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Detection of continuous variable entanglement without coherent local oscillators

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We propose three criteria for identifying continuous variable entanglement between two many-particle systems with no restrictions on the quantum state of the local oscillators used in the measurements. Mistakenly asserting a coherent state for the local oscillator can lead to incorrectly identifying the presence of entanglement. We demonstrate this in simulations with 100 particles and also find that large number fluctuations do not prevent the observation of entanglement. Our results are important for quantum information experiments with realistic Bose-Einstein condensates or in optics with arbitrary photon states.

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The study of the quantum properties of matter waves is a rapidly developing field known as quantum atom optics [1,2]. Already several experiments have observed nonclassical effects in ultracold gases, including the Hanbury Brown-Twiss effect for bosons [3], antibunching for fermions [4], sub-Poissonian number fluctuations [5], and density correlations from molecular dissociation [6] and in the Mott-insulator regime in an optical lattice [7]. Although impressive achievements, the experimental techniques utilized in these observations are insufficient to detect quantum squeezing or entanglement. The demonstration of entanglement—which Schrödinger described as being the central mystery of quantum mechanics [8]—will be an important step toward quantum information applications of ultracold atomic systems.

In quantum optical systems continuous variable (CV) entanglement can be demonstrated experimentally by measuring certain correlation functions of electromagnetic field quadratures and finding that these violate an inequality for separability [9,10] or an inequality [11] for demonstrating the Einstein-Podolsky-Rosen (EPR) paradox [12]. For a number of systems, these quadratures can only be determined using homodyne or heterodyne measurement techniques and require a coherent state local oscillator—a highly occupied mode of the electromagnetic field that is a good approximation to the output of many lasers [13]. Such measurements have led to the observation of optical squeezing [14,15] and the EPR paradox with photons [16], and have been used to perform quantum-state tomography [17] and continuous variable teleportation [18].

In principle, squeezed or entangled atomic fields can be generated by atomic four-wave mixing [19–21], molecular disassociation [22], or mapping photon statistics onto atoms [23]. Entanglement generated in these situations can potentially be used as a resource for quantum information [24]. However, to unequivocally demonstrate entanglement between matter waves, it will be necessary to perform measurements sensitive to the relative phase of atomic wave packets. In principle, it is possible to measure matter-wave quadratures in direct analogy to the optical case using atomic measurements with a suitable local oscillator (phase reference) [25,26], of which the matter-wave equivalent is a Bose-Einstein condensate. However, these are typically not large, with a maximum of 10^8 particles [27], and the phase stability is compromised compared to a laser, as atomic interactions result in a mode shape and energy that depend on the particle number. Unfortunately, the usual correspondence between homodyne measurements and the field quadratures that is ubiquitous in quantum optics may be lost if the local oscillator is neither large nor coherent.

In this paper we show how to detect CV entanglement without a coherent state as a phase reference. We introduce three entanglement criteria based on homodyne measurement that require no assumptions about the quantum state of the local oscillator. These allow for the demonstration of inseparability and the EPR paradox with measurements that are feasible with current or foreseeable technologies with ultracold atoms. We give a numerical demonstration of the application of these criteria in an experiment with 100 particles.

We begin by reviewing homodyne measurement methods. The two quadrature operators for the mode with annihilation operator $\hat{a}$ are canonically defined to be

$$\hat{X}_c = \hat{a} + \hat{a}^\dagger, \quad \hat{Y}_c = i(\hat{a} - \hat{a}^\dagger). \tag{1}$$

These operators do not conserve boson number and are not detectable directly by intensity measurements. In optics, homodyne and heterodyne measurements allow access to these observables by using a beam splitter to interfere the signal with a local oscillator with well-defined phase and amplitude followed by intensity measurement(s) as illustrated in Fig. 1. Using the Heisenberg picture, if $\hat{a} \equiv \hat{a}_{\rm in}$ and $\hat{b} \equiv \hat{b}_{\rm in}$ describe

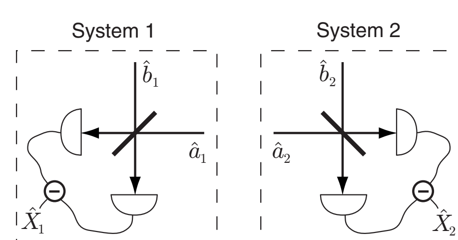

FIG. 1. Homodyne measurements of the two systems. Signal modes 1 and 2 are mixed with local oscillators; the resulting number differences are a measurement of the quadratures $\hat{X}_1$ and $\hat{X}_2$.

