delta\mu(\Delta B)_{in}$, where $(\Delta B)_{in}/\Delta B=(\Delta\mu\Delta B)/[\hbar^2/(2ma_{bg}^2)]$ [20,21]. g is related to the ratio between ϵ_R and the Fermi energy ϵ_F via $g=[\epsilon_R/\epsilon_F]^{1/4}$. Hence, if the Fermi energy is well within the characteristic energy scale of resonances—i.e., $g\gg 1$—we have a broad resonance. The opposite limit of $g\leqslant 1$ defines a narrow one. We note also that it is possible to tune the value of the intrinsic width of resonances by changing the density of the gas, as $g\propto k_F^{-1/2}\propto n_{tot}^{-1/6}$.

SUPERFLUID TRANSITION TEMPERATURE T_c

The superfluid transition temperature can be conveniently determined by the Thouless criterion

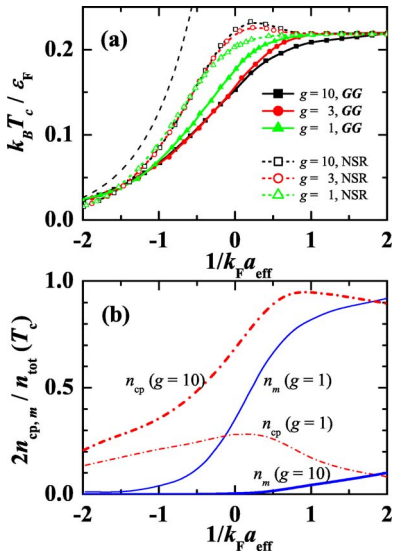

FIG. 2. (Color online) (a) The superfluid transition temperature T_c as a function of the effective coupling strength v_{eff} for dimensionless atom-molecule couplings $g=10$, 3, and 1. The solid lines with solid symbols represent the numerical results of our self-consistent theory. The dotted lines with open symbols are obtained by the NSR method. (b) The population of bare Feshbach molecules n_m at T_c and the population of preformed Cooper pairs $2n_{\mathrm{cp}}=n_F-n_{F0}$ at T_c, where $n_{F0}=2\Sigma_{\mathbf{k}}f(\epsilon_{\mathbf{k}}-\mu)$ and $f(x)$ is the Fermi distribution function.

$$[\Gamma(\mathbf{q}=\mathbf{0},i\nu_n=0)]^{-1}_{T=T_c}=0, \tag{8}$$

which describes the instability of the normal phase of the gas towards the formation of Cooper pairs of atoms [13]. In our T-matrix approximation, the Thouless criterion is consistent with the Hugenholtz-Pines condition for Feshbach molecules such that the molecular Green function (6) must develop a pole at $\mathbf{q}=i\nu_n=0$ as $T\rightarrow T_c$. As the four-point vertex function represents roughly the propagator of Cooper pairs, this consistency implies that the superfluid phase transition is accomplished by dressed molecules that involve both the preformed Cooper pairs and the Feshbach molecules [14].

In Fig. 2(a) we present our results for T_c as a function of an effective coupling constant $v_{eff}=1/(k_Fa_{eff})$ as solid symbols, where $4\pi\hbar^2a_{eff}/m=U_0+g_0^2/(-\nu_0+2\mu)$. For comparison the NSR predictions are also plotted as open symbols. Three values of the dimensionless atom-molecule coupling $g$have been considered, corresponding to situations with broad ($g=10$), medium ($g=3$), and narrow ($g=1$) resonances. For all these intrinsic resonance widths, it is obvious that our results of T_c increase monotonically with increasing the effective coupling constant. In contrast, for the broad or medium resonance, the NSR theory predicts a slight hump structure at the crossover. This maximum presumably is an artifact of the lack of self-consistency. As stated by Haussmann [12], a repulsive interaction between the pairs leads to a depression of T_c due to the composite nature of pairs. The approximating use of the free-fermionic Green function in the NSR scheme neglects the possible repulsion between dressed molecules and therefore overestimates the transition temperature. Nev-

ertheless, in two limits of weak and strong interactions, both theories give almost the same results: In the weak-coupling limit of $v_{eff} \to -\infty$, they give back the standard BCS result of $k_B T_{BCS}/\epsilon_F \to 0.614\exp[\pi v_{eff}/2]$, as shown by the dashed line. On the other hand, in the strong-coupling limit of $v_{eff} \to +\infty$, they correctly reproduce the BEC temperature of $k_B T_{BEC}/\epsilon_F \approx 0.218$.

