Correlations and fluctuations in atom-molecular BEC

P. D. Drummond, K. Kheruntsyan
University of Queensland/Universitaet Erlangen-Nurenberg

2nd July 2002

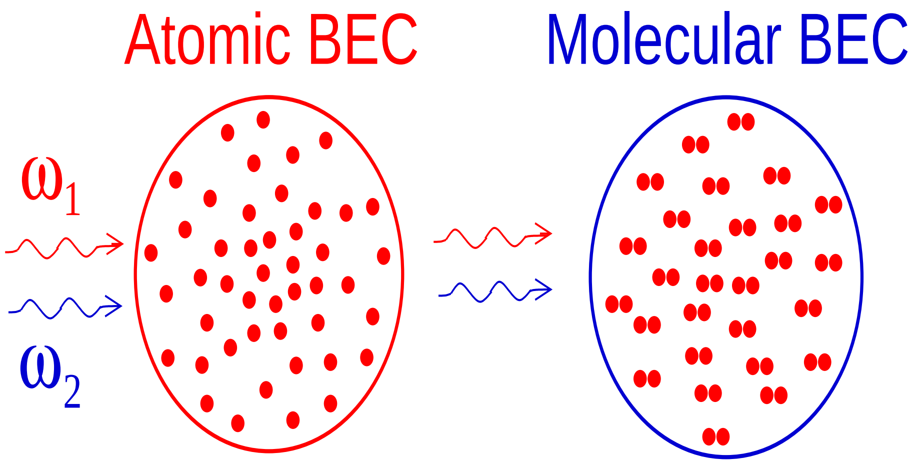
Correlations and fluctuations in atom-molecular BEC

- ***Coupled atom-molecular BECs*** - mechanisms
- ***Dressed molecules*** - exact low-density solutions
- ***Superchemistry*** - stimulated molecule emission
- ***Solitons*** - *stable* in 2D & 3D space.
- ***Quantum squeezing*** - entangled atom laser beams?
- ***Conclusions***

COUPLED ATOM-MOLECULAR BEC

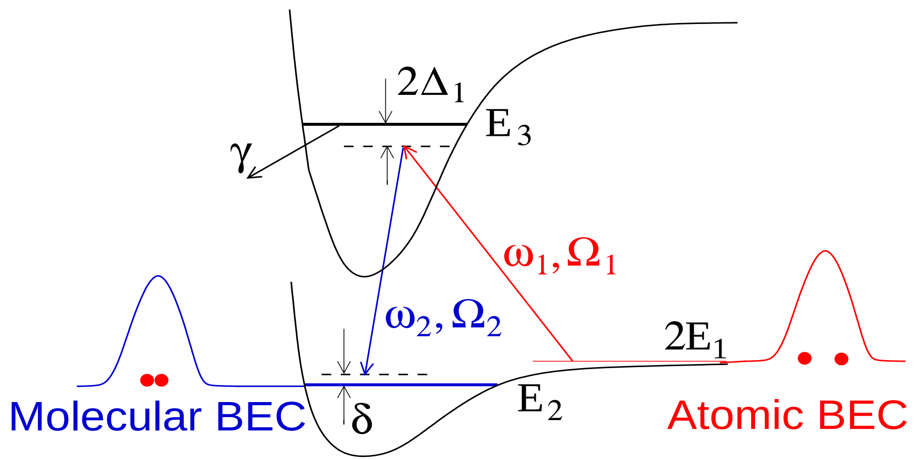
- Can we BOSE-ENHANCE molecule formation in a BEC?

- Analogous to superconductivity, except chemical reactions are enhanced!



