Quantum superpositions and correlations in coupled atomic-molecular BECs

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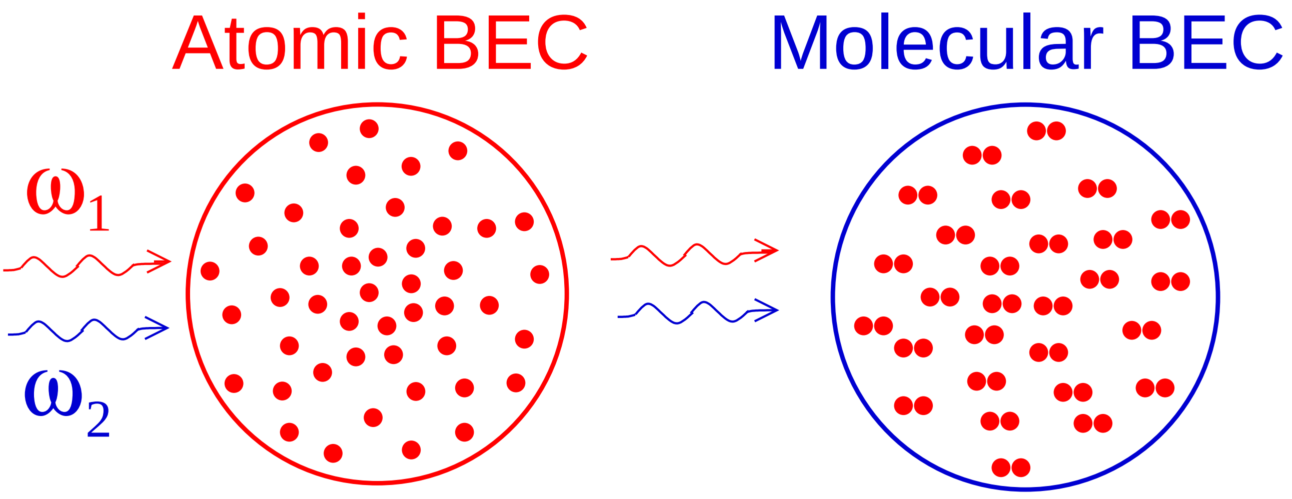
Outline

- Coupled atom-molecular BECs – mechanisms
- Recent experiments at JILA
  - coherent oscillations in a BEC
  - quantum superposition of atoms and molecules
- Theory behind the superposition
  - predictions of 1998 revisited...
- Quantum correlated twin atom-laser beams

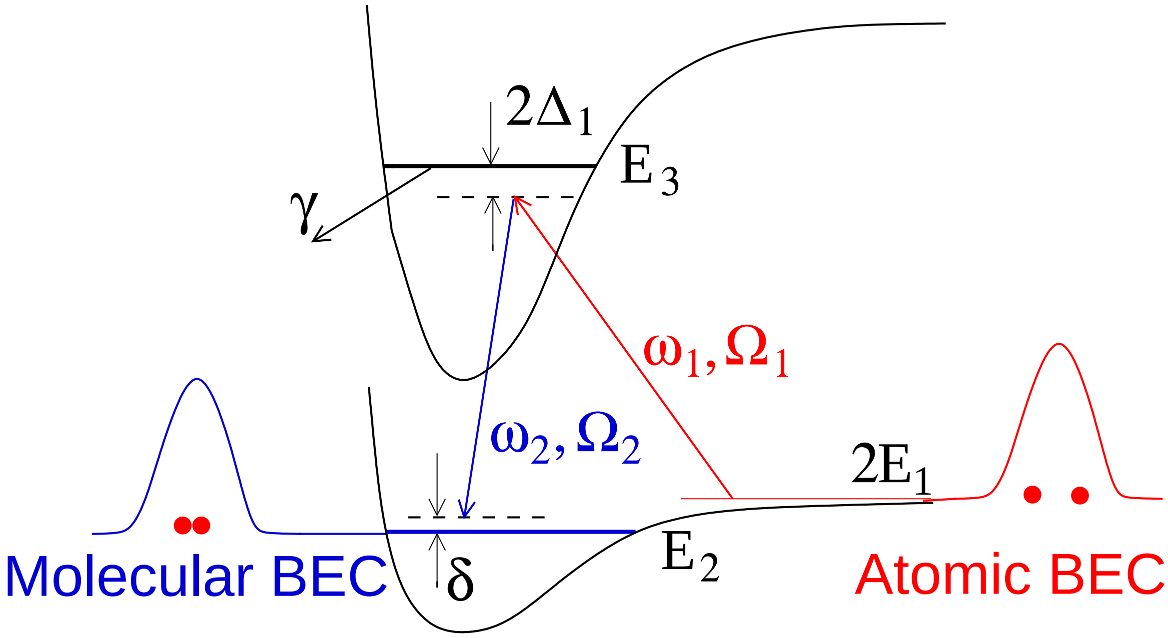
Why coupled AM BECs

- Can a molecular BEC form **coherently** from an atomic BEC?
- New regimes of non-linear atom optics
- BOSE-ENHANCED chemistry or **‘superchemistry’** at ultralow T
- New microscopic BEC physics to emerge



Route 1: Raman Photoassociation

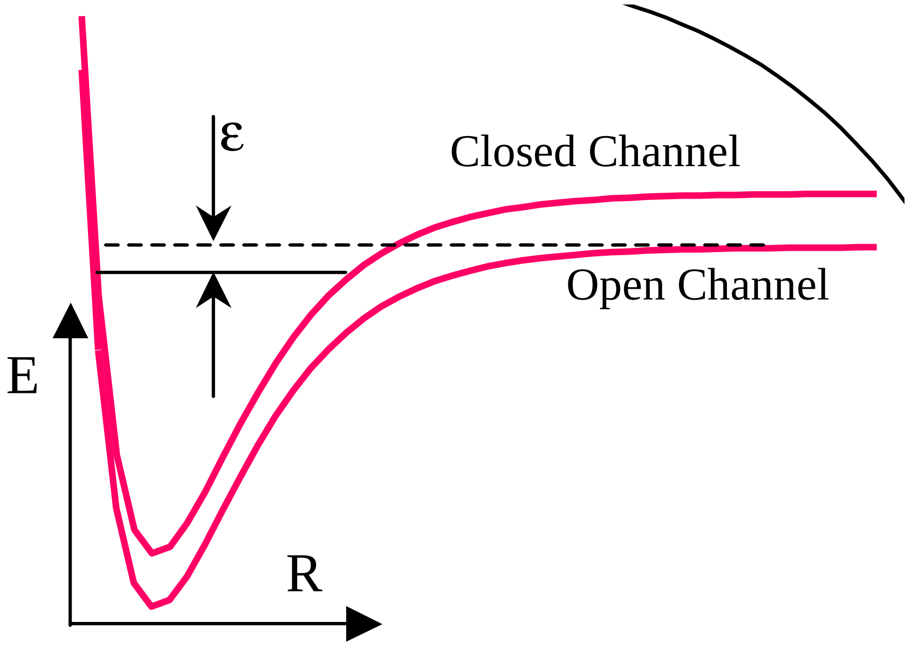
- Pairs of atoms collide, absorb a photon *and* emit one, giving a ground molecular state
- Phase locking the two lasers ensures that the conversion is coherent (A BEC → M BEC)
- Drawback: spontaneous emission losses; week free-bound couplings



