Quantum phase-space analysis of the pendular cavity

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(Received 19 April 2004; published 21 October 2004)

We perform a quantum-mechanical analysis of the pendular cavity, using the positive-*P* representation, showing that the quantum state of the moving mirror, a macroscopic object, has noticeable effects on the dynamics. This system has previously been proposed as a candidate for the quantum-limited measurement of small displacements of the mirror due to radiation pressure, for the production of states with entanglement between the mirror and the field, and even for superposition states of the mirror. However, when we treat the oscillating mirror quantum mechanically, we find that it always oscillates, has no stationary steady state, and exhibits uncertainties in position and momentum which are typically larger than the mean values. This means that previous linearized fluctuation analyses which have been used to predict these highly quantum states are of limited use. We find that the achievable accuracy in measurement is far worse than the standard quantum limit due to thermal noise, which, for typical experimental parameters, is overwhelming even at 2 mK

DOI: 10.1103/PhysRevA.70.043815 PACS number(s): 42.50.Lc, 42.50.−p, 42.50.Pq, 03.65.Yz

I. INTRODUCTION

The pendular cavity, or Fabry-Perot cavity in which one of the mirrors is free to oscillate, has previously been investigated by a number of researchers, both experimentally [1–3] and theoretically [4–13]. Closely related schemes have been theoretically proposed to entangle mirrors [14] and create superposition states of a mirror [15]. Common to almost all the theoretical treatments is a linearization of quantum Langevin equations around their classical steady-state solutions [16]. This then allows for the relatively simple calculation of spectral quantities which may be measured outside the cavity. Results obtained in this way have been used to analyze the sensitivity of gravity wave interferometers [17], predict the suppression of quantum noise [4,5], propose quantum nondemolition measurements of photon number [6], analyze the quantum limits to measurements with an atomic force microscope [7], analyze the quantum noise in position measurements of the oscillating mirror [9], calculate the phase noise in the cavity field [11], predict the entanglement of macroscopic oscillators via radiation pressure [14], and propose the quantum locking of interferometer mirrors [13]. Using the state-vector approach so common in quantum computing theory, it has been proposed that quantum superpositions, entanglement, and near-number states of the cavity field, along with superposition states of the mirror, can be produced with this system [8]. Using a similar state-vector approach, it has been proposed that quantum superpositions of a mirror may be created by the interaction with a single photon [15].

It is well known that the linearized fluctuation analysis used in the majority of the theoretical papers cited above is limited in its applicability. It has been shown, for example, that the mean-field equations derived in this way can give misleading results for traveling-wave second harmonic generation [18–20] and for the intracavity interaction between light and condensed atoms [21–23] as well as in Raman photoassociation of atomic Bose-Einstein condensates [24–27]. The spectra calculated via this method are also known to not be accurate near any critical points of the system, as has been shown with the optical parametric oscillator [28,29]. There are two conditions which must be fulfilled for a linearized analysis to be trustworthy. The first has to do with the sign of the real part of the eigenvalues of the drift matrix of the equations written for the fluctuations; if these have the wrong sign the fluctuations can grow exponentially and the analysis loses its validity. The second has to do with the size of the fluctuations themselves, in relation to the classical steady-state values. As we will show, this second condition is not fulfilled for this system, as the quantum state of the mirror, or more accurately, of the mirror phonons, is reasonably expected to be thermal. A characteristic of a thermal state is that the variance is larger than the mean value, which therefore makes any expression of the mirror phonons as having some well-defined classical mean value plus small fluctuations rather dubious. Even though cooling of the mirror via feedback mechanisms has been achieved [30], and analyzed theoretically [31], due to the nature of the coupling between the electromagnetic field and the mirror phonons, it seems that all that can be achieved is a thermal state at a lower temperature, so that the problem remains.

To treat the macroscopic mirror quantum mechanically we will begin with the Hamiltonian approach of Law [32], in the approximation that only a single optical mode is important, and extended to include cavity pumping and damping. To treat the fluctuations of the mirror which result from its coupling to a thermal reservoir, we will use the Brownian motion master equation developed by Diósi [33], which is suitable for the temperatures we will consider here. Following a common procedure in quantum optics [34], we will develop a Fokker-Planck equation in the positive-*P* representation [35]. This Fokker-Planck equation allows us to write stochas-

tic differential equations which are an exact mapping from the system master equation, and which can be used to calculate any desired normally ordered operator moments.

II. SYSTEM AND HAMILTONIAN

We consider a system of a pumped Fabry-Perot cavity in which one of the mirrors is free to undergo oscillatory motion due to both the light pressure and thermal fluctuations. We use the standard annihilation $\hat{a}$ and creation, $\hat{a}^\dagger$ operators for the electromagnetic field, and the operators $\hat{x}$ and $\hat{p}$ for the displacement from the equilibrium position and the momentum of the mirror, which will be treated as a harmonic oscillator. Neglecting the coupling of the mirror to its bath for the moment, we can write the Hamiltonian as

$$\hat{H} = \hat{H}_{free} + \hat{H}_{int} + \hat{H}_{pump} + \hat{H}_{bath}, \tag{1}$$

where

$$\hat{H}_{free} = \hbar\omega_0\hat{a}^\dagger\hat{a} + \frac{1}{2m}\hat{p}^2 + \frac{1}{2}m\omega_m^2\hat{x}^2,$$

$$\hat{H}_{int} = -\hbar g\hat{a}^\dagger\hat{a}\hat{x},$$

$$\hat{H}_{pump} = i\hbar\epsilon(\hat{a}^\dagger e^{i\omega_0 t} - \hat{a}e^{-i\omega_0 t}),$$

$$\hat{H}_{bath} = \Gamma\hat{a}^\dagger + \Gamma^\dagger\hat{a}. \tag{2}$$

In the above, ω_0 is the field frequency, ϵ represents the classical real pump, m is the mass of the mirror, ω_m is the mirror oscillation frequency, and $g=\omega_0/L$ is the coupling between the mirror and the cavity field, with L being the length of the cavity. The Γ's represent optical bath operators. The damping of the mirror, which we will treat as Markovian, will be included at the next step.