the signal and local oscillator before the beam splitting, then output modes $\hat{a}_{\text{out}}$ and $\hat{b}_{\text{out}}$ are given by

$$\hat{a}_{\text{out}} = t\hat{a}_{\text{in}} + r\hat{b}_{\text{in}}, \quad \hat{b}_{\text{out}} = r^*\hat{a}_{\text{in}} - t^*\hat{b}_{\text{in}}, \tag{2}$$

where $|t|^2+|r|^2=1$. Here we focus on balanced homodyning using a 50-50 beam splitter with $t=r=1/\sqrt{2}$. The final step of a balanced homodyne measurement is to measure the difference in the number of particles exiting the beam splitter ports $\hat{a}_{\text{out}}$ and $\hat{b}_{\text{out}}$. The measured quadrature is rescaled by the size of the local oscillator, according to

$$\hat{X}_{\text{m}} = \frac{\hat{a}^\dagger_{\text{out}}\hat{a}_{\text{out}} - \hat{b}^\dagger_{\text{out}}\hat{b}_{\text{out}}}{\langle \hat{b}^\dagger_{\text{in}}\hat{b}_{\text{in}}\rangle^{1/2}} = \frac{\hat{a}^\dagger\hat{b} + \hat{a}\hat{b}^\dagger}{\langle \hat{b}^\dagger\hat{b}\rangle^{1/2}}. \tag{3}$$

If the local oscillator is a coherent state $|\beta\rangle$, where β is real and positive, and the coherent state is large $\langle \hat{b}^\dagger\hat{b}\rangle=|\beta|^2 \gg \langle \hat{a}^\dagger\hat{a}\rangle$, then the difference between the two quadrature operators $\hat{X}_{\text{c}}$ and $\hat{X}_{\text{m}}$ will be negligible. For a signal that is comparable in size to a coherent local oscillator, $|\beta|^2 \sim \langle \hat{a}^\dagger\hat{a}\rangle$, the moments of the canonical quadrature observables can be determined from the measured observables using the result [13,15]

$$\langle \hat{X}_{\text{m}}\rangle = \langle \hat{X}_{\text{c}}\rangle, \quad \langle \hat{X}^2_{\text{m}}\rangle = \langle \hat{X}^2_{\text{c}}\rangle + \langle \hat{a}^\dagger\hat{a}\rangle/\langle \hat{b}^\dagger\hat{b}\rangle. \tag{4}$$

However, for local oscillators that are not coherent states, the difference between $\hat{X}_{\text{m}}$ and $\hat{X}_{\text{c}}$ cannot be neglected.

For matter waves the equivalent of the beam splitter operation is performed by Bragg, rf, or Raman pulses [1] to interfere atoms with different momenta and/or internal states. However, the best Bose-Einstein condensate (BEC) phase references will typically have less than 10^7 atoms. With the additional effects of atomic interactions and finite temperatures, it can be expected that typical BECs in the laboratory will have greater phase and/or amplitude fluctuations than the coherent output of a laser. Therefore we must take full account of the difference between the canonical and measured quadrature variables when detecting entanglement in condensate experiments.

The observation of CV entanglement requires the simultaneous measurement of two quadratures, and so balanced homodyning requires two phase references. We follow the general method of Duan *et al.* [9], Simon [10], and Hofmann and Takeuchi [28] to derive a separability criterion for the two systems of interest, depicted in Fig. 1. System 1 consists of the signal $\hat{a}_1$ and phase reference $\hat{b}_1$, and System 2 has signal $\hat{a}_2$ and phase reference $\hat{b}_2$. Measurements are made of the (number-conserving) quadratures

$$\hat{X}_j = \frac{\hat{a}_j\hat{b}^\dagger_j + \hat{a}^\dagger_j\hat{b}_j}{\langle \hat{b}^\dagger_j\hat{b}_j\rangle^{1/2}}, \quad \hat{Y}_j = i\frac{\hat{a}_j\hat{b}^\dagger_j - \hat{a}^\dagger_j\hat{b}_j}{\langle \hat{b}^\dagger_j\hat{b}_j\rangle^{1/2}}, \tag{5}$$

with commutator $[\hat{X}_j,\hat{Y}_k]=-2i\delta_{j,k}(\hat{b}^\dagger_j\hat{b}_j-\hat{a}^\dagger_j\hat{a}_j)/\langle \hat{b}^\dagger_j\hat{b}_j\rangle$.

