Phase Waves in Mode-Locked Superfluorescent Lasers

P. D. Drummond
Department of Physics, University of Queensland, St Lucia, Queensland, Australia

J. D. Harvey, J. M. Dudley, and D. B. Hirst
Department of Physics, University of Auckland, Private Bag 92019, Auckland, New Zealand

S. J. Carter
Kellogg Radiation Laboratory, 106-38, California Institute of Technology, Pasadena, California 91125
(Received 26 March 1996; revised manuscript received 25 September 1996)

We present results from both theoretical and experimental studies of the noise characteristics of mode-locked superfluorescent lasers. The results show that observed macroscopic broadband amplitude noise on the laser pulse train has its origin in quantum noise-initiated "phase-wave" fluctuations, and we find an associated phase transition in the noise characteristics as a function of laser cavity detuning. [S0031-9007(96)02057-1]

PACS numbers: 42.50.Dv

The physical description of any laser system is governed by the dynamic interaction between the intracavity laser field and the atomic gain medium, and lasers can be classified according to the dominant dynamic time scale of interest. In the majority of lasers, the coherence decay times of the laser gain medium are much faster than any other interaction time scale, and the laser dynamics are accurately described by rate equations [1].

Mode-locked gas lasers, however, when operated in a high-Q cavity, can exhibit a variety of characteristics which are not explained using a simple rate-equation description of the resonant laser dynamics. The resonant interaction in these lasers is dominated by the coherent Rabi cycling of atomic dipoles in the gain medium, and the physics of the laser operation in this coherent-coupling regime is fundamentally different from that encountered in other laser systems. The combination of a periodic loss due to an active mode-locking element, and an inversion recovery time shorter than the cavity period, creates a situation where a pulse circulating in the laser encounters an identical gain at every point in the atomic medium. In this regime, the operation of the laser is equivalent to that of a *swept-gain laser amplifier*, where each point in an atomic amplifying medium is excited sequentially by a traveling-wave excitation at the speed of light. In particular, experiments have shown that the high-Q operation of mode-locked gas lasers approaches the superfluorescent limit of such a swept-gain amplifier at high gain to loss ratios [2]. Previously reported characteristics of pulses from a mode-locked argon-ion laser in such a high-Q configuration have included superfluorescent effects such as pulse durations shorter than the inverse gain bandwidth of the medium [3], and coherent ringing on the pulse trailing edge [4].

Although this coherent-coupling regime of a mode-locked superfluorescent laser (MSL) has only been studied in detail in mode-locked gas lasers, the limit is a universal one for any mode-locked laser in which the intracavity Rabi frequency is sufficiently large compared to the atomic decay rates of the atomic gain medium. One feature of particular interest in this regime is the amplification of microscopic quantum noise to the macroscopic domain in the form of large-amplitude fluctuations on the pulse envelope, first predicted by Hopf and Overman in 1979 [5]. These fluctuations have their origin in the extreme sensitivity of a pulse propagating in a superfluorescent swept-gain amplifier to microscopic instabilities in the phase of the atomic polarization ahead of it in the amplifying medium. Hopf and Overman showed that this quantum-initiated phase fluctuation took the form of a "phase wave" which could propagate into the pulse in the amplifier and cause macroscopic episodic fluctuations in pulse energy.

In this Letter, we report the first quantum theoretical model for the mode-locked superfluorescent laser, and we compare the results of the model with the first experimental quantum noise data for a swept-gain laser. We find that phase-wave fluctuations play an important role in MSL operation, and a novel and unexpected feature of the MSL is the presence of a distinct phase transition in the phase-wave fluctuations as a function of the laser cavity length. This transition is from a noisy regime, where spontaneously initiated phase waves disrupt the regular mode-locked pulse train, to a coherently stimulated quiet regime where the phase waves are suppressed. We obtain good agreement between theory and experiment. Our results are the first direct observations of phase-wave effects in a laboratory laser system.