[Wynar et al., Science 287,1016 (2000)]

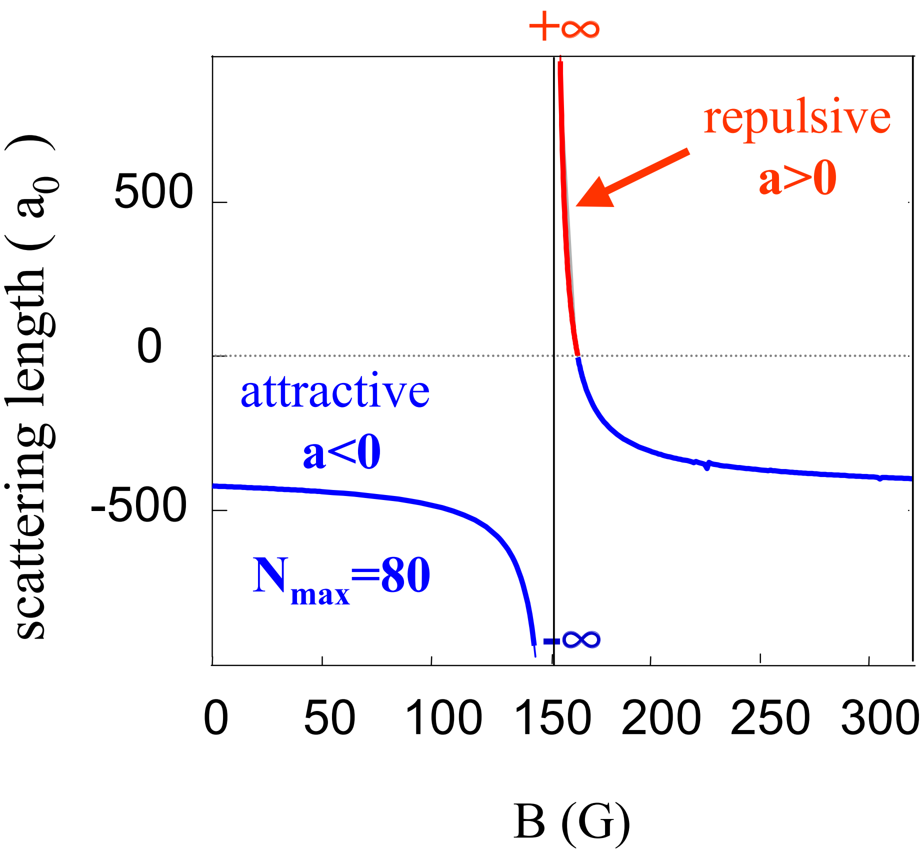
Route 2: Feshbach Resonance

- Tunable with an external magnetic field
- Bound molecular levels become resonant with the energy of free atoms
- Drawback: large losses due to inelastic collisions
- Advantages: stronger couplings; tunable atom-atom interactions



JILA experiment

- Rb^{85} BEC with $a > 0$ near Feshbach resonance: $a \to a(B) = a_{bg}\left(1 - \frac{\Delta B}{B - B_0}\right)$

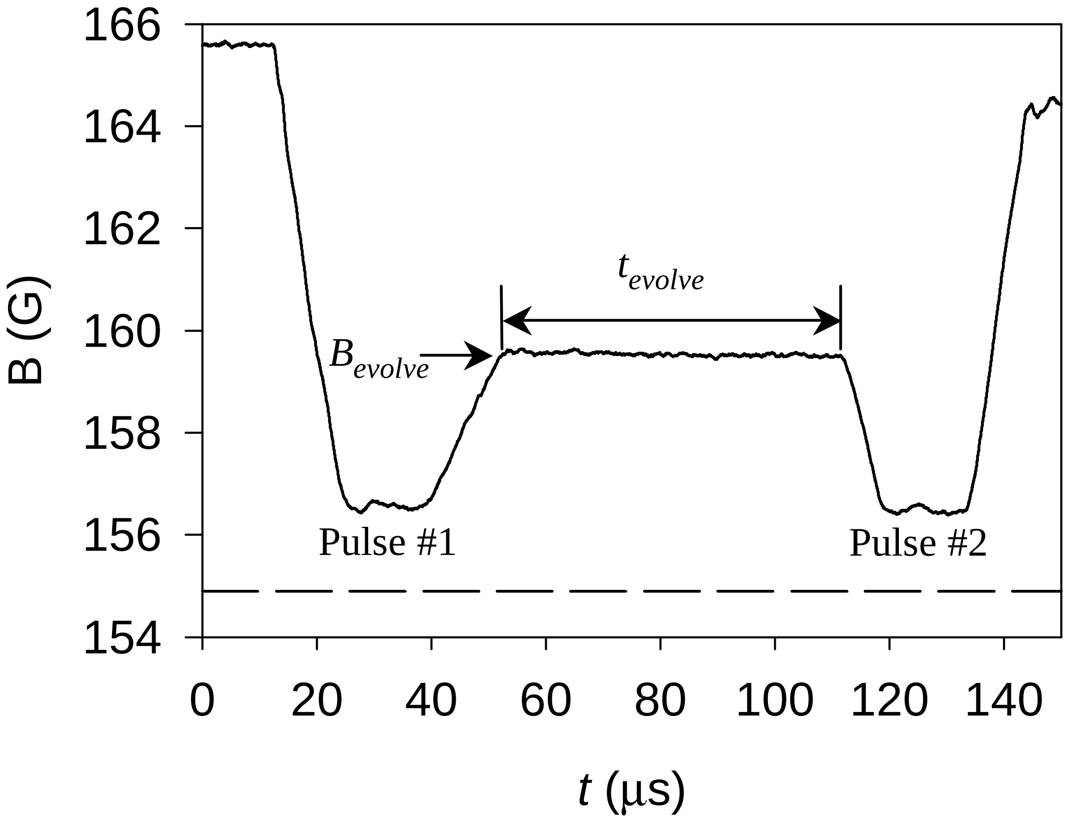


- For $na^3 \sim 1$ (rather than $\ll 1$), the customary mean-field theory description breaks down