The transition temperature depends also on the intrinsic width of Feshbach resonances in a nontrivial way. As shown in Fig. 2(b), theoretically, with decreasing the intrinsic width the character of dressed molecules changes from the large object of Cooper pairs to the small point structure of "bare" Feshbach molecules. As the fact that the transition temperature of noninteracting "bare" molecules is larger than that of performed Cooper pairs, the less population of Cooper pairs tends to enhance our predicted transition temperature. This feature should be universal with respect to the inclusion of *repulsive* molecule-molecule interactions [22] in the atom-molecule model (1), because, contrasted to the composite Cooper pairs, the transition temperature of a weakly interacting gas of *true* molecules increases with increasing repulsions between molecules [23] and thereby is always larger than that of Cooper pairs. We note that, interestingly, the NSR results go exactly in the opposite direction as the intrinsic width decreases, which is again due to the lack of self-consistency.

MOLECULAR PROBE OF THE BCS-BEC CROSSOVER

At the crossover the dressed molecules are extremely weakly bound objects, depending strongly on many-body effects. They can hardly be detected by standard techniques such as the time-of-flight expansion. In contrast, the "bare" Feshbach molecules are well-defined point structures and can easily be characterized by the usual optical spectroscopy methods. Therefore it is possible to probe the crossover indirectly through the measurement of "bare" molecules. As the dressed molecules can be expressed as a superposition of Cooper pairs in the open channel and of "bare" molecules in the closed channel [16,18]—i.e., $|\text{dressed}\rangle \simeq e^{i\phi}\sqrt{1-Z_m}|\text{open}\rangle + \sqrt{Z_m}|\text{closed}\rangle$—a candidate observable would be the amplitude Z_m of the bare molecular state. For an ultracold gas of ^{6}Li atoms, the closed (open) channel, to an excellent approximation, corresponds to the atomic singlet (triplet) state. As a result, the value of Z_m can directly be determined by selectively projecting out the singlet component of the dressed molecules with optical spectroscopy. This is exactly what was done recently by Partridge *et al.* [16], where a tiny value of Z_m ($\sim 10^{-4}$) has been observed in the unitary limit ($k_F a_{eff} \to \infty$) right on resonance.

In our theory Z_m is associated with the residue of the pole of the molecular Green function (6), which, in the spirit of the Landau quasiparticle concept, takes the form

$$D(\mathbf{q}, i\nu_n) = \frac{Z_m}{i\nu_n - (\hbar^2\mathbf{q}^2/2m_m^* - \mu_m)} \tag{9}$$

for sufficiently small $\mathbf{q}$ and $i\nu_n$. Here m_m^* is a complex effective mass and μ_m is a molecular chemical potential that goes

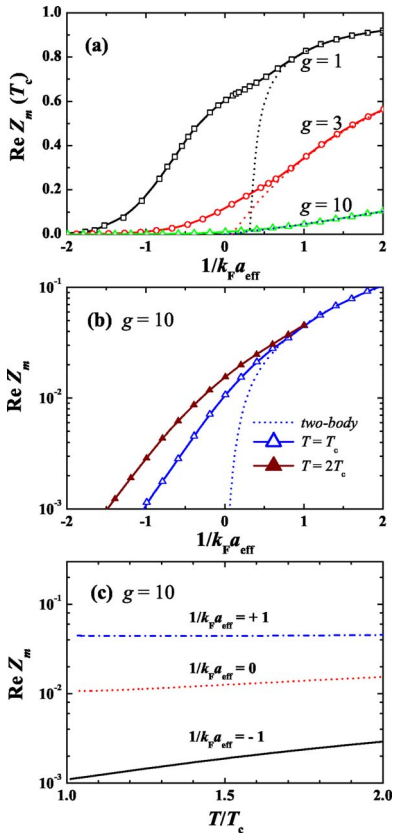

FIG. 3. (Color online) (a) The real part of Z_m at the superfluid transition as a function of v_{eff}. (b) Re $Z_m(T_c)$ and Re $Z_m(2T_c)$ at the crossover for $g=10$ in logarithmic scale. (c) Temperature dependence of Re Z_m at $v_{eff}=-1$, 0, and +1 for $g=10$.