The original system studied theoretically by Hopf and Overman in Ref. [5] consists of an infinitely extended, homogeneously broadened medium of two-level atoms in which the gain is swept at the speed of light by a traveling-wave excitation pulse. This allows the atoms

 0031-9007/97/78(5)/836(4)$10.00

to be fully inverted, as seen by a quantum-initiated pulse traveling a short distance behind the excitation. In the case of a large gain and a small value of linear loss, the steady-state superfluorescent pulse in the amplifier has an area of π [2]. The steady state in the amplifier is maintained because the pulse gains energy from coherent superfluorescent emission from the atoms at the same rate that it loses energy to the medium through linear losses. We note that this swept-gain superfluorescent behavior is distinct from recent proposals of stationary intracavity superfluorescent lasers [6].

In the swept-gain system, the sensitivity of the propagating steady-state pulse to the initial polarization of the atoms arises because of the coherent transfer of energy to the field from the atomic population. In particular, when the leading edge of the steady-state pulse initially begins to interact with the atomic population, a weak-field period of interaction exists where the value of the field can be significantly influenced by any quantum fluctuations in the atomic polarization. These can spontaneously destabilize the pulse by completely reversing the phase of the leading edge of the electric field. This transient phase difference between the leading and trailing parts of the field prevents the coherent transfer of energy to the field, and the pulse rapidly loses energy—until its phase evolves and is restored to a uniform value. Now it can again induce coherent superfluorescent emission, and is amplified to its original steady-state value. The difference in phase between the leading and trailing edges of the electromagnetic field which sweeps through the pulse and then disappears, causing the macroscopic collapse followed by the revival of the steady-state pulse, is what is referred to as a phase-wave fluctuation.

We have extended the theory of Hopf and Overman to accurately describe our experimental MSL system based on an actively mode-locked argon-ion laser. The usual two-level plane wave model of the amplifying medium using the Maxwell-Bloch equations [4] greatly overestimates atomic cooperativity in a realistic experimental system, thus predicting phase-wave fluctuations that are orders of magnitude greater than experimental observations. We use a set of modified coupled atom-field equations [7] which include the realistic effects of inhomogeneous Doppler broadening, the presence of multiple magnetic sublevels due to Zeeman splitting in the gain medium, multilevel atomic pumping effects, and the radial variations of the intracavity mode intensity. The dominant quantum noise source is polarization noise, and we include this using a normally ordered representation [1,8] similar to that employed in superfluorescence calculations [9,10]. Finally, to simulate the MSL operation, the theoretical equations are solved numerically subject to the constant loss of the laser cavity mirrors, and the time-varying transmission of the active mode-locker [11].

The resulting field and atomic equations in the rotating-wave and paraxial approximations are

$$\left[\frac{\partial}{\partial z} + \left(\frac{1}{c} - \frac{1}{v}\right)\frac{\partial}{\partial t'}\right]\Omega(t', \mathbf{x}) = \sum_{\mathbf{m},\mathbf{j}} g_{\mathbf{j}}(\mathbf{x}) R^{-}_{\mathbf{mj}}(t'), \quad (1)$$

with

$$\begin{aligned}
\frac{\partial}{\partial t'} R^{-}_{\mathbf{mj}}(t') &= -\gamma_n R^{-}_{\mathbf{mj}} + F_m \Omega[R^{(2)}_{\mathbf{mj}} - R^{(1)}_{\mathbf{mj}}] \\
&\quad + \Gamma_{\mathbf{mj}}(t'), \\
\frac{\partial}{\partial t'} R^{(2)}_{\mathbf{mj}}(t') &= -\gamma_p [R^{(2)}_{\mathbf{mj}} - 1/\mathcal{M}] - \mathrm{Re}[\Omega^* R^{-}_{\mathbf{mj}}] \\
&\quad \times U(\mathbf{x_j})/2, \\
\frac{\partial}{\partial t'} R^{(1)}_{\mathbf{mj}}(t') &= -\gamma_l R^{(1)}_{\mathbf{mj}} + \mathrm{Re}[\Omega^* R^{-}_{\mathbf{mj}}] U(\mathbf{x_j})/2. \quad (2)
\end{aligned}$$

