



EPR CORRELATIONS VIA BEC DISSOCIATION

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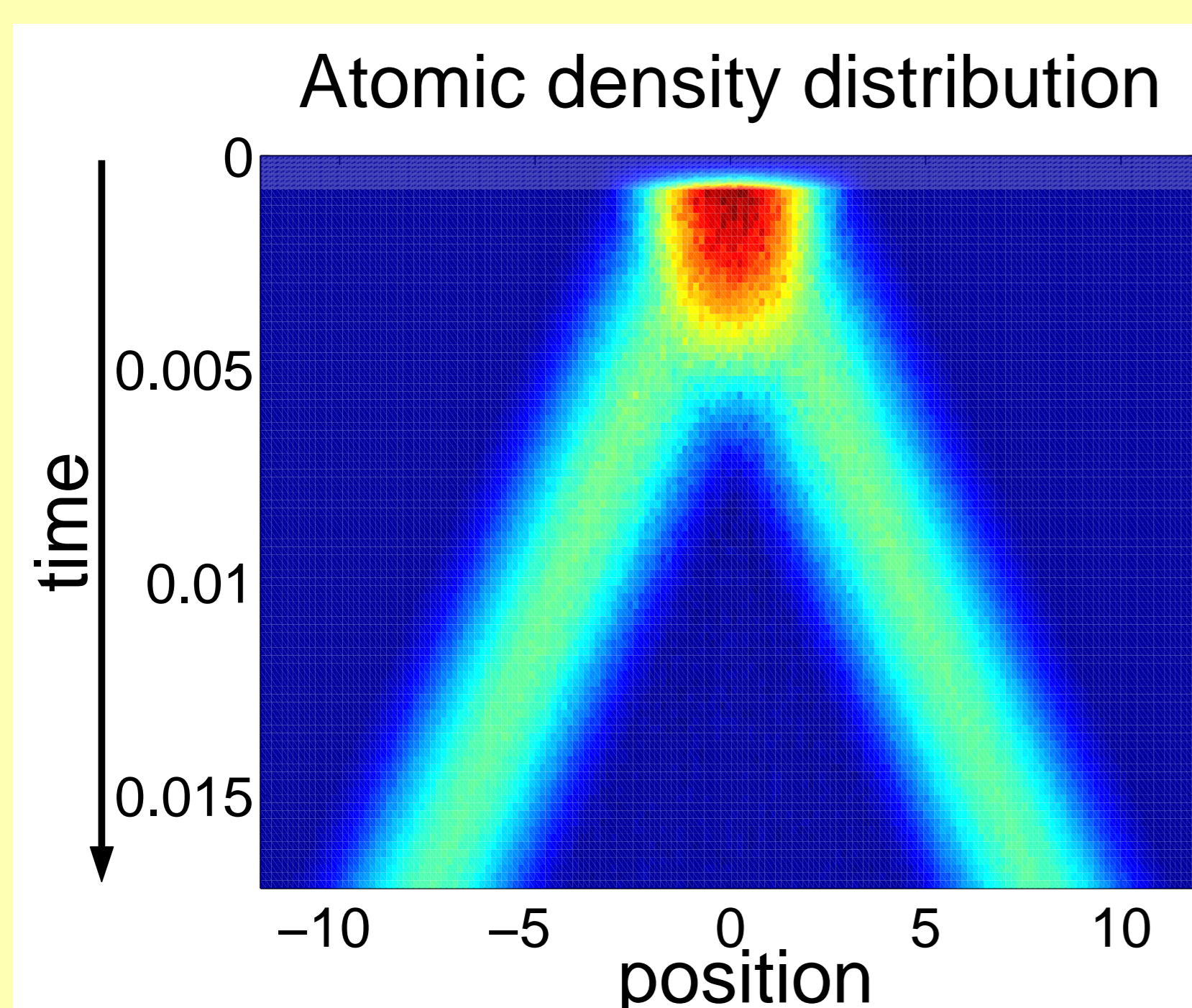


Introduction

- Continuous variable demonstration of quantum mechanical paradoxes with massive particles.
- Will correlations be the same for massive atoms and massless photons?
- We use field quadratures: directly analogous to position and momentum of original paradox.
- We extend quantum optical experiments into the atom-molecular domain: will interactions and increased noise destroy correlations?.