It has been shown [29] that for any local observables on the two systems, $\hat{A}_j$ and $\hat{B}_j$, with commutator $[\hat{A}_j,\hat{B}_j]=i\hat{C}_j$, all separable states obey

$$\text{Var}[\hat{A}_1+\hat{A}_2] + \text{Var}[\hat{B}_1+\hat{B}_2] \geq |\langle \hat{C}_1\rangle| + |\langle \hat{C}_2\rangle|, \tag{6}$$

where $\text{Var}[\hat{A}]$ is the variance of the observable corresponding to the operator $\hat{A}$. For our system, it follows that the two systems are entangled when the inequality

$$\begin{aligned}\text{Var}[\hat{X}_1 \pm \hat{X}_2] + \text{Var}[\hat{Y}_1 \mp \hat{Y}_2] \geq{}& 2|1-\langle \hat{a}^\dagger_1\hat{a}_1\rangle/\langle \hat{b}^\dagger_1\hat{b}_1\rangle| \\ &+ 2|1-\langle \hat{a}^\dagger_2\hat{a}_2\rangle/\langle \hat{b}^\dagger_2\hat{b}_2\rangle|\end{aligned} \tag{7}$$

is violated. For semiclassical local oscillators, the right-hand side (RHS) of Eq. (7) approaches 4, as originally derived by Duan *et al.* [9] and Simon [10]. Note that this result is not the same as applying the coherent state corrections [Eq. (4)] to the derivation of Duan *et al.* [9] and Simon [10]—for further discussion, see [30].

The value of a quadrature of System 2 may be estimated from measurements of System 1 due to the existence of correlations between them. For separable states, the accuracy of this estimation will be restricted by the Heisenberg uncertainty limit. However, entangled systems may allow one to make predictions seemingly better than this fundamental limit, a fact related to the EPR paradox [12] by Reid [11].

We denote the inferred value of $\hat{X}_2$ by the measurement result of $\hat{X}_1$ as $\hat{X}^{\text{inf}}_2=f(\hat{X}_1)$ and similarly $\hat{Y}^{\text{inf}}_2=g(\hat{Y}_1)$, with the error of these estimates given by $\Delta^2_{\text{inf}}[\hat{X}_2]=\langle(\hat{X}_2-\hat{X}^{\text{inf}}_2)^2\rangle$ and $\Delta^2_{\text{inf}}[\hat{Y}_2]=\langle(\hat{Y}_2-\hat{Y}^{\text{inf}}_2)^2\rangle$, respectively. The EPR paradox is demonstrated when the inferred predictions have better accuracy than allowed by the Heisenberg uncertainty principle [11]. This occurs when the inequality

$$\Delta^2_{\text{inf}}[\hat{X}_2]\Delta^2_{\text{inf}}[\hat{Y}_2] \geq (1-\langle \hat{a}^\dagger_2\hat{a}_2\rangle/\langle \hat{b}^\dagger_2\hat{b}_2\rangle)^2 \tag{8}$$

is violated. Although the optimal inferred quadrature values could be in principle any function of the measured values, for simplicity we restrict ourselves to linear functions—i.e. $\hat{X}^{\text{inf}}_2=a+b\hat{X}_1$ and $\hat{Y}^{\text{inf}}_2=c+d\hat{Y}_1$. The simplest guess can be motivated by the inseparability criterion above; if the variances in Eq. (7) were vanishingly small, then we would infer that $\hat{X}^{\text{inf}}_2=\mp\hat{X}_1\pm\langle \hat{X}_1\rangle+\langle \hat{X}_2\rangle$ and $\hat{Y}^{\text{inf}}_2=\pm\hat{Y}_1\mp\langle \hat{Y}_1\rangle+\langle \hat{Y}_2\rangle$. This results in inference errors

$$\begin{aligned}\Delta^2_{\text{inf}}[\hat{X}_2] &= \text{Var}[\hat{X}_1 \pm \hat{X}_2], \\ \Delta^2_{\text{inf}}[\hat{Y}_2] &= \text{Var}[\hat{Y}_1 \mp \hat{Y}_2].\end{aligned} \tag{9}$$

Together with Eq. (8), this implies

$$\text{Var}[\hat{X}_1 \pm \hat{X}_2] + \text{Var}[\hat{Y}_1 \mp \hat{Y}_2] \geq 2\left|1-\frac{\langle \hat{a}^\dagger_2\hat{a}_2\rangle}{\langle \hat{b}^\dagger_2\hat{b}_2\rangle}\right|. \tag{10}$$

This is the same result as for the separability criterion above, Eq. (7), except the RHS is twice as small (typically). Thus it is more difficult to demonstrate the EPR paradox than inseparability, but it proves the stronger result that either local causality is violated or quantum mechanics provides an incomplete description of System 2 (for a discussion of this hierarchy, see [31,32]). The RHS of Eq. (10) becomes 2 for semiclassical local operators, which agrees with the work of Reid *et al.* [33]. This inequality is a special case of a more general EPR criterion for arbitrary observables [34].