Here we use a co-moving reference frame, defined by $t' = t - z/v$, as the time coordinate, and the $\mathbf{x}$ coordinate defines a transverse spatial position in the atomic medium. The field variable in these equations is the scaled Rabi frequency $\Omega(t', \mathbf{x})$, and the atomic variables are the scaled polarization and population amplitudes $R^{-}_{\mathbf{mj}}$ and $R^{(1,2)}_{\mathbf{mj}}$, respectively. To account for the radial variation of the intracavity laser intensity, the spatial coordinate $\mathbf{x}$ is divided into a lattice of points $\mathbf{x_j}$. The atomic populations are similarly divided into discrete classes, with a combined class index $\mathbf{m} = (m, n)$, indicating both a particular Zeeman-split magnetic sublevel (m) and a particular value of Doppler detuning (n). The decay rates here are the usual polarization decay rate $\gamma_\perp$, the lower-level decay rate γ_l, and the pumping equilibration rate γ_p.

The scaled Rabi frequency $\Omega(t', \mathbf{x})$ represents the corresponding plane-polarized electric field operator, where

$$\Omega(t', \mathbf{x}) \sim (2\overline{\mu}/\hbar)\hat{E}^{(+)}(t, \mathbf{x})/u(\mathbf{x}). \quad (3)$$

Here $u(\mathbf{x})$ is the amplitude of the normalized fundamental transverse mode at the laser resonance frequency ω_0, and $U(\mathbf{x}) = |u(\mathbf{x})|^2$. The dipole moment $\overline{\mu}$ is an rms value, averaged over the $\mathcal{M}$ coupled magnetic sublevels involved in the $\Delta m = 0$ lasing transition. The relative dipole moment squared of the mth magnetic sublevel is $F_m = (\mu_m/\overline{\mu})^2$. We include the diffraction of the fundamental cavity mode, but neglect the additional effects of higher-order modes during propagation. Spatial filtering is included via a projection onto the fundamental cavity mode after each pass of length L, so that $\Omega(t', \mathbf{x})$ is uniform at the start of each pass.

The scaled polarization and population amplitudes are defined in terms of transition operators representing the average behavior for a group of atoms in a particular magnetic sublevel, with a particular Doppler detuning, and at a particular spatial position. The possible Doppler detunings are divided into groups of equal atomic population, each of width $\Delta\omega_n$ and center frequency ω_n, and with decay rate $\gamma_n = \gamma_\perp + i(\omega_n - \omega_0)$. The individual transition operators for the νth atom are defined as $\hat{\sigma}^{-}_{m\nu} =$

$|1,m\rangle_\nu\langle 2,m|_\nu$ and $\hat{\sigma}^{(l)}_{m\nu} = |l,m\rangle_\nu\langle l,m|_\nu$, with $l = (1,2)$. The cell-averaged polarization and population amplitudes are therefore defined:

$$R^-_{\mathbf{mj}}(t') \sim (\mu_m/\overline{\mu}N_{\mathbf{j}}) \sum_{\nu\in S(n,\mathbf{j})} 2\hat{\sigma}^-_{m\nu}(t')/u(\mathbf{x_j}),$$
$$R^{1,2}_{\mathbf{mj}}(t') \sim (1/N_{\mathbf{j}}) \sum_{\nu\in S(n,\mathbf{j})} \hat{\sigma}^{1,2}_{m\nu}(t'), \quad (4)$$

where $N_{\mathbf{j}} = \rho(\mathbf{x_j},\omega_0)\Delta\omega_0\Delta V_{\mathbf{j}}$ is the equilibrium number of inverted atoms in each lattice cell of volume $\Delta V_{\mathbf{j}}$. The density (at line center) of lasing atoms is $\rho(\mathbf{x},\omega_0)$ per unit volume and frequency, and the mean coupling coefficient $g_{\mathbf{j}}(\mathbf{x})$ has the standard form of

$$g_{\mathbf{j}}(\mathbf{x}) = \frac{\omega_0|\bar{\mu}|^2\rho(\mathbf{x_j},\omega_0)\Delta\omega_0\Delta V_{\mathbf{j}}}{2\epsilon_o\hbar c}\,\delta^3(\mathbf{x}-\mathbf{x_j}). \quad (5)$$