We now wish to write a master equation for the density matrix of our combined system in a frame rotating at ω_0. To do this, we will make two different, but consistent approximations for the damping of the cavity and the mirror. The cavity reservoir will be considered to be at zero temperature, which is consistent with the very high temperatures necessary to produce thermal photons at the frequencies involved. The mirror reservoir will be treated as being at a finite temperature, which is necessary because of the number of thermal phonons which will be present in the system. As the temperatures required to create these respective excitations differ by many orders of magnitude, these approximations are not contradictory. This process gives us

$$\begin{aligned} i\hbar\frac{\partial\hat{\rho}}{\partial t} &= [\hat{H},\hat{\rho}] + \hat{\mathcal{L}}\hat{\rho} \\ &= \left[\frac{1}{2m}\hat{p}^2 + \frac{1}{2}m\omega_m^2\hat{x}^2 - \hbar g\hat{a}^\dagger\hat{a}\hat{x} - i\hbar\epsilon(\hat{a}-\hat{a}^\dagger),\hat{\rho}\right] \\ &\quad + i\hbar\gamma(2\hat{a}\hat{\rho}\hat{a}^\dagger - \hat{a}^\dagger\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^\dagger\hat{a}) + D_m\hat{\rho}, \end{aligned} \tag{3}$$

where γ represents the loss rate through the fixed mirror and $D_m\hat{\rho}$ represents the mirror damping, using the Brownian motion master equation developed by Diósi [33],

$$D_m\hat{\rho} = \gamma_m[\hat{x},\{\hat{p},\hat{\rho}\}] - \frac{i\hbar\gamma_m}{2\lambda_{dB}^2}[\hat{x},[\hat{x},\hat{\rho}]] - \frac{i\kappa\gamma_m\lambda_{dB}^2}{\hbar}[\hat{p},[\hat{p},\hat{\rho}]]. \tag{4}$$

In the above, γ_m is the mirror damping rate, which depends on temperature through the mechanical quality factor, $\lambda_{dB} = \hbar/\sqrt{4mk_BT}$, the thermal de Broglie wavelength of the mirror, with k_B Boltzmann's constant and T the temperature. κ is a numerical factor which must be greater than 1 for this master equation to be of the Lindblad form, but is not important here as it leads to terms of the order $\hbar\omega_m/k_BT$, which we will show to be insignificant at the temperatures we consider.

III. EXPANSION OF THE MIRROR IN COHERENT STATES

Rather than writing the Heisenberg equations of motion, which are difficult to solve, we will make use of the original definition [34] of the annihilation and creation operators in terms of $\hat{x}$ and $\hat{p}$ and develop stochastic differential equations in the positive-P representation [35]. This allows us to use c-number equations which describe all the quantum properties of the mirror dynamics contained in the original master equation. We will describe the operators $\hat{x}$ and $\hat{p}$, in terms of the operators $\hat{b}$ and $\hat{b}^\dagger$, where

$$\hat{x} = A(\hat{b} + \hat{b}^\dagger), \quad \hat{p} = B(\hat{b} - \hat{b}^\dagger), \tag{5}$$

with

$$A = \sqrt{\frac{\hbar}{2m\omega_m}}, \quad B = -i\sqrt{\frac{\hbar m\omega_m}{2}}, \tag{6}$$

and $[\hat{b},\hat{b}^\dagger]=1$. Writing the equations using these variables is advantageous because it allows us to automatically define a P-representation of the density matrix in terms of an expansion in the minimum uncertainty (coherent) states $|\beta\rangle$, defined as $\hat{b}|\beta\rangle = \beta|\beta\rangle$. It also means that the mirror quadrature variances have a coherent state or vacuum value of 1, the same as for the electromagnetic field. In fact, A and $|B|$ represent the standard quantum limits (SQL's) for measurementof the mirror position and momentum, respectively. We note that these phonon annihilation and creation operators have previously been used to describe the mirror, but not in the context of developing phase-space representation stochastic differential equations [8,36,37].

In terms of these new variables, the master equation for the mirror damping is now written as

$$\begin{aligned} D_m\hat{\rho} &= \gamma_m AB[\hat{b}+\hat{b}^\dagger,\{\hat{b}-\hat{b}^\dagger,\hat{\rho}\}] - \frac{i\hbar\gamma_m A^2}{2\lambda_{dB}^2}[\hat{b}+\hat{b}^\dagger,[\hat{b}+\hat{b}^\dagger,\hat{\rho}]] \\ &\quad - \frac{i\kappa\gamma_m\lambda_{dB}^2}{\hbar}B^2[\hat{b}-\hat{b}^\dagger,[\hat{b}-\hat{b}^\dagger,\hat{\rho}]]. \end{aligned} \tag{7}$$

IV. STOCHASTIC EQUATIONS

Using the well-known operator correspondences for the P representation [34], we may map the master equation onto a partial differential equation for the P function of the system,

$$\begin{aligned}\frac{dP}{dt} = \Bigg(&- \Bigg[\frac{\partial}{\partial\alpha}(\epsilon - \gamma\alpha + igA\alpha[\beta+\beta^*]) + \frac{\partial}{\partial\alpha^*}(\epsilon^* - \gamma\alpha^* - igA\alpha^*[\beta+\beta^*]) + \frac{\partial}{\partial\beta}(-i\omega_m\beta + igA|\alpha|^2 - \gamma_m[\beta-\beta^*]) \\ &+ \frac{\partial}{\partial\beta^*}(i\omega_m\beta^* - igA|\alpha|^2 - \gamma_m[\beta^*-\beta]) \Bigg] \\ &+ \frac{1}{2}\Bigg\{ \frac{\partial^2}{\partial\alpha\partial\beta}(-igA\alpha) + \frac{\partial^2}{\partial\beta\partial\alpha}(-igA\alpha) + \frac{\partial^2}{\partial\alpha^*\partial\beta^*}(igA\alpha^*) + \frac{\partial^2}{\partial\beta^*\partial\alpha^*}(igA\alpha^*) \\ &+ \frac{\partial^2}{\partial\beta^2}\left(\gamma_m\left[1 - \frac{2k_BT}{\hbar\omega_m} + \frac{\kappa\hbar\omega_m}{4k_BT}\right]\right) + \frac{\partial^2}{\partial\beta^{*2}}\left(\gamma_m\left[1 - \frac{2k_BT}{\hbar\omega_m} + \frac{\kappa\hbar\omega_m}{4k_BT}\right]\right) \\ &+ \frac{\partial^2}{\partial\beta\partial\beta^*}\left(-\gamma_m\left[1 - \frac{2k_BT}{\hbar\omega_m} - \frac{\kappa\hbar\omega_m}{4k_BT}\right]\right) + \frac{\partial^2}{\partial\beta^*\partial\beta}\left(-\gamma_m\left[1 - \frac{2k_BT}{\hbar\omega_m} - \frac{\kappa\hbar\omega_m}{4k_BT}\right]\right)\Bigg\}\Bigg) P(\alpha,\alpha^*,\beta,\beta^*,t). \end{aligned} \tag{8}$$

The diffusion matrix of the above equation is

$$D = \begin{bmatrix} 0 & 0 & -igA\alpha & 0 \\ 0 & 0 & 0 & igA\alpha^* \\ -igA\alpha & 0 & \gamma_m\left(1 - \frac{2k_BT}{\hbar\omega_m} + \frac{\kappa\hbar\omega_m}{4k_BT}\right) & -\gamma_m\left(1 - \frac{2k_BT}{\hbar\omega_m} - \frac{\kappa\hbar\omega_m}{4k_BT}\right) \\ 0 & igA\alpha^* & -\gamma_m\left(1 - \frac{2k_BT}{\hbar\omega_m} - \frac{\kappa\hbar\omega_m}{4k_BT}\right) & \gamma_m\left(1 - \frac{2k_BT}{\hbar\omega_m} + \frac{\kappa\hbar\omega_m}{4k_BT}\right) \end{bmatrix}. \tag{9}$$

We note here that this drift matrix has diverging terms as $T\to 0$, but this is not a problem as the Diósi master equation is valid in the limit where $k_BT \gg \hbar\omega_m$. As a physical example, in Ref. [2], we find $\omega_m = 1.6\times 10^5\ \mathrm{s}^{-1}$, so that $\hbar\omega_m = 1.72\times 10^{-29}$ J, whereas $k_BT = 5.8\times 10^{-23}$ J at 4.2 K, the temperature which we will mainly use in our investigations.