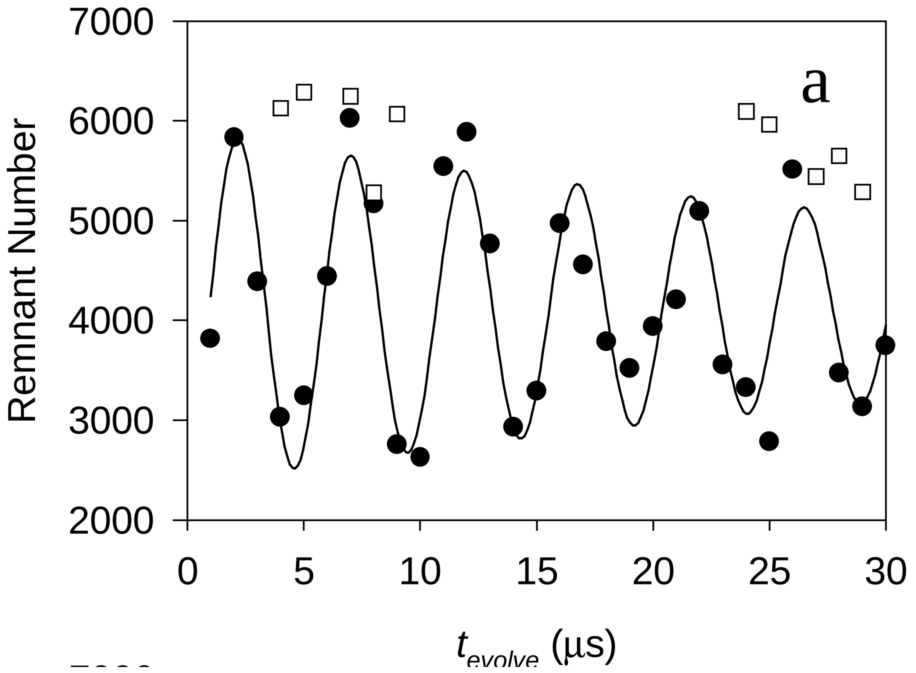
- New microscopic BEC physics to be expected!

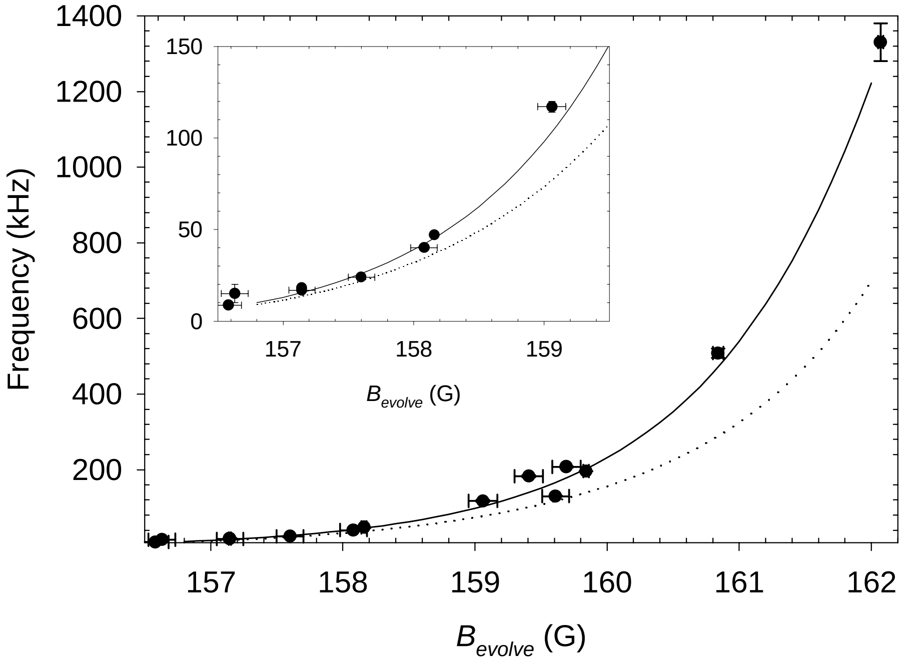
Experimental procedure

- Prepare an atomic BEC at $B \simeq 166$ G, $a > 0$
- Apply a sequence of two fast magnetic-field pulses
- Measure the number of atoms remaining as a function of t_{evolve}, for different B_{evolve}



Results [Nature, 417, 529 (May 2002)]





Atomic Ramsey interferometry

- Consider two-level atom ($|g\rangle$, $|e\rangle$); Apply two successive $\pi/2$ pulses, R_1 and R_2, at $\omega \simeq \omega_{ge}$.