Finally, we note that the quantum noise term $\Gamma_{\mathbf{mj}}(t)$ must have nonvanishing correlations [7], and is given by

$$\langle\Gamma^*_{\mathbf{mj}}(t)\Gamma_{\mathbf{m'j'}}(t')\rangle = \frac{8\gamma_\perp F_m\delta_{\mathbf{jj'}}\delta_{\mathbf{mm'}}}{U(\mathbf{x_j})\mathcal{M}N_{\mathbf{j}}}\,\delta(t-t'). \quad (6)$$

We have simulated our argon-ion laser in a high-Q cavity configuration at an operating wavelength of 514.5 nm. This is a $J = 3/2 \rightarrow J = 5/2$ lasing transition, with Zeeman multiplicity of $\mathcal{M} = 4$, and known dipole moments of 4.3×10^{-30} C m and 5.3×10^{-30} C m for the $m = \pm3/2$ and $m = \pm1/2$ transitions. The atomic lifetimes were estimated as $\gamma_p = 1.25$ ns, $\gamma_l = 0.36$ ns, and $\gamma_\perp = 0.64$ ns [12]. The measured small-signal intensity gain per pass was 15%, and the measured output-coupler loss was 4%. The Doppler line had a measured FWHM of 4.1 THz. The cavity round-trip time was $T_{\text{cav}} = 2L/c = 13.2$ ns, and the mode waist was 0.64 mm in the center of the gain region. These characteristics define the model, since the atomic inversion density can be calculated from the measured small-signal gain.

The simulation results show that quantum-initiated phase-wave fluctuations play an important role in the behavior of the MSL, and that their characteristics are a sensitive function of the detuning between the round-trip time of the passive laser cavity, T_{cav}, and the modulation period of the active mode-locker, T_{ml}.

When the detuning $\Delta T = T_{\text{cav}} - T_{\text{ml}}$ is positive or zero, we have found that no stable steady-state pulse ever develops in the MSL, because the pulse evolution is episodically disrupted by phase-wave events. A typical simulation result for $\Delta T = 0$ is shown in Fig. 1(a). Here, the pulse evolution in the laser is plotted as a function of the number of cavity round trips and the intracavity pulse intensity is plotted in the co-moving time frame described above. This figure clearly shows how a phase wave (initiated from quantum noise on the pulse leading edge) causes a macroscopic change in the pulse intensity. Note also the oscillatory structure on the pulse trailing