If we wish to treat Eq. (8) as a genuine Fokker-Planck equation which we may map onto stochastic differential equations, the matrix D must be positive-definite. Numerical investigations using typical parameters show that this is not the case, therefore for quantum calculations we will have to use the positive-P representation [35]. The positive-P representation equations in a doubled phase space can be found by the simple change of variables $\alpha^* \to \alpha^+, \beta^* \to \beta^+$, so that (noting that $\overline{\alpha^+} = \overline{\alpha^*}$ only in the mean and similarly for β^+), we now have four independent stochastic variables. Ignoring the terms proportional to $\hbar\omega_m/k_BT$ due to their small relative size, one possible factorization of the diffusion matrix, $D = NN^{\mathrm{T}}$, of Eq. (9) is

$$N = \begin{bmatrix} 0 & \sqrt{\frac{-igA\alpha}{2}} & \sqrt{\frac{igA\alpha}{2}} & 0 & 0 \\ 0 & 0 & 0 & \sqrt{\frac{igA\alpha^+}{2}} & -\sqrt{\frac{-igA\alpha^+}{2}} \\ -\sqrt{\gamma_m\left(1 - \frac{2k_BT}{\hbar\omega_m}\right)} & \sqrt{\frac{-igA\alpha}{2}} & -\sqrt{\frac{igA\alpha}{2}} & 0 & 0 \\ \sqrt{\gamma_m\left(1 - \frac{2k_BT}{\hbar\omega_m}\right)} & 0 & 0 & \sqrt{\frac{igA\alpha^+}{2}} & \sqrt{\frac{-igA\alpha^+}{2}} \end{bmatrix}, \tag{10}$$

which allows us to write a set of four stochastic differential equations (note that the Itô form and the Stratonovich form of these equations are identical),

$$\frac{d\alpha}{dt} = \epsilon - \gamma\alpha + igA\alpha(\beta+\beta^+) + \sqrt{\frac{-igA\alpha}{2}}(\eta_2 + i\eta_3),$$

$$\frac{d\alpha^+}{dt} = \epsilon^* - \gamma\alpha^+ - igA\alpha^+(\beta+\beta^+) + \sqrt{\frac{igA\alpha^+}{2}}(\eta_4 - i\eta_5),$$

$$\frac{d\beta}{dt} = -i\omega_m\beta - \gamma_m(\beta-\beta^+) + igA\alpha^+\alpha - \sqrt{\gamma_m\left(1-\frac{2k_BT}{\hbar\omega_m}\right)}\eta_1 + \sqrt{\frac{-igA\alpha}{2}}(\eta_2 - i\eta_3),$$

$$\frac{d\beta^+}{dt} = i\omega_m\beta^+ + \gamma_m(\beta-\beta^+) - igA\alpha^+\alpha + \sqrt{\gamma_m\left(1-\frac{2k_BT}{\hbar\omega_m}\right)}\eta_1 + \sqrt{\frac{igA\alpha^+}{2}}(\eta_4 + i\eta_5). \tag{11}$$

In the above, the real Gaussian noise terms have the correlations

$$\overline{\eta_i(t)} = 0, \quad \overline{\eta_i(t)\eta_j(t')} = \delta_{ij}\delta(t-t'). \tag{12}$$

The set of coupled equations (11) may be integrated numerically, with averages taken over a large number of stochastic trajectories, which allows for the probabilistic calculation of any desired normally ordered operator moments. As an example, with N trajectories, we have

$$\langle \hat{a}^{\dagger m}\hat{a}^n\rangle = \lim_{N\to\infty}\frac{1}{N}\sum_{j=1}^{N}\alpha^{+m}\alpha^n, \tag{13}$$

where j labels the results from the jth trajectory.

V. CLASSICAL ANALYSIS

A. Steady-state solutions

Before we return to the full stochastic equations, which we will solve numerically, we will investigate some of the classical properties of the system, which allow for analytical insights. From the drift part of Eq. (8), we can immediately write the mean-field equations using the notation $\overline{z}$ for the classical mean-field value of z,

$$\frac{d\overline{\alpha}}{dt} = \epsilon - \gamma\overline{\alpha} + igA\overline{\alpha}(\overline{\beta}+\overline{\beta^*}),$$

$$\frac{d\overline{\beta}}{dt} = -i\omega_m\overline{\beta} - \gamma_m(\overline{\beta}-\overline{\beta^*}) + igA\overline{|\alpha|^2}, \tag{14}$$

from which we may find the classical steady-state solutions.

Solving Eqs. (14) for the steady states, we find that β_{ss} is real, which means that the steady-state momentum is zero. (Note that this will not be the prediction of stochastic integration of the full equations.) However, using this fact we may write the solutions as

$$\beta_{ss} = \beta_{ss}^* = \frac{gA}{\omega_m}|\alpha_{ss}|^2, \quad \alpha_{ss} = \frac{\epsilon}{\gamma - 2igA\beta_{ss}}.$$

Although the solutions above are not closed (the solution for α_{ss} is a function of β_{ss}, etc.), we can make an iterative expansion, beginning with the result for fixed mirrors,

$$\alpha_{ss}^0 = \frac{\epsilon}{\gamma}, \tag{16}$$

and substitute this into the solution for β_{ss}. This can then be substituted into the solution for α_{ss}, the process being repeated until we attain the required degree of convergence. We note here that there are parameter regimes for which this expansion does not converge and that these are regions where we do not find classical steady-state solutions, but rather a limit-cycle, self-pulsing behavior [4].

Although we will demonstrate below that the classical steady-state solutions, especially for x and p, are not accurate in any parameter regime, they do allow for some insight into which property of the electromagnetic field outside the cavity is most likely to allow for an inference of the mirror position. The usual candidates are the intensity, which may be measured by photodetection, and the quadratures, which may be measured by homodyne detection. Defining the intracavity quadratures as $\hat{X}_a = \hat{a} + \hat{a}^\dagger$ and $\hat{Y}_a = -i(\hat{a} - \hat{a}^\dagger)$, with their classical equivalents written in terms of α_{ss} and α_{ss}^*, we find

$$X_a = \frac{2\epsilon\gamma}{\gamma^2 + g^2x^2}, \quad Y_a = \frac{2g\epsilon x}{\gamma^2 + g^2x^2},$$

$$|\alpha|^2 = \frac{\epsilon^2}{\gamma^2 + g^2x^2}, \tag{17}$$

where we have set ϵ as real and used $x = 2A\beta$, with β real. As γ^2 is typically much larger than g^2x^2, we may make a series expansion of these expressions. We find that

$$X_a \approx \frac{2\epsilon}{\gamma}\left(1 - \frac{g^2x^2}{\gamma^2}\right),$$

$$Y_a \approx \frac{2g\epsilon x}{\gamma^2}\left(1 - \frac{g^2x^2}{\gamma^2}\right),$$

$$|\alpha|^2 \approx \frac{\epsilon^2}{\gamma^2}\left(1 - \frac{g^2x^2}{\gamma^2}\right). \tag{18}$$

It is immediately obvious that Y_a depends on x to first order, while the other two exhibit only a second-order dependence. This shows that homodyne measurements of the Y_a quadrature will be more sensitive to variations in the position of the mirror than will be the other two measurements, as previously noted by Vitali *et al.* [31], although the X_a quadrature will show a weaker dependence rather than being totally independent of x as in the linearized analysis of Ref. [31].