- The first R_1 pulse transforms:

$$|g\rangle \rightarrow \left(|g\rangle + |e\rangle\right)/\sqrt{2}$$
$$|e\rangle \rightarrow \left(-|g\rangle + |e\rangle\right)/\sqrt{2}$$

- After a time delay t_{evolve}, R_2 transforms:

$$|g\rangle \rightarrow \left(|g\rangle + |e\rangle\, e^{i\theta}\right)/\sqrt{2}$$
$$|e\rangle \rightarrow \left(-|g\rangle\, e^{-i\theta} + |e\rangle\right)/\sqrt{2}$$

$\theta = (\omega - \omega_{ge})t_{evolve}$ - accumulated phase difference

- Probability of $|g\rangle \rightarrow |g\rangle$ or $|g\rangle \rightarrow |e\rangle$:

$$P_{g\rightarrow g} = (1 + \cos\theta)/2$$
$$P_{g\rightarrow e} = 1 - P_{g\rightarrow g} = (1 - \cos\theta)/2$$

Effective quantum field theory

$$H_0 = \sum\nolimits_{i=1,2} \int d\mathbf{x} \left[\frac{\hbar^2}{2m_i} |\nabla \hat{\Psi}_i|^2 + V_i(\mathbf{x}) \hat{\Psi}_i^\dagger \hat{\Psi}_i \right]$$

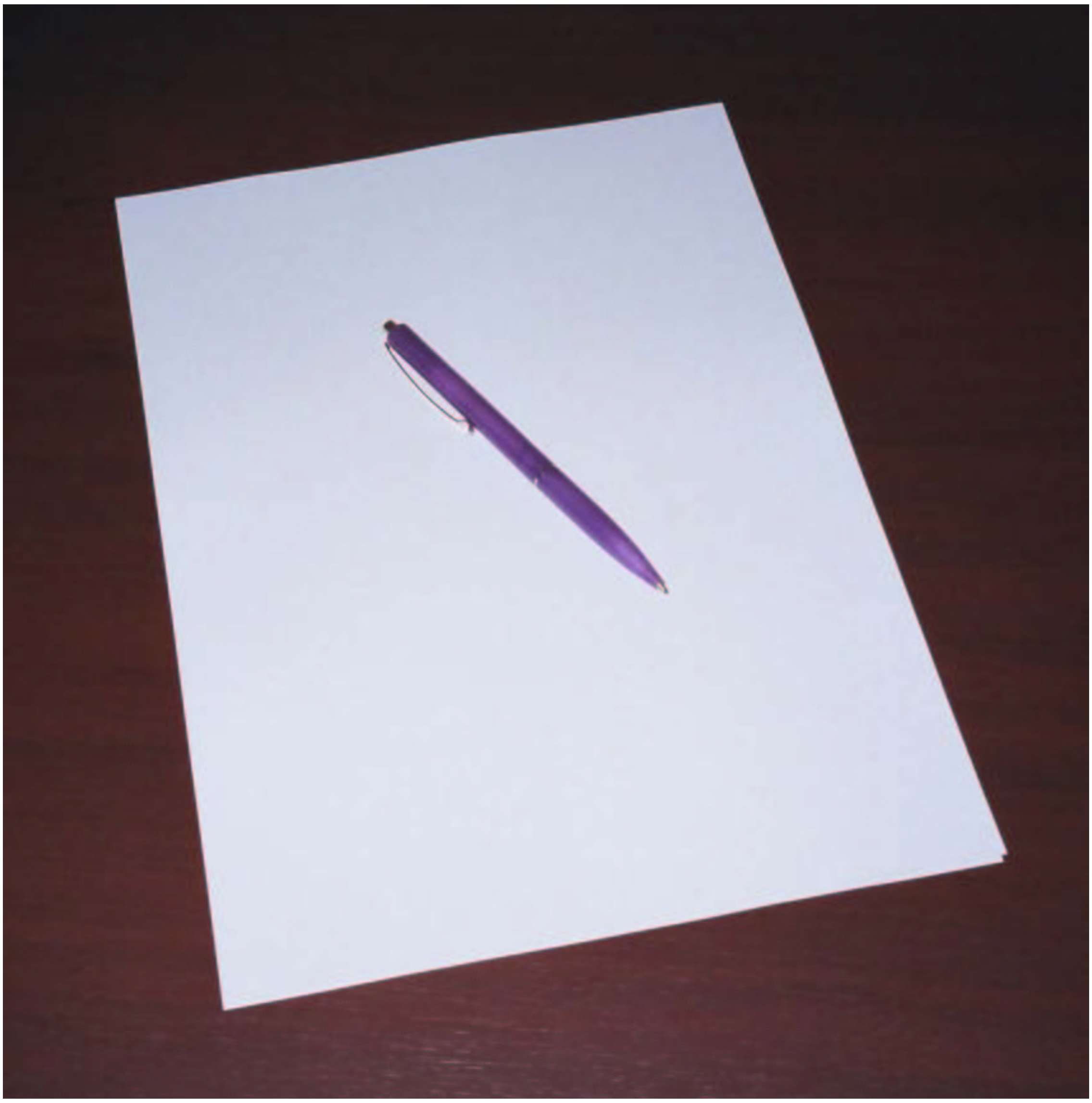
$$H_{int} = \frac{\hbar\chi}{2} \int d\mathbf{x} \left[\hat{\Psi}_2 \hat{\Psi}_1^\dagger \hat{\Psi}_1^\dagger + H.c. \right]$$

$$H_{self} = \sum\nolimits_{ij} \frac{\hbar U_{ij}}{2} \int d\mathbf{x} \hat{\Psi}_i^\dagger \hat{\Psi}_j^\dagger \hat{\Psi}_j \hat{\Psi}_i$$