Route I: Raman photo-dissociation

- Two atoms absorb a photon and then emit one
- This ensures energy is conserved in a coherent, phase-conserving interaction
- Drawback: spontaneous emission dephasing



Route II: Feshbach resonance

- Bound levels become resonant with free atomic levels
- Also ensures energy is conserved
- Tunable with external magnetic fields
- Drawback: large collisional losses

Effective Quantum Field Theory

'Superchemistry' parametric field theory (3D):

$$
\begin{aligned}
H_0 &= \sum \int d^3x \left[\frac{\hbar^2}{2m_i} |\nabla \hat{\Psi}_i|^2 + \hbar \Delta \hat{\Psi}_1^\dagger \hat{\Psi}_1 + V_2 \hat{\Psi}_2^\dagger \hat{\Psi}_2 \right] \\
H_{int} &= \frac{\hbar \chi}{2} \int d^3x \left[\hat{\Psi}_2 \hat{\Psi}_1^\dagger \hat{\Psi}_1^\dagger + H.c. \right] \\
H_{self} &= \sum \frac{\hbar U_{ij}}{2} \int d^3x \hat{\Psi}_i^\dagger \hat{\Psi}_j^\dagger \hat{\Psi}_j \hat{\Psi}_i
\end{aligned}
$$

- $U_{11} = \kappa = 4\pi\hbar a/m_1$, a – 1D S-wave scattering length
- $(V_2 - 2V_1) = E_m$ – bare atom-molecular 'detuning'

EXACT two-particle quantum solution

There is an exact ground state: $\left|\psi^{(2)}\right\rangle$ is a superposition state:

$$\left|\psi^{(2)}\right\rangle = \left[\int d^3\mathbf{x}\hat{\Psi}_2^\dagger(\mathbf{x}) + \int\int d^3\mathbf{x}d^3\mathbf{y}\; g(\mathbf{x}-\mathbf{y})\;\hat{\Psi}_1^\dagger(\mathbf{x})\hat{\Psi}_1^\dagger(\mathbf{y})\right]|0\rangle$$

- This is a type of dressed state - only exists when the coupling is turned on.
- Low-density, strong-coupling solution
- ***Predicted:*** Drummond et al, Physical Review Letters **81**, 3055-3058 (1998).

EXACT 2N-particle quantum ground state

Another ground state: $\left|\psi^{(2N)}\right\rangle$ is a superposition state:

$$\left|\psi^{(2N)}\right\rangle = \left[\int d^3\mathbf{x}\hat{\Psi}_2^\dagger(\mathbf{x}) + \int\int d^3\mathbf{x}d^3\mathbf{y}\ g(\mathbf{x}-\mathbf{y})\ \hat{\Psi}_1^\dagger(\mathbf{x})\hat{\Psi}_1^\dagger(\mathbf{y})\right]^N |0\rangle$$

- EXACT ground-state for infinite momentum cutoff!
  $g(0) = -\chi/2\kappa; \quad g(r) = 0; r \neq 0$
- Bose condensate of dressed molecules
- APPROXIMATE low-density solution at finite cut-off, $k_m \simeq 1/a$.