to zero as $T \to T_c$. In Fig. 3(a), we report our self-consistent calculations for Re Z_m at T_c against the coupling v_{eff}, together with the two-body results in dashed lines [17,18]. In the strong-coupling BEC region ($v_{eff} \geqslant 1$) these results are identical as expected. However, near the Feshbach resonance they do show a large difference, which should be attributed to many-body effects.

The value of Z_m decreases significantly with increasing the atom-molecule coupling. In the large-g limit where the Cooper pairs dominate at the crossover, we find approximately $Z_m(g, v_{eff}, T) \approx Z_m(v_{eff}, T)/g^2$. For ^{6}Li gas at the broad resonance $B_0 = 834$ G, we estimate $g \approx 120$ [16]. Therefore, from our data at $g=10$, the expected value of Z_m in the unitary limit is of the order of 10^{-4}, which agrees qualitatively well with the experimental findings by Partridge *et al.* [16]. A full comparison between our theoretical results and experimental data over the entire crossover regime is shown in Fig. 4. We note that this qualitative agreement is robust against the unknown temperature in the experiment. In Figs. 3(b) and 3(c), we show the finite-temperature effect on Z_m in a logarithmic scale for $g=10$. As the temperature doubles, Z_m increases, but its order is essentially unchanged. We note also that in our calculations we do not consider the effect of a harmonic trap present in experiments, which tends to quantitatively decrease the value of Z_m.

At the crossover, the Feshbach molecules can transfer or break up into free fermions. This mechanics causes the finite

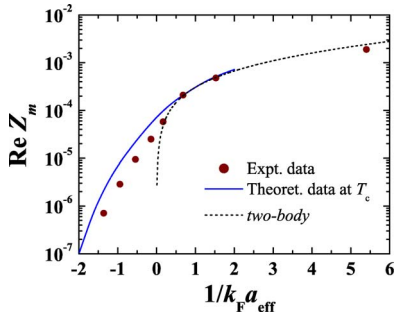

FIG. 4. (Color online) Comparison between our theoretical predictions and the experimental data (Fig. 5 in Ref. [16]) for the residue of the pole of the ^{6}Li Feshbach molecules at the broad resonance $B_0=834$ G.

lifetime of the molecules, as evidenced by the nonzero imaginary part of the residue Z_m, or of the effective mass m_m^*. We present in Fig. 5 the results for m_m^* at T_c. In the deep BEC limit, the molecules are stable so that the imaginary part of m_m^* is nearly zero and the real part Re $m_m^* \approx 2m$. While in the opposite limit of weak coupling—i.e., $v_{eff} \lesssim -1$—the imaginary part is much larger than the real part, which suggests that the molecules are overdamped. In addition, as the dimensionless atom-molecule coupling g decreases, the value of Im m_m^* becomes smaller, as expected.

CROSSOVER FROM THE NARROW TO BROAD RESONANCES

It was suggested that for a broad Feshbach resonance, the gas should be well described by the single-channel model with a single parameter (i.e., the s-wave scattering length)

