Introduction
- Continuous variable demonstration of quantum mechanical paradoxes with massive particles.
- Will correlations be the same for massive atoms and massless photons?
- We use field quadratures: directly analogous to position and momentum of original paradox.
- We extend quantum optical experiments into the atom-molecule domain: will interactions and increased noise destroy correlations?

BEC Dissociation
- Molecular BEC via Feshbach resonance or photoassociation of atomic BEC.
- Dissociation (with appropriate detunings) gives correlated atomic outputs.
- Previously shown to be strongly correlated in intensities [K. V. Kheruntsyan and P. D. Drummond, PRA 66, 031602 (2002)]

Quantum Optics Approach
- Undepleted molecular field in $k = 0$ mode.
- Only two atomic momentum modes populated, at $\pm \hbar k_0$.
- EXACT analytical solutions possible, but miss some physics.

Quantum Hamiltonian
\[
\hat{H} = H_{\text{kin}} + i \int dt \left\{ \frac{1}{2} \sum_{i,j} U_{ij} \hat{\psi}_i^\dagger \hat{\psi}_j \hat{\psi}_i^\dagger \hat{\psi}_j + \sum_i \omega_i \hat{\psi}_i^\dagger \hat{\psi}_i \right\},
\]
- $H_{\text{kin}}$ is kinetic energy.
- $\hat{\psi}_i(z,t)$ - field operators for bosonic atoms (1) and bosonic dimers (2).
- $V_i(x)$ is harmonic trap for molecules.
- $U_{ij}$ - $\pi$-wave scatterings.
- $\gamma$ - atom-molecule coupling ($2A = M$).
- Atoms not trapped.
- Similarities with well-known optical parametric amplifier Hamiltonian.

DIFFERENCES ARE IMPORTANT

EPR with Quadratures
- Two quadrature correlated (entangled) outputs with different momenta.
- We can choose noncommuting properties to measure.
- Field quadratures:
\[
\begin{align*}
\hat{X}_q &= a_q + \hat{a}_q^\dagger, \\
\hat{Y}_q &= -i \left( a_q - \hat{a}_q^\dagger \right).
\end{align*}
\]
- $[a_q, \hat{a}_q^\dagger] = \delta(q - q') \rightarrow V(\hat{X})V(\hat{Y}) \geq 1$.
- $V^{\text{inf}}(\hat{X}_q)|V^{\text{inf}}(\hat{Y}_q) < 1$.

[ M. D. Reid, PRA 40, 913 (1989)]

Mode matching
- Naive calculation as in single-mode case shows no EPR correlations.
- Different momentum modes are mixed.
- Solution: mode-matched local oscillators.
- Four mode-matched quadrature operators:
\[
\begin{align*}
\hat{X}_{\pm}(\tau) &= \int d\xi \left[ \phi^*_\pm(\xi,\tau) \hat{\psi}_1(\xi,\tau) + \hbar c \right], \\
\hat{Y}_{\pm}(\tau) &= -i \int d\xi \left[ \phi^*_\pm(\xi,\tau) \hat{\psi}_1(\xi,\tau) - \hbar c \right].
\end{align*}
\]
- $\phi_{\pm}(\xi) = |\phi_{\pm}(\xi)| \exp(\mp i q_0 \xi)$ are Gaussian local oscillator fields.

One Dimensional Analysis
- Positive-P representation equations:
\[
\begin{align*}
\frac{\partial \psi_1}{\partial \tau} &= i \frac{\partial^2 \psi_1}{\partial x^2} - (\gamma_1 + i \delta) \psi_1 + \kappa \psi_1 \psi_1^* + \sqrt{\kappa} \psi_1 \eta_1, \\
\frac{\partial \psi_1^*}{\partial \tau} &= -i \frac{\partial^2 \psi_1^*}{\partial x^2} - \kappa \psi_1^* \psi_1 + \sqrt{\kappa} \psi_1^* \eta_1.
\end{align*}
\]
- Plus equations for $\psi_2^*$, $\psi_2^\dagger$.
- Four independent variables: can model QM with c-number fields.
- Use stochastic integration; take trajectory averages.
- Stochastic averages give normally-ordered operator moments.

RESULTS
- Inset: dissociation coupling on.
- Clear demonstration of EPR paradox.

Summary
- Homodyne measurements of BEC could open new lines of investigation.
- Macroscopic test of quantum mechanics with massive particles.
- Important to use realistic condensates.

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