B. Bistability

It has been predicted that, with a nonzero detuning between the field and the cavity resonance, this system can exhibit bistability in the optical intensity [5]. To find the condition for bistability, we start with the classical equations with detuning Δ included,

$$\frac{d\overline{\alpha}}{dt} = \epsilon - (\gamma + i\Delta)\overline{\alpha} + igA\overline{\alpha}(\overline{\beta} + \overline{\beta^*}),$$

$$\frac{d\overline{\beta}}{dt} = -i\omega_m\overline{\beta} - \gamma_m(\overline{\beta} - \overline{\beta^*}) + igA\overline{|\alpha|^2}. \tag{19}$$

These have the steady-state solutions,

$$\beta_{ss} = \beta_{ss}^* = \frac{gA}{\omega_m}|\alpha_{ss}|^2,$$

$$|\alpha_{ss}|^2 = \frac{\epsilon^2}{\gamma^2 + (\Delta - 2gA\beta_{ss})^2}, \tag{20}$$

which immediately leads to the cubic equation in $I = |\alpha_{ss}|^2$,

$$\frac{4g^4A^4}{\omega_m^2}I^3 - \frac{4g^2A^2\Delta}{\omega_m}I^2 + (\gamma^2 + \Delta^2)I - \epsilon^2 = 0. \tag{21}$$

We find the condition for bistability by differentiating this expression with respect to I, which gives

$$\frac{12g^4A^4}{\omega_m^2}I^2 - \frac{8g^2A^2\Delta}{\omega_m}I + (\gamma^2 + \Delta^2) = 0. \tag{22}$$

The condition for bistability is that this quadratic equation has two positive real roots. The roots are written as

$$r_\pm = \frac{\Delta\omega_m}{3g^2A^2} \pm \frac{\omega_m}{6g^2A^2}\sqrt{\Delta^2 - 3\gamma^2}, \tag{23}$$

giving the inequality

$$\Delta \pm \frac{1}{2}\sqrt{\Delta^2 - 3\gamma^2} > 0. \tag{24}$$

A necessary, but not sufficient, condition is that Δ be positive, as I must be positive. This immediately contradicts the condition given by Mancini and Tombesi [5], $|\Delta| > \sqrt{3}\gamma$, which allows for negative intensities. As I must also be real, we find the condition for bistability as $\Delta > \sqrt{3}\gamma$.

VI. STOCHASTIC RESULTS

A. Initial conditions

To numerically integrate Eq. (11), we make use of the fact that the Itô and Stratonovich forms are identical so that we may use a standard three-step predictor-corrector method. The convergence of the algorithm was checked by comparison with a four-step method, and also by halving the time step. In all the quantities shown, the sampling errors are comparable to the thickness of the plotted lines.

We will use the published experimental parameters of Ref. [2], and make comparisons with the theoretical predictions reported elsewhere. The oscillating mirror is considered as being perfectly reflecting, with a mass of $m = 10^{-5}$ kg, a mechanical quality factor of $Q = 4 \times 10^6$ at 4.2 K, decreasing to 2.25×10^6 at 70 K, and a resonance frequency $\omega_m/2\pi = 26$ kHz. The damping rate of the mirror is $\gamma_m = 0.5\omega_m/Q = 0.0363\ s^{-1}$ at 4.2 K. We consider a cavity length of $L = 1$ cm, with cavity finesse $\mathcal{F} = 15 \times 10^3$, which gives $\gamma = \pi c/2\mathcal{F}L = 3.14 \times 10^6\ s^{-1}$. We consider an optical wavelength of $\lambda = 1064$ nm, which gives $\omega_0 = 1.77 \times 10^{15}\ s^{-1}$, and a coupling between the light and the mirror of $g = \omega_0/L = 1.77 \times 10^{17}\ m^{-1}\ s^{-1}$. The optical pumping of the cavity is $\epsilon = \sqrt{\gamma\mathcal{P}/\hbar\omega_0}$, where $\mathcal{P}$ is the laser power in W.

In stochastic integration of the equations which describe an intracavity optical system, the standard approach is to begin with the state inside the cavity as vacuum so that, with a continuous pump, the system enters the steady state (or its limit cycle behavior in the case of self-pulsing) after a few cavity lifetimes. In the present case the situation is somewhat different, as not only the electromagnetic field, but also the oscillating mirror, has to reach the steady state. As the relaxation time of the mirror can be orders of magnitude larger than that of the intracavity field, and we need to average over a large number of trajectories to obtain reliable results, it is not practical to begin the integration with an arbitrary initial condition for the mirror. Naively beginning with $\beta(0) = \beta^+(0) = 0$, the ground state of the mirror, leads to extremely long-lived transients, as this is far from the equilibrium state at finite temperature. To give some idea, even at 4.2 K, which is perhaps thelowest easily achievable temperature, the average number of mirror quanta becomes $|\overline{\beta}|^2 = k_BT/\hbar\omega_m = 3.36 \times 10^6$. For purposes of comparison, we will therefore choose the initial mirror state in two different ways and integrate the equations without any pumping of the cavity. First, as a (real) coherent state, which has uncertainties in position and momentum at the SQL, with the P function

$$P(\beta) = \delta(\beta - \sqrt{k_BT/\hbar\omega_m}), \tag{25}$$

and, second from the thermal distribution

$$P(\beta) = \frac{1}{\pi\overline{n}}e^{-|\beta|^2/\overline{n}}, \tag{26}$$

where $\overline{n} = k_BT/\hbar\omega_m$. Note that, at the beginning of each trajectory, $\beta = (\beta^+)^*$ and the phase is completely random for the thermal distribution. We stress here that the variance in the number of mirror phonons for a thermal state is $V(n) = \overline{n}^2 + \overline{n}$, which is very much larger than $\overline{n}$. Using the Planck distribution,

$$\overline{n} = (e^{\hbar\omega_m/k_BT} - 1)^{-1}, \tag{27}$$

we find that to achieve $\overline{n} = 1$, we would need $T = 1.8\ \mu$K, and even then the variance would be 2, or twice the mean value.