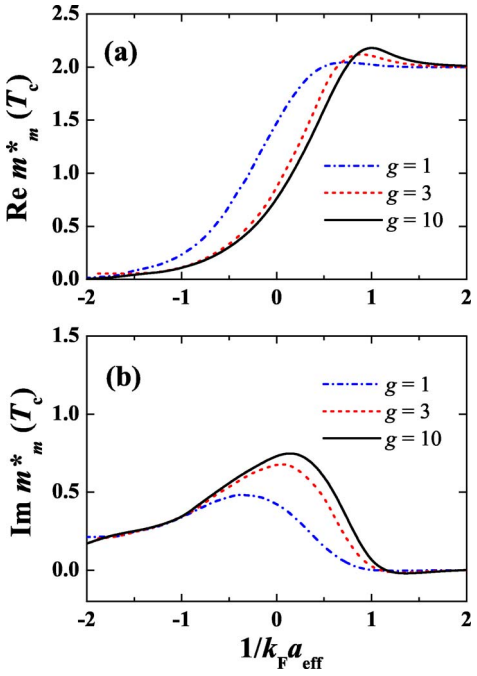

FIG. 5. (Color online) The real part (a) and imaginary part (b) of the effective mass of the bare molecules at the crossover.

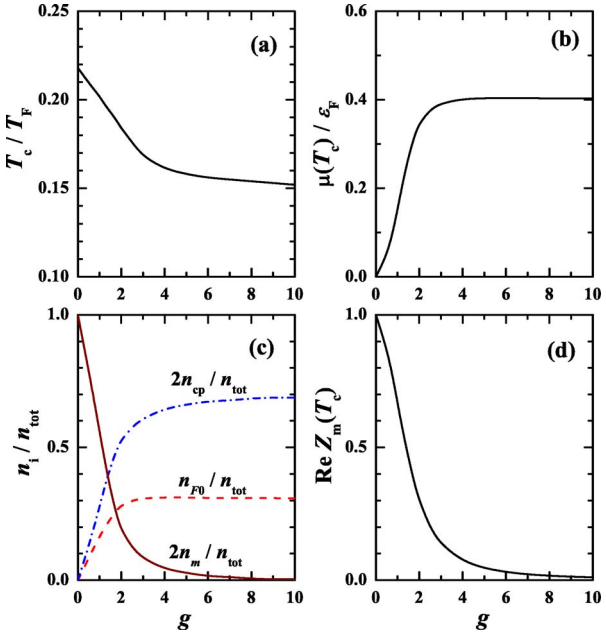

FIG. 6. (Color online) Illustration of the crossover from narrow to broad resonances on resonance $\nu_0=0$. (a) The transition temperature T_c, (b) the chemical potential at T_c, (c) the fractions of particles, and (d) Re $Z_m(T_c)$ as a function of the dimensionless atom-molecule coupling g.

[20]. This suggestion was supported from a coupled-channels calculation in the context of two-body physics [24] or from a mean-field calculation at zero temperature [21]. However, the situation is less clear beyond the mean-field approximation. In Fig. 6, we show our many-body results on resonance ($\nu_0=0$) for the critical temperature T_c, the chemical potential at T_c, and the residue of the pole of "bare" Feshbach molecules Z_m, as well as the fractions of particles. As g increases, T_c and $\mu(T_c)$ come to saturate and Z_m and n_m drop as $1/g^2$. Therefore, using our self-consist theory we verified numerically that in the broad resonance limit the behavior of the gas indeed becomes universal, towards the prediction of the single-channel model.

A possible candidate to observe the crossover from narrow to broad resonances is a gas of ^{40}K atoms at $B_0=202$ G, where we find $g \approx 10$ under the current experimental situation [1]. According to the relation $g \propto n^{-1/6}$, it is possible to obtain a narrow resonance by increasing the center density of the gas, n. This might be achieved by using more tight harmonic traps or trapping more atoms in the experiment.

CONCLUSIONS

In this paper we have developed a many-body theory of the two-channel model for an interacting Fermi gas with a Feshbach resonance. The theory includes the strong fluctuations in a self-consistent manner that is necessary at the BCS-BEC crossover. As an application, we have studied in

detail the normal-phase properties of the crossover, with special attention on the effects of intrinsic resonance widths. A reasonable superfluid transition temperature has been predicted. Our predictions for the residue of the pole of "bare" molecules agree qualitatively well with the experimental findings. We expect, therefore, that the present theory gives a reliable description of current BCS-BEC crossover physics in atomic normal Fermi gases. The generalization of our theory to the superfluid phase is under investigation and will be reported elsewhere.

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