Ground state energy: finite cutoff

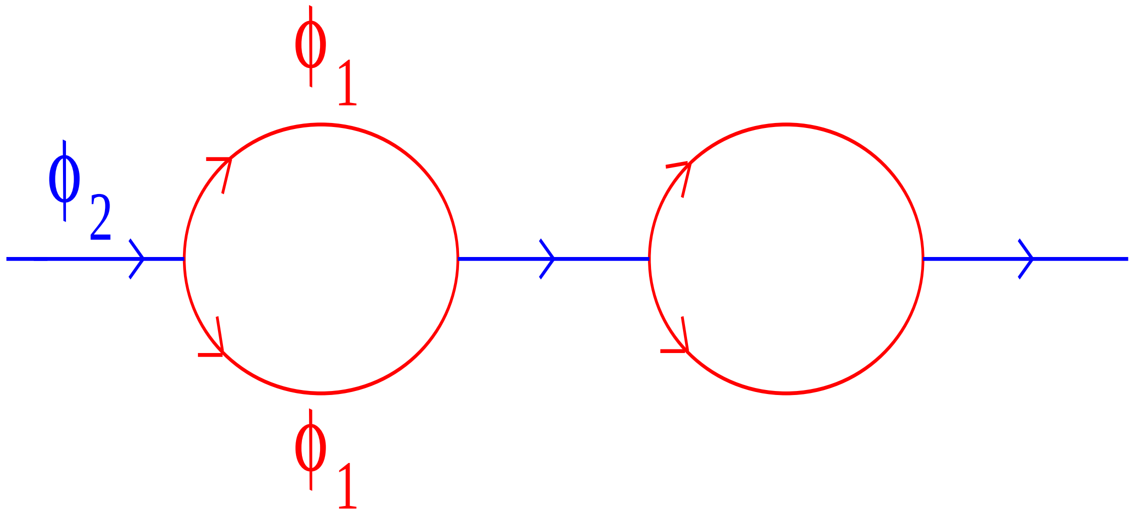
- Must use momentum cutoff with $k_m < 1/|a|$, or else renormalize couplings
- Correlation function: $\tilde{g}(\mathbf{k}) = \frac{-m_1}{\hbar(k^2+\mu^2)}\left[\frac{\chi}{2} + \kappa g(0)\right]$,
- Dressed binding energy: $E_b = -E_m + \frac{\chi^2 m}{8\pi\hbar}\left(a + \frac{\pi}{2f_D(E_b)}\right)^{-1}$
- Define a correlation length: $r_c = \hbar/\sqrt{-E_b m_1}$
- In 3D, $f_3(E_b) = [k_m - \tan^{-1}(r_c k_m)/r_c] \approx [k_m - \pi/(2r_c)]$

Approximate behaviour for small a

- Energy: $E_0 - E_m = E_b + C\sqrt{E_b}$
- Where: $C = \left(\frac{m^{3/2}\chi^2}{8\pi\hbar}\right)$,
- Feshbach resonance -let: $E_m - E_0 = \mu\Delta B$
- For small E_b, $E_b = \mu^2\Delta B^2/C^2$
- Quadratic (not linear) binding energy dependence!
- Similar result in full coupled channels calculation (Kokkelmans).

Schroedinger kitten:

- The molecule exists in a superposition with a pair of atoms in the ground state!



Predicted energy corresponds to observed JILA Feshbach energy: cond-mat/0201400 & CM2002.

Low-density dynamics of atom-molecular BEC

Physical understanding of Jila interference experiments (simplified): see Holland et al paper for alternate (Heiseenberg) picture!

1. Approach to Feshbach resonance PREPARES dressed molecules

2. Rapid ramp of field DECOUPLES atomic and molecular fields

3. Approach to Feshbach resonance RECOMBINES dressed molecules if in phase!

Expect: interference between the 'dead' and 'alive' kittens! Note: this is not a true Schroedinger cat - not MACRO superposition, but condensate of MICRO superpositions.

I: Preparation phase

If the Feshbach resonance is approached slowly, must produce the interacting ground state: $\left|\psi^{(B)}\right\rangle$

$$\left|\psi^{(B)}\right\rangle = \left[\psi^{(B)}\right]^N |0\rangle = \left[\psi^{(F)} + \psi^{(C)}\right]^N |0\rangle$$

:Linear combination of molecular and atomic states:

$$\psi^{(F)} = \left[\int d^3\mathbf{x}\hat{\Psi}_2^\dagger(\mathbf{x})\right]$$

$$\psi^{(C)} = \left[\int\int d^3\mathbf{x}d^3\mathbf{y}\ g(\mathbf{x}-\mathbf{y})\ \hat{\Psi}_1^\dagger(\mathbf{x})\hat{\Psi}_1^\dagger(\mathbf{y})\right]$$

Note: this should be a SLOW ramp - in fast ramp experiments, non-adiabatic behavior gives excess excited atoms (as observed).