It is possible to derive a second EPR criterion that is violated for a greater range of states than Eq. (10). The optimal linear inference of $\hat{X}_2$ based on the measurement of $\hat{X}_1$ is [11]

$$\hat{X}_2^{\mathrm{inf}} = \langle \hat{X}_2 \rangle + \frac{\mathrm{Var}[\hat{X}_1, \hat{X}_2]}{\mathrm{Var}[\hat{X}_1]}(\hat{X}_1 - \langle \hat{X}_1 \rangle), \tag{11}$$

where the covariance $\mathrm{Var}[\hat{A},\hat{B}] = \langle \hat{A}\hat{B} \rangle - \langle \hat{A} \rangle \langle \hat{B} \rangle$. In separate measurements we can estimate $\hat{Y}_2$ after measuring $\hat{Y}_1$ in the same way. The inference errors from this method are

$$\Delta^2_{\mathrm{inf}}[\hat{X}_2] = \mathrm{Var}[\hat{X}_2] - \mathrm{Var}[\hat{X}_1, \hat{X}_2]^2/\mathrm{Var}[\hat{X}_1],$$

$$\Delta^2_{\mathrm{inf}}[\hat{Y}_2] = \mathrm{Var}[\hat{Y}_2] - \mathrm{Var}[\hat{Y}_1, \hat{Y}_2]^2/\mathrm{Var}[\hat{Y}_1], \tag{12}$$

which are less than or equal to the comparable results in Eqs. (9). Substituting Eqs. (12) directly into Eq. (8) results in an EPR criterion that is superior to Eq. (10). This result agrees with that of Reid [11] in the limit that the phase reference is large and the RHS of the inequality is unity.

We illustrate the use of these criteria in degenerate four-wave mixing with atoms [19,20], where condensate atoms in mode 0 collide and populate modes 1 and 2, respectively. The Hamiltonian is

$$\hat{H} = i\chi(\hat{a}_0^2 \hat{a}_1^\dagger \hat{a}_2^\dagger - \hat{a}_0^{\dagger 2} \hat{a}_1 \hat{a}_2), \tag{13}$$

where $\hat{a}_j$ is the annihilation operator for mode j and χ represents the strength of the coupling. This model can be realized with a moving condensate in an optical lattice, and the outgoing modes 1 and 2 are predicted to be entangled [21].

First, we consider an initial state with N bosons in mode 0 and vacuum in 1 and 2 [i.e., $|\Psi(0)\rangle = |N\rangle|0\rangle|0\rangle$]. The Hamiltonian then ensures that the state can be written in the form

$$|\Psi(t)\rangle = \sum_m c_m(t)|N-2m\rangle|m\rangle|m\rangle \tag{14}$$

at all times. The Hamiltonian in this basis can be numerically diagonalized (for $N \lesssim 10^4$), providing a solution for the exact dynamics of the system. Figure 2(a) shows the number of atoms in each mode as a function of time for $N=100$.

To measure the quadratures of the signal modes (1 and 2), we require a phase reference for each. We can use the remaining condensate in mode 0 as our phase reference, but must first divide it in two with a beam splitter according to

$$\hat{b}_1 = (\hat{a}_0 + \hat{a}_3)/\sqrt{2}, \quad \hat{b}_2 = (\hat{a}_0 - \hat{a}_3)/\sqrt{2}, \tag{15}$$

where the auxiliary mode 3 is initially in the vacuum state. We then perform the quadrature measurements as expressed by Eq. (5) and compare the results with the separability criterion, Eq. (7), and the EPR criteria, Eq. (10) and Eqs. (8) and (12). The results are plotted in Figs. 2(b) and 2(c) where we can see that all the inequalities are violated at some stage in this experiment. After some time the number of particles in the signal beams grow larger than the phase references, and this measurement scheme is no longer optimal to detect the entanglement.