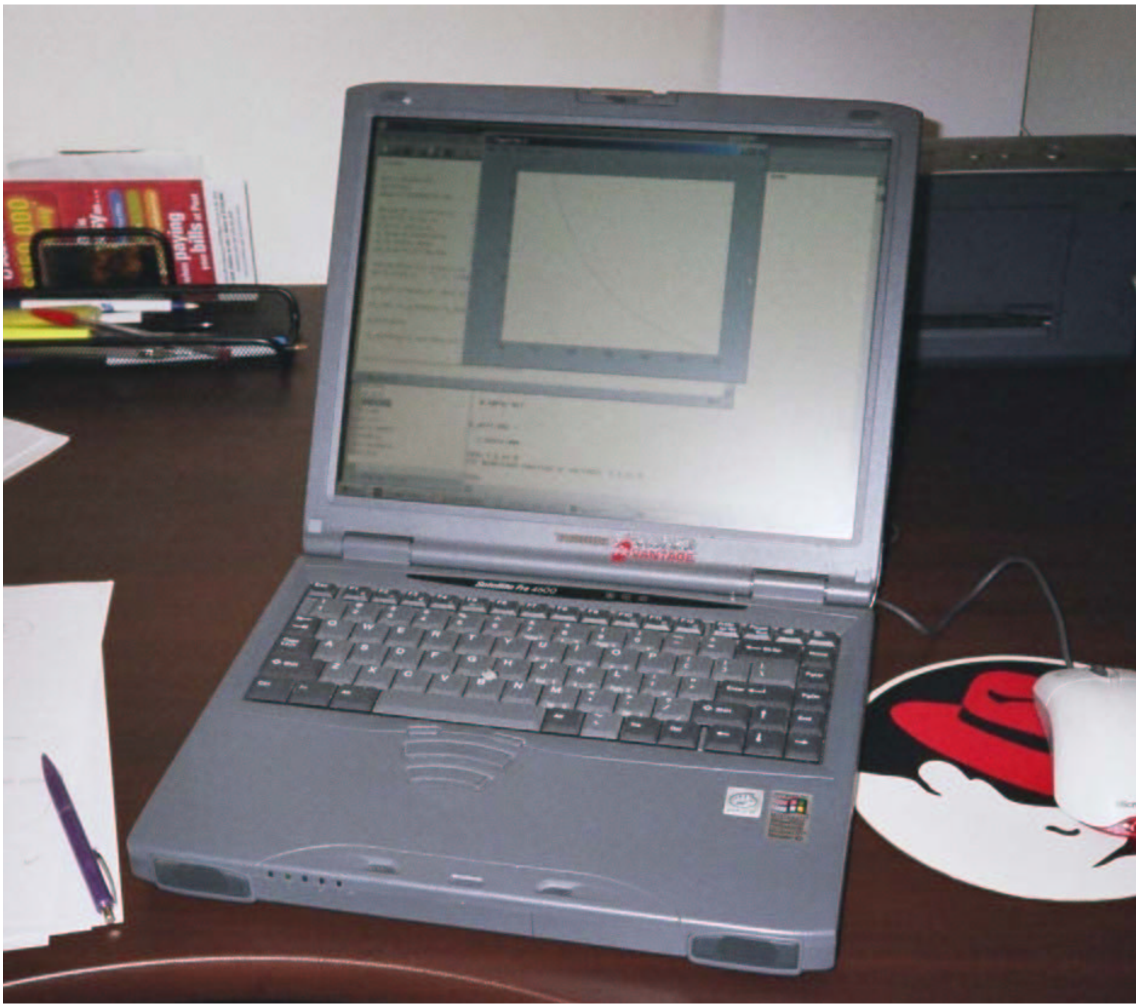
- $\hat{\Psi}_{1/2}(t, \mathbf{x})$ – atomic/molecular field operators
- $U_{11} \propto a$ – atom-atom S-wave scattering

  – comes from $\tilde{U}_{11}(x - y) \rightarrow U_{11}\delta(x - y)$, together with a UV momentum cutoff $k_{\max}$
- χ – atom-molecule coupling ($A + A \rightleftarrows A_2$)
- V_i – trap potentials, including internal energies E_i

... apparatus 1 ...



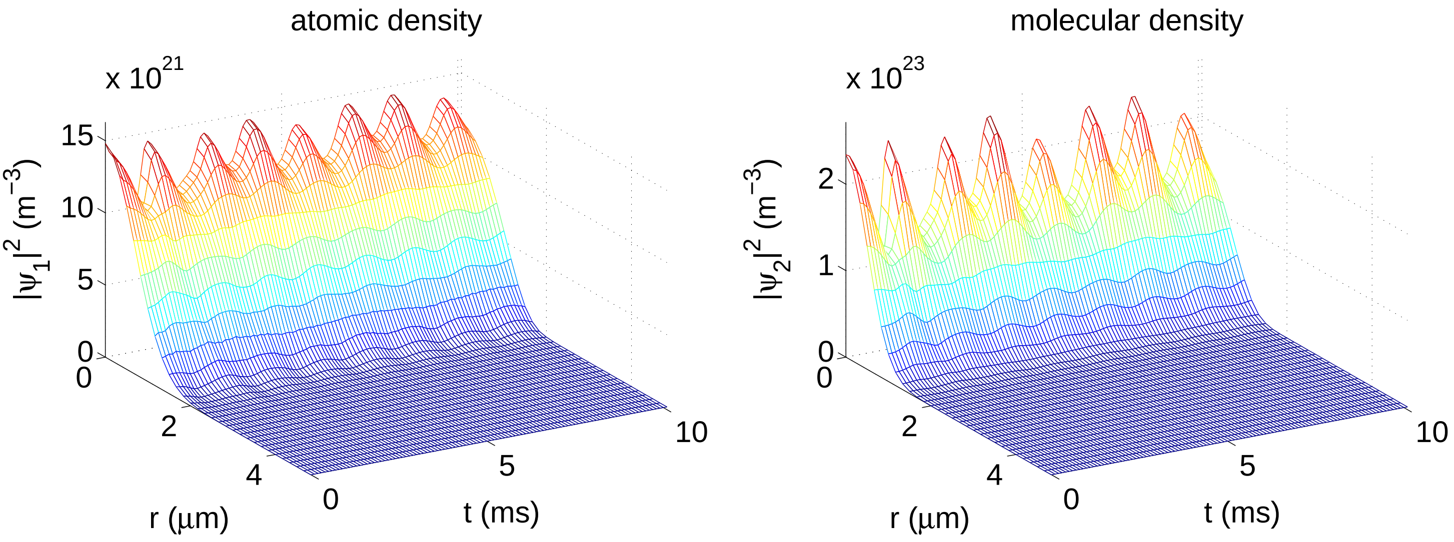
… apparatus 2 …



High density regime: 3D matter wave solitons

- At high densities (mean-field theory applies) wave-like interactions between the entire atomic and molecular BECs are favored