It is important to note here that the number of phonons does not enter into the equations, but rather the quadratures $\hat{X}_b$ and $\hat{Y}_b$. In a linearized approach using our equations, it is the uncertainties in these which are important. We can easily calculate these for a thermal state of the mirror with an unpumped cavity. A simple integration gives

$$V(\hat{X}_b) = V(\hat{Y}_b) = 1 + 2\sqrt{\pi}\left(\frac{k_BT}{\hbar\omega_m}\right)^{3/2}, \tag{28}$$

equal to 2.2×10^{10} at $T = 4.2$ K for our system. This is in stark contrast to a coherent state of the mirror, sometimes

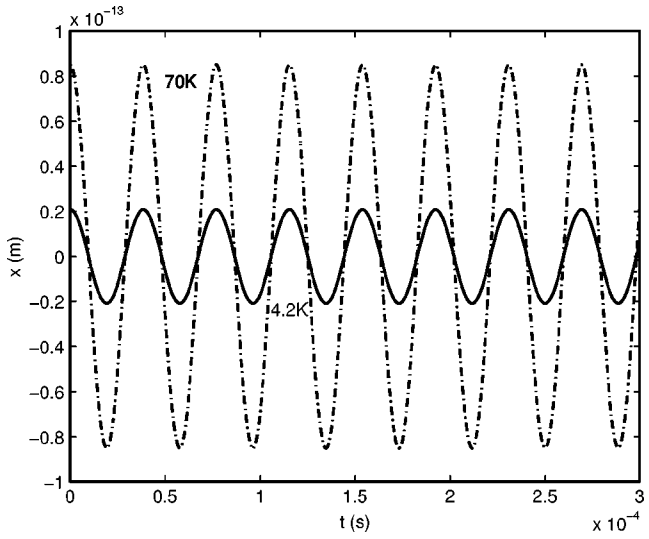

FIG. 1. Mean values for x at $T=4.2$ K and $T=70$ K for an initial coherent state of the mirror and no optical pumping. These results are the averages of 7.6×10^3 and 2.7×10^4 trajectories, respectively. Note that, unless otherwise stated, the values plotted here and in subsequent graphs are dimensionless.

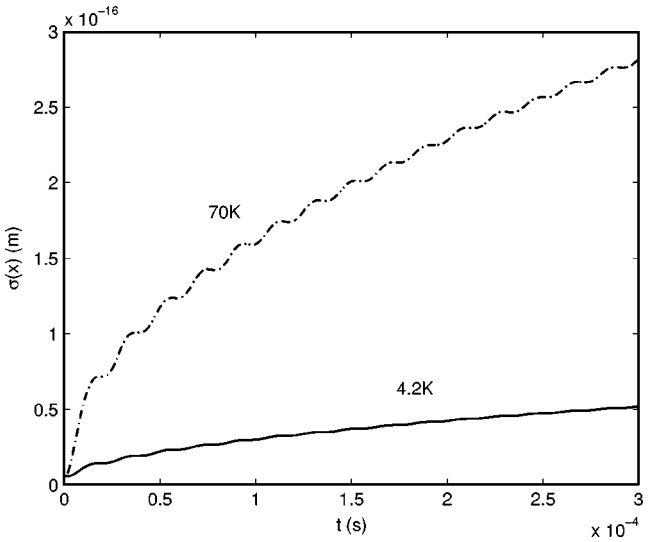

FIG. 2. Standard deviations for x at $T=4.2$ K and $T=70$ K for an initial coherent state of the mirror and no optical pumping. Note that these quantities were still increasing at twice the time shown here and would be expected to eventually attain the thermal values.

used to facilitate the mathematics of a linearized analysis, and for which $V(\hat{X}_b)=V(\hat{Y}_b)=1$.

We have calculated the stochastic results for the means and standard deviations of the mirror position, without any optical pumping and for initial thermal and coherent states of the mirror at temperatures of 4.2 and 70 K. In Fig. 1 we show the stochastic results for the position x of the mirror, with initial coherent states at these two temperatures. Considering only these mean values could give the erroneous impression that the mirror is in a nonstationary steady state, which we can immediately see is not the case when we look at Fig. 2, which shows the standard deviations $\sigma(x)$ for the same parameters. Although we have shown the standard deviations here, the variances for the initial coherent state continue to increase linearly for more than twice the time shown, which was as far as we continued the integration. In contrast, for an initial thermal state, the mean value of the mirror displacement is, by definition, zero as can be seen from the equation for the P function (26). We note here that our stochastic results over more than 2×10^6 trajectories still showed oscillations of the order of 10^{-16} m, but that we are confident that this small, but nonzero, value is due to the difficulty of sampling the distribution with a finite number of trajectories. By comparison with the coherent state values, for an initial thermal state $\sigma(x)\approx1\times10^{-14}$ m at $T=4.2$ K, and is almost constant, indicating that this is a good choice of initial condition. This value agrees well with the expression given in Ref. [2] for the thermal noise, $\sigma(x)=\sqrt{k_BT/m\omega_m^2}$, which gives a value of 1.47×10^{-14} m. Note that, over the time scales shown in Fig. 1 we do not see any decay in the oscillations towards the thermal state values, as this would be expected to happen on a time scale of $1/\gamma_m$, which is approximately 50 s for the parameters used here. In fact, although an initial coherent state of the mirror has been used in theoretical analyses (see, for example, Bose *et al.* [8]), it is not at all obvious how this particular state may be constructed experimentally. A thermal state arises naturally, and will be equal to a coherent state for $T=0$ K, but absolute zero cannot be reached experimentally. In optics, a coherent state can be described theoretically as a displacement of the vacuum by the displacement operator $D(\alpha)=\exp(\alpha\hat{a}^\dagger-\alpha^*\hat{a})$, which we can see has some relation to the optical pumping term of the Hamiltonian, H_{pump}. Therefore an ideal empty cavity with this pumping term will naturally develop an intracavity coherent state. We are not aware of any similar candidate for the mirror, even if it could begin in the $T=0$ vacuum state. Therefore we will use an initial thermal state in our investigations.

