Growth dynamics of a Bose-Einstein condensate

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Outline

- Brief introduction to BEC.
- Kinetic theory.
- Extension to quantum kinetic theory: QKT™
- Application to condensate growth
- Latest calculations
- What is a quasi-condensate?
Bose-Einstein Condensation

- Macroscopic occupation of a single quantum level

\[ n \Lambda_{dB}^3 \geq 2.612 \]

- Created for the first time in 1995 in atomic Rb
- More than 30 groups have now observed BEC
- Atomic interactions relatively weak

⇒ Calculations from first principles are feasible

Testing ground for computational quantum field theory
How to make a BEC

- Laser cooling produces cloud $\sim 10$–$100 \mu K$
- Transfer to a magnetic trap (harmonic potential)
- Evaporative cooling with rf “scalpel”
- Atoms are lost, but temperature decreases more rapidly.
- Phase space density increases until $n \Lambda^3_{dB} \geq 2.612$.

Question: How exactly does a condensate form?
Condensate formation
Condensate fraction

Will the condensate fraction follow this curve adiabatically?

\[ \frac{N_0}{N_{\text{tot}}} = 1 - \left( \frac{T}{T_c} \right)^{3/2} \]

Or will there be a lag between a change in temperature and condensate growth?
Introduction to kinetic theory

Apologies:

\[
\hat{H} = \int d^3x \hat{\Psi}^\dagger(x, t) \left[ -\frac{\hbar^2}{2m} \nabla^2 + V_{\text{trap}}(x) \right] \hat{\Psi}(x, t) \\
+ \frac{1}{2} \int d^3x \int d^3x' \hat{\Psi}^\dagger(x, t) \hat{\Psi}^\dagger(x', t) V(x - x') \hat{\Psi}(x', t) \hat{\Psi}(x, t).
\]

- Field operator: \( \hat{\Psi}(x, t) = \sum_n \hat{a}_n(t) \phi_n(x) \),

- At high temperature, interaction term is a perturbation.

- Induces transitions between eigenstates

- No coherences between levels
Quantum Boltzman equation

\[ \frac{dn_p}{dt} = \frac{4\pi U_0^2}{\hbar} \sum_{qmn} g(\epsilon_{\text{min}}) \delta(\epsilon_p + \epsilon_q - \epsilon_m - \epsilon_n) \]

\[ \times \left\{ (n_p + 1)(n_q + 1)n_m n_n - n_p n_q (n_m + 1)(n_n + 1) \right\}. \]

- \( n_p \): Occupation of quantum level \( p \)
- \( g(\epsilon) \): Density of states
- \( U_0 = 4\pi \hbar^2 a/m \): Strength of atomic interaction

Equilibrium:

\[ n_p = \left\{ \exp\left[\frac{(\epsilon_p - \mu)}{k_B T}\right] - 1 \right\}^{-1} \quad \text{— Bose distribution!} \]

Works extremely well when no condensate is present.
What is the effect of a condensate?

- BEC forms a very dense core at centre of trap
- Atom experience mean field potential of BEC.
- “Best” eigenstates of Hamiltonian change rapidly
Quantum kinetic theory

• Formalism developed by Gardiner and Zoller.

• Applies the methods of quantum optics to kinetic theory.

• Non-condensate band $R_{NC}$: described by a QBE.
  – Levels which are not significantly affected by BEC

• Condensate band $R_C$: described by a master equation.
Condensate growth — first approximation: 1997

- Full master equation is horrendous. Simplify:
  - Take mean value.
  - Include condensate level only in $R_C$.
  - Assume condensate wave function grows adiabatically.
  - Take equilibrium Boltzmann distribution for $R_{NC}$.