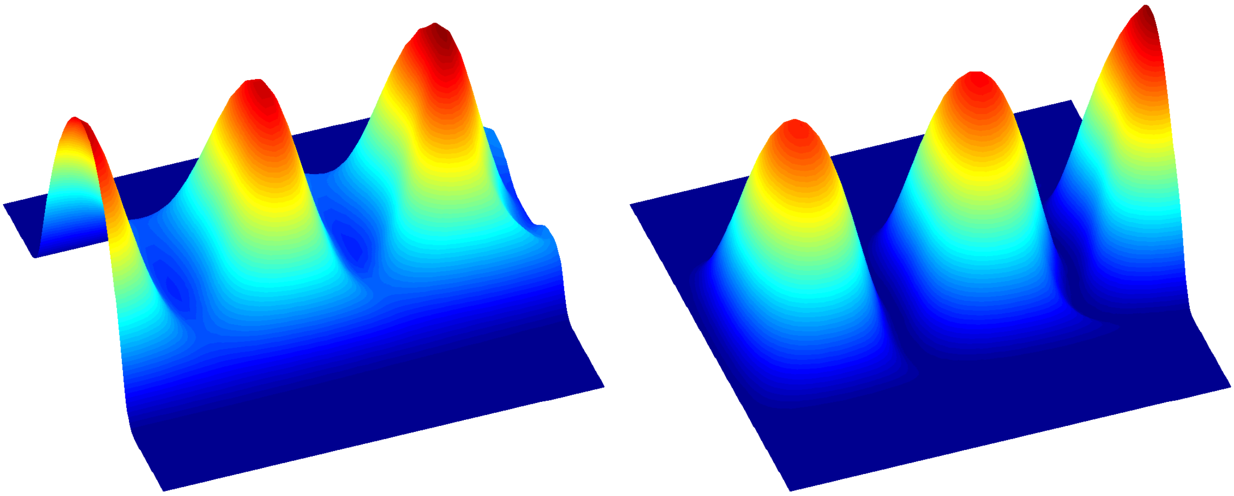
- Stable 3D matter-wave solitons can form in free space!



[PRL **81**, 3055 (1998)]

‘Superchemistry’ oscillations

- Start with a pure atomic BEC; switch on the coupling
- Giant coherent oscillations are observed between the atomic and molecular BECs
- Enhanced chemical reaction rates due to bosonic stimulation at $T \to 0$!



[PRL **84**, 5029 (2000)]

Low-density regime: Coherent quantum superposition

- EXACT quantum ground state (for $N=2$):

$$\left|\Psi^{(2)}\right\rangle = \left[\hat{b}^{\dagger}(0) + \int_{|\mathbf{k}|=0}^{k_{\max}} d\mathbf{k} G(\mathbf{k}) \hat{a}^{\dagger}(\mathbf{k}) \hat{a}^{\dagger}(-\mathbf{k})\right] |0\rangle$$

– the eigenstate is a coherent superposition of a molecule with a pair of atoms: “dressed” molecule

- Energy eigenvalue

$$E = E_m - \frac{\hbar\chi^2}{2}\left[U_{11} + \frac{2\pi^2\hbar R/m}{Rk_{\max} - \tan^{-1}(Rk_{\max})}\right]^{-1}$$

$$= -\frac{\hbar^2}{mR^2}.$$

R - correlation radius; $E_b = -E$ - binding energy.

Approximate quantum many-body solutions

Quantum many-body ($N > 2$) ground state is well approximated by:

$$\left|\Psi^{(N)}\right\rangle = \left[\hat{b}^\dagger(0) + \int_{|\mathbf{k}|=0}^{k_{\max}} d\mathbf{k} G(\mathbf{k}) \hat{a}^\dagger(\mathbf{k}) \hat{a}^\dagger(-\mathbf{k})\right]^{N/2} |0\rangle$$

- Corresponds to a gas of $N/2$ independent "dressed" molecules, with total energy

$$E^{(N)} = \frac{N}{2} E.$$

- The two-particle solution $\left|\Psi^{(2)}\right\rangle$ and the corresponding energy E are keys to understanding the JILA experiment!

Binding energy E_b vs B ?

- $E_b(B)/\hbar$ should give the frequency of oscillations observed (analogous to Ramsey fringes), as interference between atoms and "dressed" molecules.

$$E_b = -E_m + \frac{\hbar\chi^2}{2}\left[U_{11} + \frac{2\pi^2\hbar R/m}{Rk_{\max} - \tan^{-1}(Rk_{\max})}\right]^{-1}$$
$$= -\frac{\hbar^2}{mR^2}.$$

– can be used for large R, with $R^{-1} \ll k_{\max} \lesssim |a_{bg}|^{-1}$

- In JILA experiment, however, $R \sim |a_{bg}|$.

- Must carry out **renormalization**, to make the theory cut-off independent.

Renormalization

- Prevents UV divergencies as $k_{\max} \to \infty$
- Relates 'bare' coupling constants in H to the **observed** values

$$\chi = \Gamma\chi_0,\ U_{11} = \Gamma U_0,\ E_m = E_0 + \alpha\Gamma\hbar^2\chi_0^2$$

where

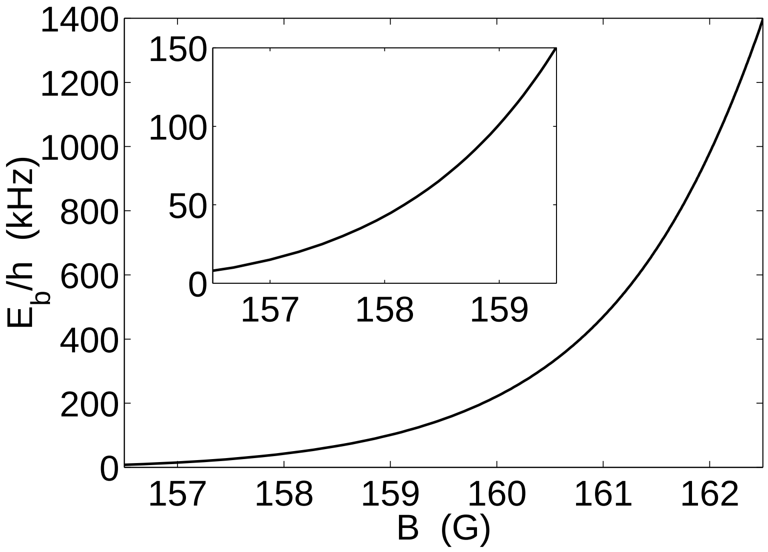
$$\begin{aligned} \Gamma &= (1 - \alpha\hbar U_0)^{-1}, \\ \alpha &= mk_{\max}/(2\pi^2\hbar^2), \\ U_0 &= 4\pi\hbar a_{bg}/m \\ E_0 &= \Delta\mu(B - B_0) \end{aligned}$$

[Kokkelmans & Holland, cond-mat/0204504]