II: Decoupling phase

If the magnetic field is rapidly changed from $B \to B'$, the dressed state is not an eigenstate of the new Hamiltonian. Some of the atoms will be at $k = 0$, but some correlated atoms have a finite momentum

$$|\psi(t)\rangle = \left[\psi^{(B')} e^{iE_b t/\hbar} + \int\int d^3\mathbf{k}\ \Delta g(\mathbf{k})\ \hat{\Psi}_1^\dagger(\mathbf{k})\hat{\Psi}_1^\dagger(-\mathbf{k}) e^{-i2E_k t/\hbar}\right]^N |0\rangle$$

- Atomic correlations will disperse slowly - this is not a true eigenstate
- Far detuned magnetic field acts like a far-detuned laser

III: Recombination phase

If the Feshbach resonance is rapidly switched on at t_0, plane-wave atomic and molecular fields recombine:

$$|\psi(t_0)\rangle = \left[\cos(E_b t/\hbar)\psi^{(B)} + (remnants)\right]^N |0\rangle$$

- If system is ramped slowly from resonance, $\left|\psi^{(B)}\right\rangle$component stays in ground state.
- other component has excess energy
- Observe Ramsey fringes if $E_b t_0 \neq n\pi\hbar$.

SUPERCHEMISTRY?

- Can we BOSE-ENHANCE molecule formation in a BEC?
- Analogous to superconductivity, except chemical reactions are enhanced!
- These equations have solitons in three dimensions
- Hence, could get a non-diverging atom laser beam
- Possible applications to lithography, nano-technology
- Quantum control of chemical reactions at $0K$!

High-density dynamics of atom-molecular BEC

- At high density, the mean-field superposition has a lower energy
- Initially only atomic species are present; then switch on molecule formation;

$$
\begin{aligned}
i\dot{\psi}_1 &= -\nabla^2\psi_1 + V_1(\mathbf{x})\psi_1 + U_{11}|\psi_1|^2\psi_1 + \psi_1^*\psi_2, \\
i\dot{\psi}_2 &= -\frac{1}{2}\nabla^2\psi_2 + V_2(\mathbf{x})\psi_2 + \frac{1}{2}\psi_1^2,
\end{aligned}
$$

- Bose-enhanced kinetics replaces chemical kinetics:
- ***Observation?:*** (small numbers only!!) Heinzen, et. al. Science **287**, 1016 (2000).

SOLITONS - *stable* in 2D & 3D space

- **Can photons or BEC matter form *stable particle-like structures*?**
- Direct simulation of coupled **parametric** equations in photonic systems:

$$
\begin{aligned}
i\frac{\partial}{\partial z}\psi_1 &= -\left[\nabla_\perp^2 + \partial^2/\partial t^2\right]\psi_1 + \psi_1^*\psi_2, \\
i\frac{\partial}{\partial z}\psi_2 &= -\frac{1}{2}\left[\nabla_\perp^2 + \delta\partial^2/\partial t^2\right]\psi_2 + \frac{1}{2}\psi_1^2,
\end{aligned}
$$

- ***2D & 3D solitons predicted*** by Drummond et. al., PRA (1997).
- ***Observed*** in photonics: Wise et. al., Phys. Rev. Lett. **82**, 4631 (1999).

QUANTUM DISSOCIATION?

- **Start with a molecular BEC, no atoms; switch on the coupling!**

- focus on short interaction times, and small numbers of atoms produced

- replace $\hat{\Psi}_2$ by a c-number; assume a Thomas-Fermi molecular density $n_2(x)$

$$\chi(t)\hat{\Psi}_2(x) \rightarrow \chi(t)\sqrt{n_2(x)} \equiv \tilde{\chi}(t,x)$$

$$\Delta + U_{12}\hat{\Psi}_2^\dagger(x)\hat{\Psi}_2(x) \rightarrow \Delta + U_{12}n_2(x) \equiv \tilde{\Delta}(x)$$