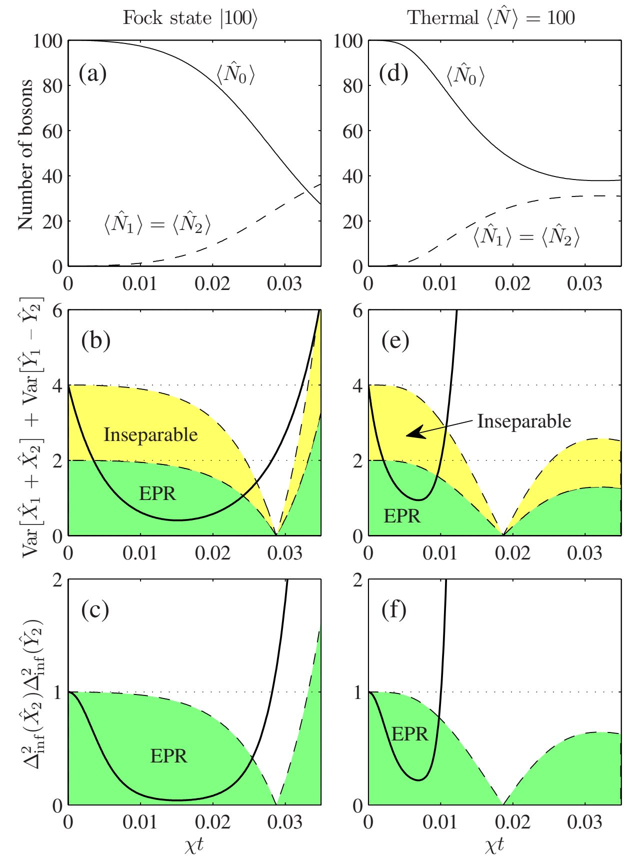

FIG. 2. (Color online) The population dynamics and entanglement criteria for the case of (a)–(c) exactly 100 particles and (d)–(f) a thermal distribution with a mean of 100 particles. In (a) and (d) we see that particles are transferred from the condensate to modes 1 and 2. In (b), (c), (e), and (f) the entanglement inequalities are violated when the solid line enters the shaded areas enclosed by the dashed lines, as described in the text. The horizontal dotted lines indicate the inequality violation boundary for the canonical quadratures [9–11].

The weaker EPR criterion, Eq. (10) in Fig. 2(b), shows a smaller region of χt where violation occurs than the stronger version, Eq. (12) in Fig. 2(c), as expected. The results in Figs. 2(b) and 2(c) identify times where it is necessary to fully treat the quantum nature of the quadrature measurement. Performing these measurements and then using the criteria derived by Duan *et al.* [9], Simon [10], and Reid [11] would lead to falsely identifying entanglement when the size of the phase references and signal beams become comparable.

A particularly interesting result is that the inseparability criterion [Eq. (7)] identifies entanglement in a small region [the top-right corner of Fig. 2(b)] when the local oscillator is actually smaller than the signal modes and has sufficient fluctuations that the original Duan criterion [9,10] would not detect entanglement. This effect becomes more prominent for larger numbers of particles (see [30] for further details). In this regime it is not possible to identify the signal state with a simple two-mode squeezed state as generated by an optical parametric oscillator. The remarkable feature of these states is that the quadrature variances are smaller than theoretically possible using a coherent-state local oscillator of the same size [cf. Eq. (4)], a fact only made possible by the entanglement between the local oscillator and signal beams (for further detail see [30]).

Number fluctuations are an important consideration in an experimental setting and will typically be at or above the shot-noise limit. We repeated the above simulation with an initial coherent state condensate with Poissonian statistics with a mean of 100 atoms. We do not plot the results here as they are essentially identical to those in Figs. 2(a)–2(c) for an initial number state. Simulations of systems beginning with a highly mixed initial state with number fluctuations well above the shot-noise limit also demonstrate entanglement. We plot the results for an initial thermal (chaotic) distribution with a mean of 100 particles in Figs. 2(d)–2(f). We observe that in this case the maximal amount of violation and the range of values of χt where entanglement is demonstrated are both reduced compared to an initial Fock state, but it is present nonetheless. In fact, for small values of χt the violation slightly *increases* with fluctuations. This result applies to both photons and atoms, and may suggest that the demonstration of entanglement in a range of quantum optics experiments are relatively unaffected by the intensity fluctuations of the laser sources.

In conclusion, we have derived three criteria for identifying continuous variable entanglement when local oscillators in arbitrary states are used in the quadrature measurements. We have shown that these criteria can be violated and entanglement demonstrated for degenerate four-wave mixing with as few as 100 particles. In this situation direct application of the criteria of Duan *et al.* [9], Simon [10], and Reid [11] is inappropriate and can lead to either falsely identifying entanglement or failing to identify entanglement when the local oscillator modes are sufficiently small. We have also shown that initial number fluctuations will not necessarily prevent the demonstration of entanglement in experiment.

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