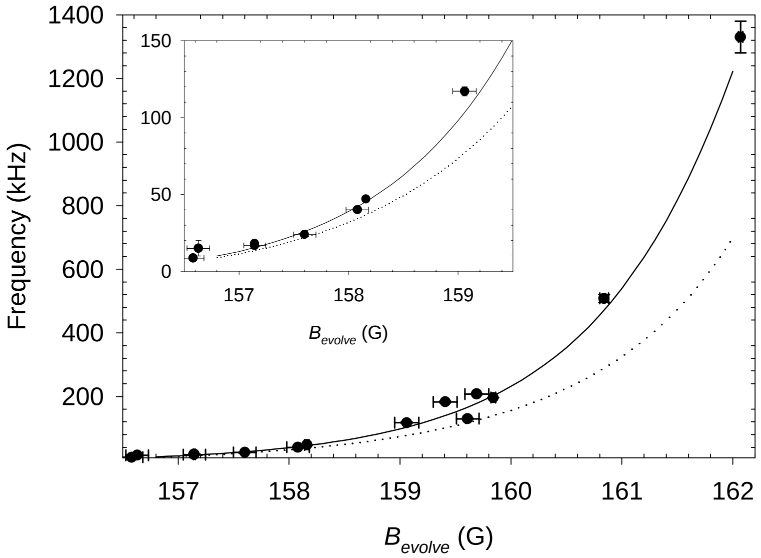
E_b vs B – final result!

$$B = B_0 + \frac{1}{|\Delta\mu|}\left(E_b + \frac{C_1\sqrt{E_b}}{1 + C_2\sqrt{E_b}}\right)$$

THEORY→



EXP. →

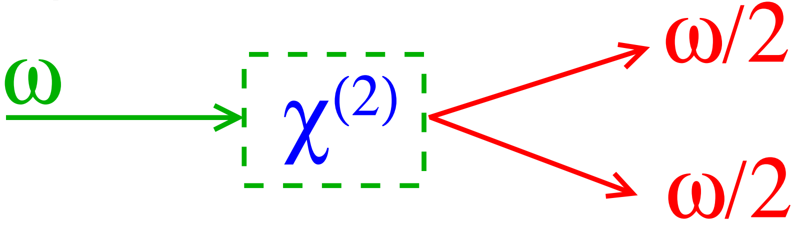


... more on quantum correlations ...

Twin atom-laser beams via dissociation of a molecular BEC

- produce quantum correlated atomic beams with entangled atom pairs

- matter-wave analog of parametric down-conversion in nonlinear optics

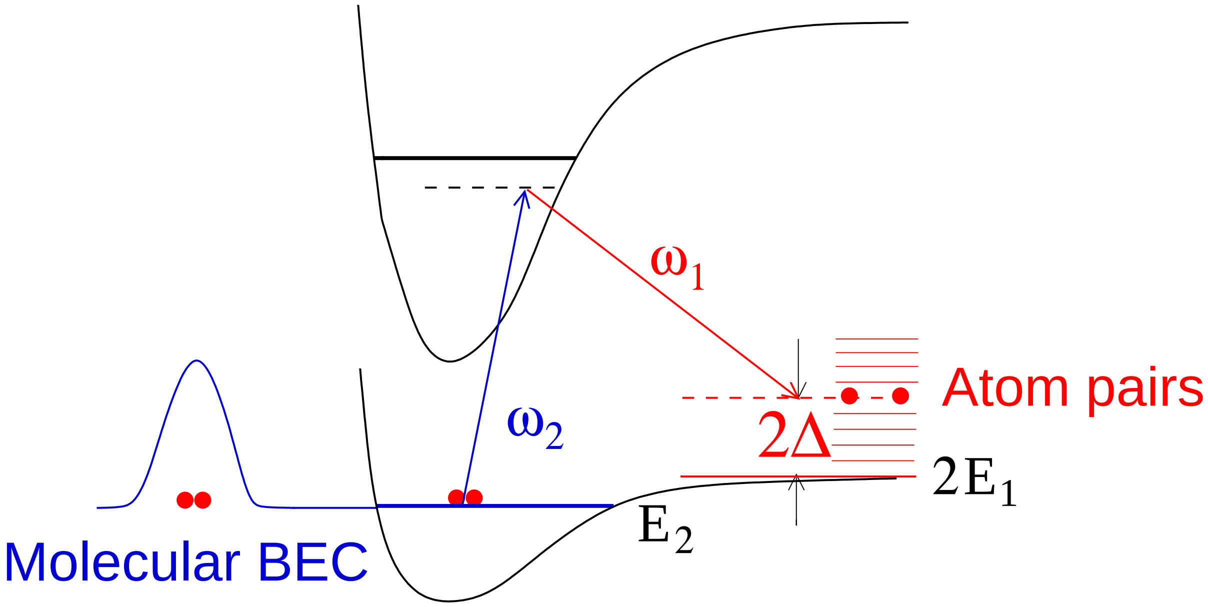


- strong particle number-difference squeezing is expected

- applications in precision (sub-shot noise) measurements; new regimes of EPR & Bell correlations with **massive** particles