- neglect atom-atom S-wave scattering

Quantum dynamics

- Stochastic P-representation equations:

$$
\begin{aligned}
\frac{\partial \psi(x,t)}{\partial t} &= i\frac{\hbar}{2m_1}\frac{\partial^2 \psi}{\partial x^2} - i\tilde{\Delta}\psi + \tilde{\chi}\psi^+ + \sqrt{\tilde{\chi}}\xi_1 \\
\frac{\partial \psi^+(x,t)}{\partial t} &= -i\frac{\hbar}{2m_1}\frac{\partial^2 \psi^+}{\partial x^2} + i\tilde{\Delta}\psi^+ + \tilde{\chi}\psi + \sqrt{\tilde{\chi}}\xi_2
\end{aligned}
$$

$$
\begin{aligned}
\tilde{\chi} &= \chi(t)\sqrt{n_2(x)} \\
\tilde{\Delta} &= \Delta + U_{12}n_2(x)
\end{aligned}
$$

Relative particle number squeezing

- Quantum correlations:

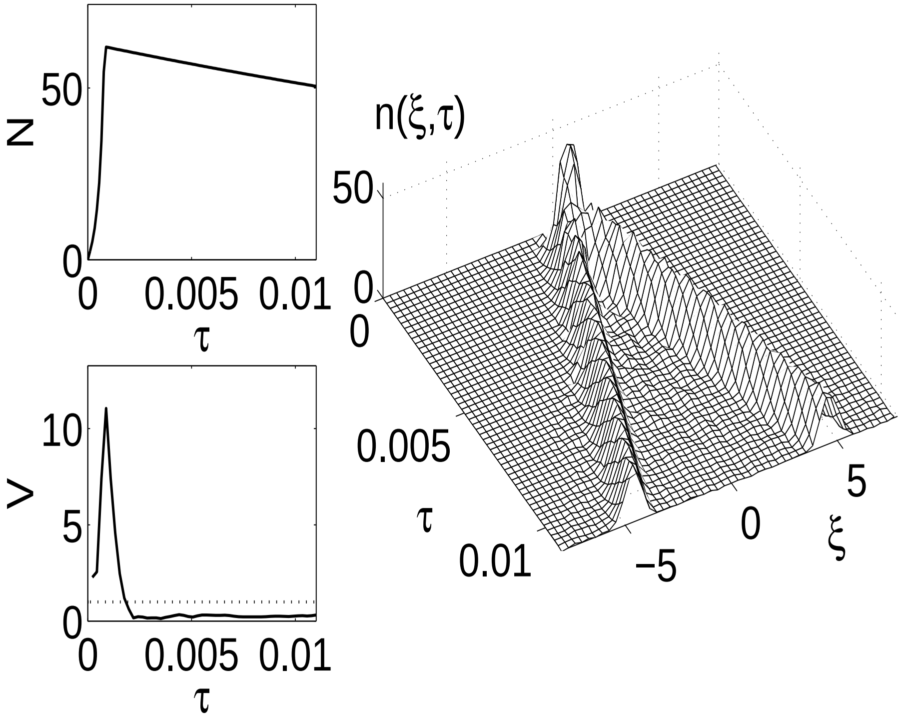
$$V = \langle[\Delta(\hat{N}_- - \hat{N}_+)]^2\rangle \Big/ (\langle\hat{N}_-\rangle + \langle\hat{N}_+\rangle)$$
$$= 1 + [\langle : (\hat{N}_+)^2 : \rangle - \langle\hat{N}_-\hat{N}_+\rangle] \Big/ \langle\hat{N}_+\rangle$$

- Particle number:

$$\hat{N}_{+(-)}(t) = \int_{0(-\infty)}^{+\infty(0)} dx \hat{\Psi}_1^\dagger(x)\hat{\Psi}_1(x)$$

- Quantum **squeezing** corresponds to: $V < 1$

BEC Quantum Down-conversion



- Phase-matching at $k^2 = -V$ gives two opposite beams.

Conclusions

- Nonlinear atom optics has quantum fluctuations!
- Strong interactions, low losses, strong correlations.
- Possible quantum phase transition, 3D solitons?
- Quantum squeezing in relative particle number
- Destructive measurement of particle number in one beam produces a single beam with a well-defined particle number
- possible applications: **EPR correlations, Bell inequalities with massive particles, quadrature squeezing, precision measurements**