When we examine the stochastic results for the intracavity field intensity, for an input power $\mathcal{P}=5$ mW, we find that the field exhibits a self-pulsing behavior at approximately the resonance frequency of the mirror, as previously predicted [4]. However, the oscillations are of very small relative amplitude, at approximately 0.2% of the average mean intensity. With increasing input power, the oscillations become larger until, for a power of 100 mW, for example, they are more than half the maximum intensity. At the lower power, the mean motion of the mirror is an oscillation between 0 and 1.2×10^{-12} m, while at the higher power oscillates between -1 and 3×10^{-11} m. Even though these displacements are truly microscopic, they have a noticeable effect on the mean intensity, which should be easily detectable experimentally. Interestingly enough, these results are almost identical to what we find by numerical integration of the classical equations (14), although these can tell us nothing about the quantum correlations which we wish to investigate. Among these quantum correlations are the variances of the intracavity field and the Fano factor, defined as $F(N_a)=V(N_a)/N_a$. These results, which we averaged over 6.71×10^5 trajectories, are shown in Fig. 3. For a coherent state, all three values are 1, which would be zero on the logarithmic vertical scale used

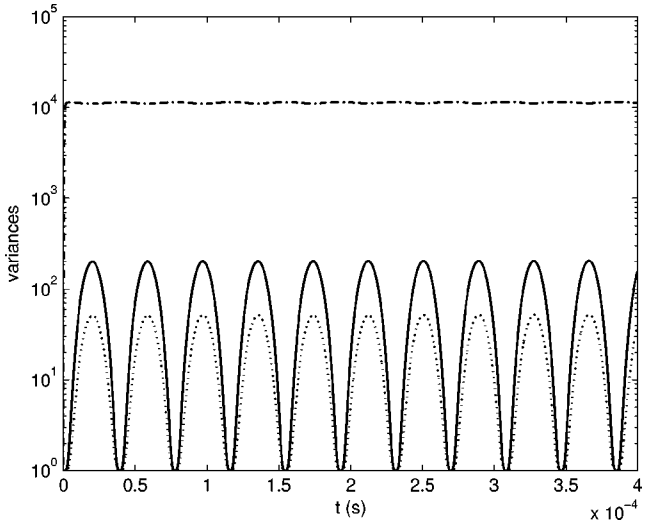

FIG. 3. The variances of the intracavity electromagnetic field for a laser power of 5 mW. The solid line is $V(\hat{X}_a)$, the dash-dotted line is $V(\hat{Y}_a)$, and the dotted line is the Fano factor.

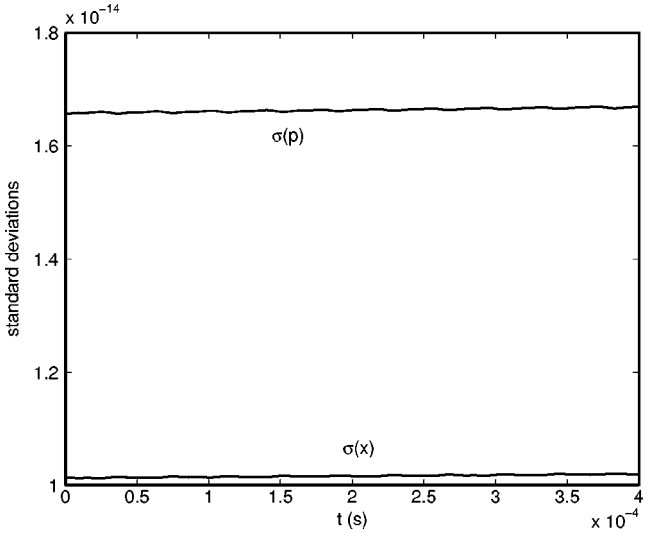

FIG. 4. The standard deviations in the mirror position and momentum for a laser power of 5 mW. The standard S.I. units for position and momentum are used. Note that a minimum uncertainty state of the mirror has $\sigma(x)=5.6824\times10^{-18}$ m and $\sigma(p)=9.283\times10^{-18}$ kg ms^{-1}.

here. As all three correlations are greater than or equal to 1, we do not see any squeezing of the field in the time domain, but do see excess noise in all three quantities. As is common with Kerr media, there is more excess noise in the $\hat{Y}_a$ quadrature than in either the $\hat{X}_a$ quadrature or in the intensity.

B. Position measurements

Much of the theoretical and experimental interest in this system has been in indirectly measuring small displacements or small forces which act on the oscillating mirror, by means of measurements on the optical field. Generally theoretical results are presented in terms of output spectra which allow for an inferred value of the mirror position at various frequencies. These spectra are simple to calculate in a linearized analysis which treats the system as an Ornstein-Uhlenbeck process [16], but we do not consider linearization valid here, for the reasons we have stated. To calculate spectra from the results of stochastic integration is also possible in many cases, but here is made difficult by the stiffness of our equations, where the field and the mirror oscillate on vastly different time scales. The length of time needed to integrate a large enough number of trajectories over a sufficient time to give reliable results upon Fourier transformation is prohibitive. Hence we will show results which were obtained in the time domain.

The first results we show for the mirror, in Fig. 4, are for the uncertainties in the mirror position and momentum, defined as $\sigma(x)=A\sqrt{V(X_b)}$ and $\sigma(p)=|B|\sqrt{V(Y_b)}$. The interaction with the field has not noticeably changed these quantities from the values we found via stochastic integration in the unpumped thermal state case, but has given the momentum a mean value which oscillates between approximately $\pm1\times10^{-19}$ kg ms^{-1}, whereas it was stable at zero in the unpumped case. It is readily seen that the fluctuations in momentum are several orders of magnitude larger than the mean value. We also note here that the uncertainties shown are certainly underestimated, due to the difficulties of simulating the large β values with small probabilities in the initial conditions used. This result also shows that, when the likely thermal state of the mirror is taken into account, the back-action noise of the field on the mirror has very little effect at the laser power considered here. At an increased power of 100 mW the standard deviations in both position and momentum both increase due to this back-action noise, reaching oscillatory values around means of $\approx1\times10^{-13}$, while the mean momentum oscillates between $\pm3\times10^{-17}$ kg ms^{-1}.

We next investigate the degree of correlation between the position of the mirror, the intracavity optical intensity, and the $\hat{X}_a$ and $\hat{Y}_a$ quadratures. This correlation function between two quantities w and z is defined as

$$\mathcal{C}(wz)=\frac{\langle wz\rangle-\langle w\rangle\langle z\rangle}{\sqrt{V(w)V(z)}}, \tag{29}$$

where a perfect correlation gives a value of 1, a perfect anticorrelation gives a value of −1, and zero signifies no correlation at all. In Fig. 5 we see that, as predicted by Eq. (18), the strongest correlation is between $\hat{Y}_a$ and the mirror position, these two being almost perfectly correlated. The correlation functions $\mathcal{C}(xX_a)$ and $\mathcal{C}(xN_a)$ oscillate at the frequency ω_m between a value of almost −1 and a value of approximately 0, ranging from being almost perfectly anticorrelated to almost perfectly uncorrelated. This behavior is quite different from that shown in a linearized fluctuation analysis, where we found that all three of these functions showed anticorrelation around the mirror frequency and were zero at other frequencies. The difference between our fully quantum nonlinear results and the usual linearized predictions is dramatically demonstrated when we consider $|\mathcal{C}(X_aY_b)|^2$, used to analyze a possible quantum nondemolition measurement by Jacobs *et al.* [6]. Their prediction [see their Eq. (31)] is for a

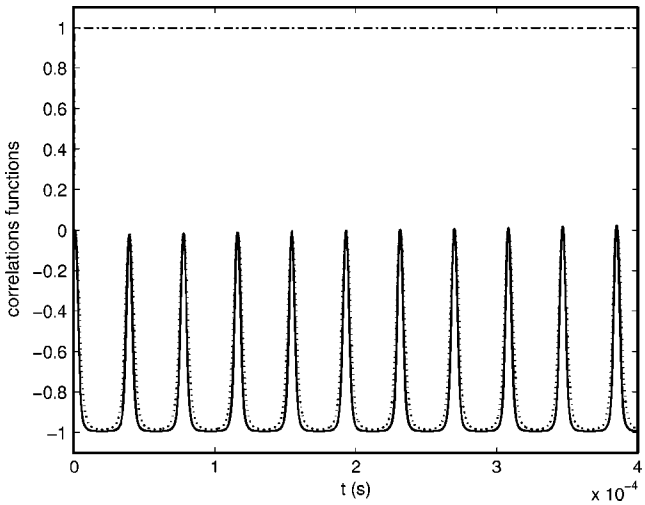

FIG. 5. The intracavity correlation functions, $\mathcal{C}(xX_a)$ (solid line), $\mathcal{C}(xY_a)$ (dash-dotted line), and $\mathcal{C}(xN_a)$ (dashed line), for an input power of 5 mW.