Photo-dissociation of a molecular BEC

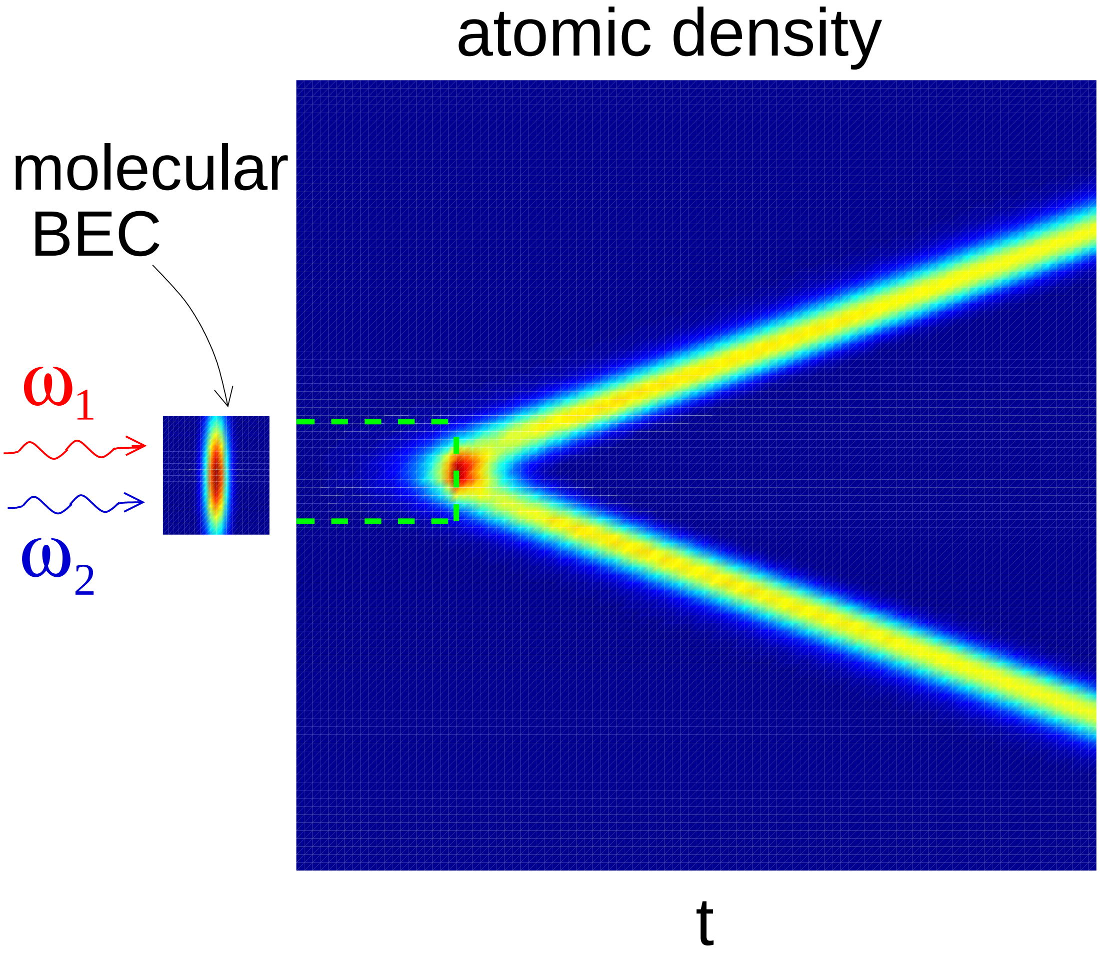
- start with a BEC of molecular dimers
- **coherently** dissociate the molecules into atom pairs, using coherent Raman transitions



- Energy conservation: $\hbar|\Delta| = \hbar^2 k^2/2m_1$

  Momentum conservation: $\pm k_0 = \sqrt{2m_1|\Delta|/\hbar}$

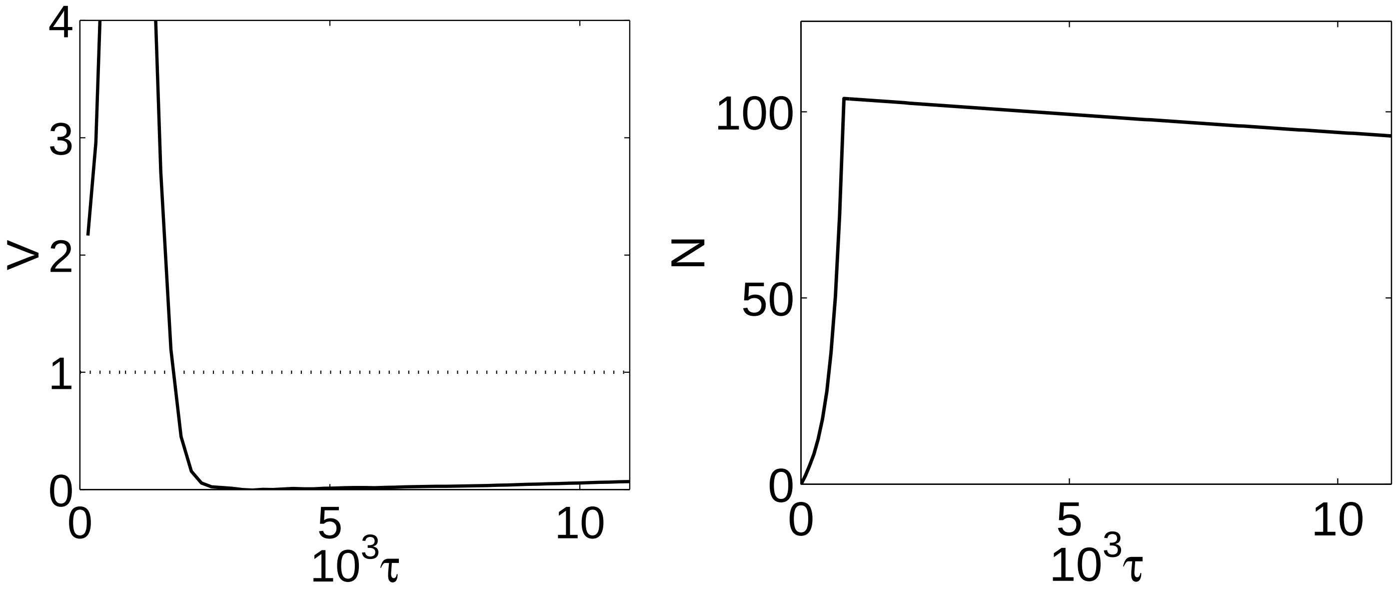
Twin atomic beams (1D)



Relative particle number squeezing

- Variance of fluctuations in particle number difference in two beams:

$$V = \langle [\Delta(\hat{N}_- - \hat{N}_+)]^2 \rangle \Big/ (\langle \hat{N}_- \rangle + \langle \hat{N}_+ \rangle)\,,$$
$$= 1 + [\langle : (\hat{N}_+)^2 : \rangle - \langle \hat{N}_- \hat{N}_+ \rangle] \Big/ \langle \hat{N}_+ \rangle$$



- $V \simeq 0.07 < 1$ (93% squeezing) in this example (losses allowed at 10%; dissociation time ~ 2 ms, $\xi_0 \simeq 30$ μm)

SUMMARY

- Coherent oscillations in BEC number in JILA Feshbach resonance experiment indicate presence of quantum superposition between atoms and molecules
- Our model quantum field theory gives explicit analytic expression for the superposition state
- The corresponding binding energy and the experimentally observed frequency of oscillations are in remarkable agreement!
- Dissociation of a molecular condensate can produce strongly correlated twin atomic beams, with almost perfect squeezing in relative particle number