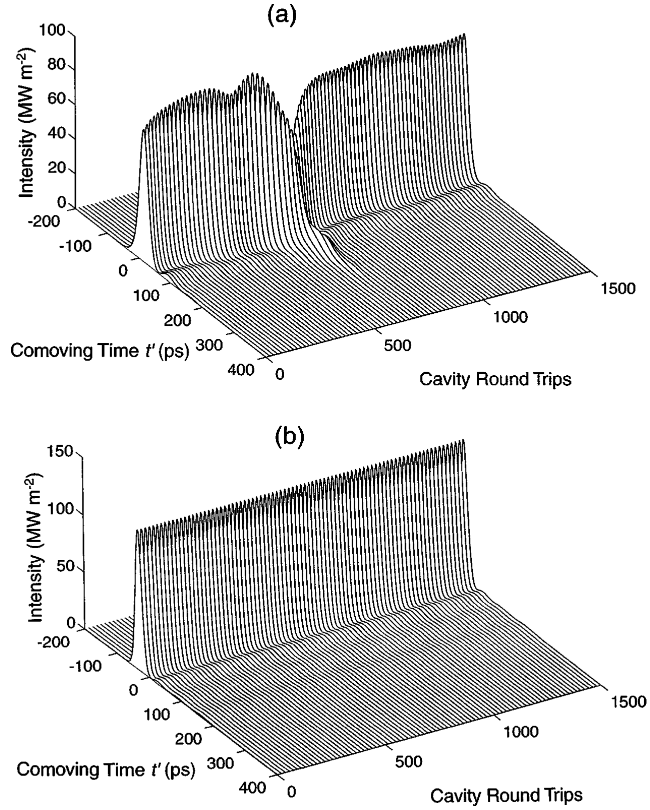

FIG. 1. Simulation results illustrating (a) effects of quantum-initiated phase waves on the peak intracavity intensity of a pulse in the MSL at zero detuning, and (b) noise free operation at a value of negative detuning ($\Delta T = -0.1$ ps).

edge due to the coherent ringing of the atomic polarization. This zero-detuning regime corresponds most closely to the original swept-gain superfluorescent amplifier studied by Hopf and Overman, but our results indicate that phase-wave events are very robust, and survive (with a reduced fluctuation intensity) even in the more realistic experimental system of the MSL. For cavity lengthening ($\Delta T > 0$), similar results to those in Fig. 1(a) are obtained, but with phase waves occurring more often and with a larger amplitude, since with a longer cavity, there is reduced loss on the pulse leading edge, and this effectively increases the duration of the weak-field initiation regime, amplifying the phase-wave effects.

In contrast, Fig. 1(b) shows simulation results for a detuning $\Delta T = -0.1$ ps, corresponding to a decrease in the laser cavity length of 15 μm. In this case, the phase waves are suppressed and a stable steady-state pulse can form in the MSL. Physically, this arises because, with this value of detuning, a pulse in the MSL arrives at the mode-locker slightly earlier so the pulse leading edge encounters more loss. This sharpens the leading edge, reducing the duration of the weak-field initiation regime and therefore suppressing the effect of phase waves on the pulse evolution. It is significant to note that, in this regime, a properly adjusted MSL can produce a train of pulses with high peak power that is shot-noise limited at frequencies in excess of a few MHz.

An unexpected result of the simulations is that this transition from the noisy phase-wave regime to the quiet regime occurs abruptly as a function of detuning, and we have carried out both experiments and more comprehensive simulations to study this in detail. In order to characterize the laser stability, we chose to measure the laser amplitude noise (referred to as the shot-noise level) at three different frequencies as a function of the cavity detuning ΔT. Typical experimental results are shown in Fig. 2(a), where the abrupt transition from a quiet to a noisy regime is very clear. We compare these experimental results with the results from numerical simulations in Fig. 2(b). The simulation results are in good agreement with experiment, clearly showing the phase-wave transition as a function of cavity length detuning, and the quantitative agreement between simulation and experiment is excellent at high frequencies. The discrepancies at lower frequencies are attributed to uncertainties in the atomic pumping parameters used in the simulations [12], and to residual mode-locker technical noise.

Although it is well known experimentally that mode-locked laser stability depends on the cavity-mode-locker detuning [13], we provide here the first evidence for phase-wave events predicted to occur in high-Q swept-gain laser amplifiers, as opposed to Raman amplifiers [14]. These experimental results relate only to one transition in an argon-ion laser, but qualitatively similar behavior has been observed in other mode-locked lines of this laser and in a mode-locked krypton ion laser.