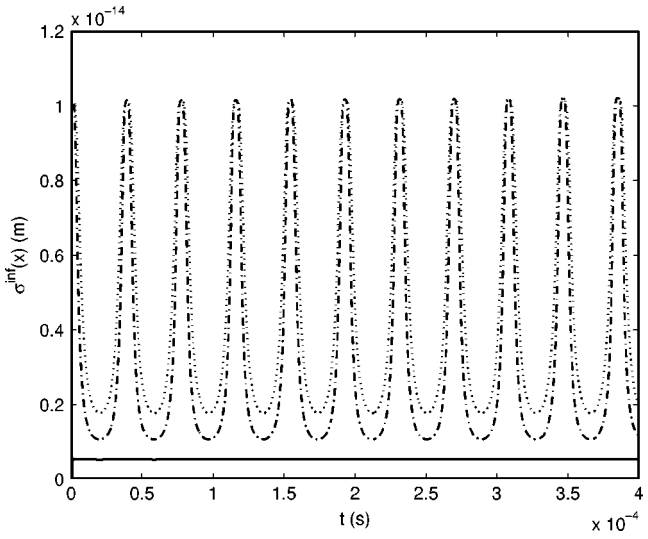

FIG. 6. The inferred uncertainties in the mirror position, $A\sqrt{V^{inf}(\hat{X}_b)}$, calculated as in Eq. (32). The solid line is the estimate using $\hat{Y}_a$, the dash-dotted line uses $\hat{X}_a$ and the dotted line uses N_a.

maximum spectral value of 0.9757 for the parameters we use in this work, whereas our stochastic prediction in the time domain oscillates between 0 and 6×10^{-3}, showing almost no correlation at all. We think that it is unlikely that this difference can be explained by the difference between a spectral measurement and a time-domain measurement, but that it is due to the inappropriateness of the linearization procedure for this system.

Another way to infer the mirror position is by linear estimation following measurements of the optical field. We follow a method proposed by Reid [38] in the context of a demonstration of the Einstein-Podolsky-Rosen paradox, and also outlined in Dechoum *et al.* [39]. We assume that a measurement of the $\hat{Y}_a$ quadrature allows for a linear estimate of $\hat{X}_b$, $\hat{X}_b^{est}=c\hat{Y}_a+d$. This is consistent with the expansion of Y_a given above, in Eq. (18). After optimizing for d, the RMS error in this estimate is given by

$$V^{inf}(\hat{X}_b)=\langle(\hat{X}_b-c\hat{Y}_a)^2\rangle-\langle\hat{X}_b-c\hat{Y}_a\rangle^2, \tag{30}$$

which we may minimize as a function of c, finding

$$c=\frac{V(\hat{X}_b,\hat{Y}_a)}{V(\hat{Y}_a)}. \tag{31}$$

We may then write

$$V^{inf}(\hat{X}_b)=V(\hat{X}_b)-\frac{[V(\hat{X}_b,\hat{Y}_a)]^2}{V(\hat{Y}_a)}, \tag{32}$$

a quantity which we may calculate via stochastic integration. The inferred uncertainty in a measurement of $\hat{x}$ will then be $\sigma^{inf}(x)=A\sqrt{V^{inf}(\hat{X}_b)}$, which will be equal to the standard quantum limit when the mirror is inferred to be in a minimum uncertainty state. We have also calculated the inferred position uncertainty using measurements of $\hat{X}_a$ and N_a in the same manner. As shown in Fig. 6, and expected from the previous discussion, using the $\hat{Y}_a$ quadrature gives better results than using either $\hat{X}_a$ or N_a, which give oscillatory inferences. As it is, all of these give inferred uncertainties well above the SQL of $A=5.68\times10^{-18}$ m for the parameters used. We note here that the actual calculated value of $\sigma(x)$, which oscillates between 1.025×10^{-14} m and 1.03×10^{-14} m, is greater than the value inferred through measurement of the $\hat{Y}_a$ quadrature, which has a steady-state value of approximately 5×10^{-16} m. This is also the case with the inferred measurements of Ref. [38], and is due to the almost perfect correlation between the position of the mirror and the $\hat{Y}_a$ quadrature.

VII. NONCLASSICAL STATE PREPARATION

The pendular cavity and variations on the theme have been proposed as useful devices for the preparation of nonclassical states of the cavity field and also of the mirror or mirrors. In this section we will give a brief review of some of these proposals, in chronological order, and explain why our results lead us to believe that they may not be as practical as the original authors suggest.

Bose *et al.* [8] suggest that this system may be used to prepare multicomponent quantum superpositions of states (Schrödinger cat states) of the field, near number states, and entangled states when two or more modes interact with the mirror, as well as Schrödinger-cat-like states of the mirror prepared via quadrature measurements of the field. Beginning with a simple Hamiltonian without any pumping or dissipation terms, they develop a time evolution operator which is seen to have a Kerr-like term, which leads them to believe that this system may exhibit some of the nonclassical features of a Kerr interaction. They then assume that both the cavity field and the mirror are initially in coherent states. While this simplifies the mathematics, and is reasonably ac-

curate for the cavity field, our calculations have shown that a coherent state is not a reasonable initial condition for the mirror. As the nonclassical features predicted depend on this initial condition, which at one stage of the paper is taken to be a vacuum state (for simplicity), and interaction of the mirror with the environment is considered to be equivalent to that of a cavity field with a zero-temperature reservoir, our results lead us to believe that these predicted nonclassical states will be, at the very least, extremely difficult to observe.