- Simple growth equation

$$\dot{N}_0 = 2W^+(N_0, T) \left\{ \left[ 1 - \exp \left( \frac{\mu_C(N_0) - \mu}{k_BT} \right) \right] N_0 + 1 \right\}$$

Derived before any experiments on condensate growth.
First approximation, continued

Prefactor is a kinetic rate:

\[ W^+(N_0, T) \approx \frac{4m(ak_B T)^2}{\pi \hbar^3} e^{2\mu/k_B T} \left\{ \frac{\mu_C(N)}{k_B T} K_1 \left( \frac{\mu_C(N)}{k_B T} \right) \right\} \]

Results: were in order of magnitude agreement!
First experiment

Ketterle performed growth experiments in late 1997.

- Evaporatively cool to just above BEC temperature.
- Suddenly remove tail of thermal distribution.
- Wait for equilibrium, watch condensate form.

Picture: Thermal cloud equilibrates before BEC grows.
Results

- Fitted data to simple growth equation.
- Shape: remarkably good. Numbers: terrible!
Extension of simple model: 1998

- More levels included in condensate band, all these levels treated dynamically.

- Bose distribution used for $W^+(N_0, T)$

- Some agreement, but experimental data not satisfactory.
Sophisticated model: 1999

- Next step: include dynamics of equilibration, i.e., solve full quantum Boltzmann equation.
- Found regimes where growth is faster than simple model predicts.
- However, this does not correspond to the regime of MIT experiment.
- Conclude: better expts required!
And then many years passed ...
Second experiment — Germans! : 2001

- Another series of experiments by Köhl, Esslinger and Hänsch, Max Planck Institute for Quantum Physics.

- This time all data was recorded, and all experimental quirks noted.

- Experiment similar to MIT:
  - Cloud cooled to near BEC temperature
  - However, this time a constant rf field was left on.
  - BEC imaged in time of flight.
  - Expts repeated many times $\Rightarrow$ statistical error bars.
Major issue: gravity

Gravity causes atom cloud to sag in magnetic field.

\[ V(\mathbf{r}) = \frac{m}{2} \left[ \omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 (z + A)^2 \right] \]

\[ A = \frac{g}{\omega_z^2} = 20.5 \mu m \]

Reduces the efficiency of evaporative cooling
Solution

- Monte Carlo simulations determine the proportion of trapped trajectories

- Also include three body loss, magnetic field drift effects.
Results I

(a)

(b)

(c)

(d)
Results II

(a) 

(b)

(c)

(d)
What is with the last figure???

Quasi-condensation: excess phase fluctuations, reduced density fluctuations for region $T_* < T < T_c$

- System trap is long and thin “cigar”
- Just below $T_c$ phase correlation length $<$ system length
- Picture: Bose stimulation with several independent condensates
- Eventually these coalesce to form a single BEC
Can theory cope?

Quasi-condensation occurs in hydrodynamic regime.

- Kinetic theory certainly not valid: $\tau_{\text{coll}} < \hbar/\epsilon$
- But: Finite temperature Gross-Pitaevskii equation should be OK.

\[
 i\hbar \frac{d\psi}{dt} = \left[ -\frac{\hbar^2}{2m} \nabla^2 + V_{\text{trap}}(x) \right] \psi + U_0 \hat{P} \left\{ |\psi|^2 \psi \right\} \\
+ U_0 \hat{P} \left\{ 2|\psi|^2 \langle \hat{\eta} \rangle + \psi^* \psi \langle \hat{\eta}^\dagger \rangle \right\} \\
+ U_0 \hat{P} \left\{ \psi^* \langle \hat{\eta} \hat{\eta} \rangle + 2\psi \langle \hat{\eta}^\dagger \hat{\eta} \rangle + \langle \hat{\eta}^\dagger \hat{\eta} \hat{\eta} \rangle \right\}
\]

$\psi$: condensate + quasiparticles, $\hat{\eta}$: thermal cloud field operator.

Plan to use this approach: but many difficulties.
Summary

- Introduced ideas behind kinetic theory.
- Outline of modifications for Bose condensation.
- Simplification to condensate growth.
- Growth experiments (MIT, MPQ).
- Theory appears to be successful?
- Quasi-condensation? — research for the future.