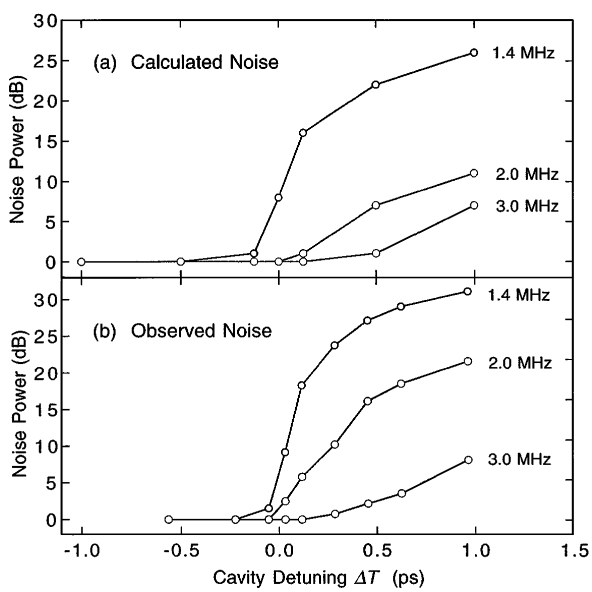

FIG. 2. Noise power relative to shot noise for the MSL as a function of cavity detuning at three characteristic frequencies; (a) from the numerical simulations, and (b) from experimental results. The detector efficiency here was 0.25%.

We expect phase waves and the corresponding phase-wave transition to be a universal characteristic of low-dispersion high-Q mode-locked lasers. Thus, we have given a first principles test of the quantum theory of a new class of laser. The phase-wave transition also shows that it is feasible to combine the high intensity of superfluorescence with a stable, low-noise output, and this could have technological applications in areas such as communications systems.

[1] H. Haken, *Laser Theory* (Springer-Verlag, Berlin, 1984).
[2] R. Bonifacio, F. A. Hopf, P. Meystre, and M. O. Scully, Phys. Rev. A **12**, 2568 (1975).
[3] J. D. Harvey, R. Leonhardt, P. D. Drummond, and S. J. Carter, Phys. Rev. A **40**, 4789 (1989).
[4] J. M. Dudley, J. D. Harvey, and R. Leonhardt, Opt. Lett. **15**, 335 (1990); J. M. Dudley, J. D. Harvey, and R. Leonhardt, J. Opt. Soc. Am. B **10**, 840 (1993).
[5] F. A. Hopf and E. A. Overman II, Phys. Rev. A **19**, 1180 (1979).
[6] F. Haake, M. I. Kolobev, and S. Reynaud, Phys. Rev. A **54**, 1625 (1996).
[7] P. D. Drummond and M. G. Raymer, Phys. Rev. A **44**, 2072 (1991).
[8] R. J. Glauber, Phys. Rev. **130**, 2529 (1963); E. C. G. Sudarshan, Phys. Rev. Lett. **10**, 277 (1963).
[9] D. Polder, M. F. H. Schuurmans, and Q. F. H. Vrehen, Phys. Rev. A **19**, 1192 (1979); F. Haake, H. King, G. Schroeder, J. Haus, and R. J. Glauber, Phys. Rev. A **20**, 2047 (1979); P. D. Drummond and J. H. Eberly, Phys. Rev. A **25**, 3446 (1982).
[10] R. H. Dicke, Phys. Rev. **93**, 99 (1954).
[11] The double-pass mode-locker intensity transmission applied after each interval of $2L$ was well approximated by $\mathcal{T}_{\mathrm{ml}}(t') = \cos^8(\pi t'/T_{\mathrm{ml}}) + 0.26|\sin^5(\pi t'/T_{\mathrm{ml}})|$.
[12] M. H. Dunn and J. N. Ross, Prog. Quantum Electron. **4**, 233 (1976).
[13] D. L. MacFarlane and L. W. Casperson, J. Opt. Soc. Am. B **4**, 1777 (1987); G. H. C. New, Opt. Lett. **15**, 1306 (1990).
[14] J. C. Englund and C. M. Bowden, Phys. Rev. A **46**, 578 (1992); D. C. MacPherson, R. C. Swanson, and J. L. Carlsten, Phys. Rev. A **40**, 6745 (1989).