In a subsequent article [36], the same authors propose that the pendular cavity may be used to probe the decoherence of a macroscopic object, namely the mirror. In this work the authors accept that the mirror will begin in a thermal state and note that a mixture of Schrödinger cat states of the mirror can therefore be produced by the interaction with the field. It is assumed that a superposition of the Fock states $|0\rangle$ and $|n\rangle$ can be prepared inside the cavity, with the most simply prepared value of $n=1$. Using the P-representation expansion of the density matrix in terms of coherent states, an expression is found for the time development of this density matrix. The field can then be measured via the interaction with a ground-state two-level atom which is passed through the cavity. The probability of the atom being excited can be related to the decoherence rate of the spatially separated superposed coherent states of the mirror's motion. Examining this scheme with our parameters, we find that, with $n=1$, the spatial separation between two of the superposed coherent states has a maximum of $\Delta x_{\max}=1.4\times10^{-22}$ m, which is less than the thermal de Broglie wavelength, $\lambda_{dB}=2.19\times10^{-21}$ m, so that it becomes difficult to claim we have a spatially separated superposition. When we take into account that the decomposition of the thermal state will include a huge number of coherent states, all with different phases, it seems that the practical realization of this scheme may be more demanding technologically than the authors had supposed.

Another proposal uses radiation pressure to entangle two macroscopic mirrors [14], but relies on Langevin equations which are linearized around their classical steady-state solutions. As we have shown above, this process is of rather doubtful validity when the mirrors are coupled to thermal reservoirs. A further idea is to produce an Einstein-Podolsky-Rosen (EPR) state in the position and momentum of two spatially separated oscillating mirrors using the output of a nondegenerate optical parametricoscillator (OPO) [37]. We note here that, although the authors call the oscillator an optical parametric amplifier, the nonlinear crystal is inside a pumped cavity, so we will follow the usual terminology (see their Fig. 2). Unlike most other treatments, this work uses an effective linear coupling between the light and the mirror. Like most of the others, the authors linearise Langevin equations around their steady-state solutions. The OPO is treated via two-mode equations which do not describe the normal well-known threshold behavior of such a device at all, and lead to the prediction of an entangled state of two combined quadratures, which is said to demonstrate an EPR correlation. The two output fields of the OPO are used to drive the two mirrors, which drives them into an EPR state of position and momentum. The mechanical damping of the mirrors introduces a noise term $V_b=1+2n_T$, where $n_T=\coth(\hbar\omega_m/2k_BT)$ is the mean thermal phonon number. This noise term seems to be the quadrature variance assuming a number state $|n_T\rangle$, of the mirror. For the parameters we use, this expression gives a value of $V_b=6.7\times10^6$, several orders of magnitude lower than our estimate in Eq. (28), which gives $V_b=2.2\times10^{10}$. Due to the issues we have raised here, we are led to believe that the EPR state of the two mirrors may not be as easily demonstrated as indicated by the authors. Mancini *et al.* [40] have also put forward a proposal to entangle two movable mirrors which form part of a four-mirror ring cavity. This work has the advantage of a description for the Brownian motion that is consistent with quantum mechanics at all temperatures, but the results presented are also obtained following a linearization process, which we have shown to not be valid for reasonable temperatures. The final proposal we will consider here is by Marshall *et al.* [15], which treats the creation of superposition states of a macroscopic mirror ($\approx10^{14}$ atoms) via the interaction with a single photon. This work again uses a Hamiltonian approach without dissipation and assumes that the mirror can be prepared in its ground state $|0\rangle$ which then allows for analytical solutions. The inclusion of decoherence and finite temperature follow the approach of Ref. [36] and we therefore have the same doubts about the physical viability of this proposal.

VIII. COOLING BY FEEDBACK

It may be thought that to reduce the thermal noise of the mirror it is sufficient to cool to a lower temperature and impressive reductions in thermal noise, of the order of 10^3, have been achieved [10,30]. However, it seems that the method of feedback cooling leads to a thermal equilibrium of the mirror at a lower temperature. For our system, using Eq. (28), a reduction by this factor leads to $V(\hat{X}_b)=V(\hat{Y}_b)\approx10^7$, which is still far above the coherent state level where linearization of the equations may be expected to work. Reference [15] suggests that a mirror may be cooled to as low as 2 mK by dilution refrigeration, which would give us $V(\hat{X}_b)=V(\hat{Y}_b)\approx2.27\times10^5$, which, if we assume that feedback cooling from this temperature is as efficient as at room temperature, would allow the variances to be further reduced to something of the order of 10^2. This is now actually smaller than the mean number of quanta, $|\beta|^2$, equal to 1.6×10^3, although the variance in the number will now be of the order of 10^6, so that linearization will still not be reliable.

Standard cooling by feedback depends on the interaction of the electromagnetic field with the mirror phonons, which is proportional to $\hat{a}^\dagger\hat{a}\hat{X}_b$, which will not drive the mirror toward a coherent state. A coupling term proportional to $\hat{a}^\dagger\hat{b}-\hat{a}\hat{b}^\dagger$ may be expected to do this, while squeezing of the mirror position would conceivably be possible with a coupling of the type $\hat{a}^{\dagger2}\hat{b}-\hat{a}^2\hat{b}^\dagger$ or $\hat{b}^{\dagger2}\hat{a}-\hat{b}^2\hat{a}^\dagger$. It is interesting to note that an effective coupling of the type required to drive the mirror toward a coherent state was used by Zhang *et al.* [37], although the physical parameters used to develop the coupling Hamiltonian are far from those we have used in our analysis. As an example in that work, $\omega_m\gg\gamma$, whereas we

have used the values $\omega_m = 1.63 \times 10^5\ s^{-1}$ and $\gamma = 3.14 \times 10^6\ s^{-1}$. Zhang *et al.* also consider that the mirror damping is of the amplitude form, which is not what we find using the Diósi master equation. Mancini *et al.* [41] have actually presented a feedback scheme based on an effective Hamiltonian which couples a light quadrature with X_b, later proposing that this method, which they call stochastic cooling, could be used to beat the SQL by achieving steady-state position squeezing of the mirror [31]. The effective coupling used again depends on the linearization of the equations of motion. We feel that whether feedback can be used to cool a macroscopic oscillator towards, or even beyond, the SQL remains an open question and subject to further research.

IX. CONCLUSION

In conclusion, we have presented a fully quantum analysis of the pumped pendular cavity which does not depend on linearization of the equations of motion. We have shown that the thermal noise of the mirror is overwhelming for typical temperatures and experimental parameters. This means that the linearization procedure commonly used is of doubtful validity. Unless ways can be found to change the quantum state of the mirror in respect to the thermal excitations, we expect that the nonclassical states predicted in a number of theoretical analyses will not be able to be demonstrated experimentally. Feedback cooling techniques, while able to lower the effective temperature to an impressive degree, serve merely to produce another thermal state of the mirror at a lower temperature. The noise is still overwhelming if we wish to reach the SQL, or even beat it, which would be necessary for the detection of gravity waves. This is also the case if we wish to observe some of the quantum superpositions and entanglement which have been theoretically predicted.

ACKNOWLEDGMENTS

This research was supported by the New Zealand Foundation for Research, Science and Technology (Grant No. UFRJ0001), the Australian Research Council, and the Brazilian agencies CNPq (Conselho Nacional de Desenvolvimento Científico e Tecnológico), and CAPES (Coordenação de Aperfeiçoamento de Pessoal de Nível Superior). Murray Olsen would like to thank Stojan Rebić for interesting discussions and for his careful reading of the text.

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