BEC Dissociation

- Molecular BEC via Feshbach resonance or photoassociation of atomic BEC.
 - Dissociation (with appropriate detunings) gives correlated atomic outputs
 - Previously shown to be strongly correlated in intensities
- [K. V. Kheruntsyan and P. D. Drummond, PRA 66, 031602 (2002)]



Quantum Hamiltonian

$$\hat{H} = \hat{H}_{kin} + \hbar \int dz \left\{ V_2(z) \hat{\Psi}_2^\dagger \hat{\Psi}_2 + \frac{1}{2} \sum_{i \geq j} U_{ij} \hat{\Psi}_i^\dagger \hat{\Psi}_j^\dagger \hat{\Psi}_j \hat{\Psi}_i - i \frac{\chi(t)}{2} \left[\hat{\Psi}_2^\dagger \hat{\Psi}_1^2 - \hat{\Psi}_1^\dagger \hat{\Psi}_2 \right] \right\}.$$

- \hat{H}_{kin} is kinetic energy
 - $\hat{\Psi}_{1,2}(z, t)$ – field operators for bosonic atoms (1) and bosonic dimers (2)
 - $V_2(z)$ is harmonic trap for molecules
 - U_{ij} – s-wave scatterings
 - χ – atom-molecule coupling ($2A \Rightarrow M$)
 - Atoms not trapped
 - Similarities with well-known optical parametric amplifier Hamiltonian
- **DIFFERENCES ARE IMPORTANT**

EPR with Quadratures

- Two quadrature correlated (entangled) outputs with different momenta
- We can choose noncommuting properties to measure
- Field quadratures:

$$\hat{X}_q = \hat{a}_q + \hat{a}_q^\dagger, \\ \hat{Y}_q = -i(\hat{a}_q - \hat{a}_q^\dagger).$$

- $[\hat{a}_q, \hat{a}_{q'}^\dagger] = \delta(q - q') \rightarrow V(\hat{X})V(\hat{Y}) \geq 1.$

$$V^{inf}(\hat{X}_{\pm q}) = V(\hat{X}_{\pm q}) - \frac{[V(\hat{X}_{+q}, \hat{X}_{-q})]^2}{V(\hat{X}_{\mp q})}.$$

- EPR signature: apparent violation of Heisenberg uncertainty principle

$$V^{inf}(\hat{X}_{\pm q})V^{inf}(\hat{Y}_{\pm q}) < 1$$

[M. D. Reid, PRA 40, 913 (1989)]

Quantum Optics Approach

- Undepleted molecular field in $k = 0$ mode
- Only two atomic momentum modes populated, at $\pm q_0$
- **EXACT** analytical solutions possible, but miss some physics

$$V^{inf}(\hat{X}_{+q_0})V^{inf}(\hat{Y}_{+q_0}) = 1/\cosh^2(2g\tau) \leq 1.$$

- **A TRAPPED CONDENSATE IS NOT A SINGLE MODE**
- Analysis is simple, but misleading

One Dimensional Analysis

- Positive-P representation equations

$$\frac{\partial \psi_1}{\partial \tau} = i \frac{\partial^2 \psi_1}{\partial \xi^2} - (\gamma_1 + i\delta)\psi_1 + \kappa \psi_2 \psi_1^\dagger + \sqrt{\kappa \psi_2} \eta_1, \\ \frac{\partial \psi_2}{\partial \tau} = \frac{i}{2} \frac{\partial^2 \psi_2}{\partial \xi^2} - i\nu \psi_2 - \frac{\kappa}{2} \psi_1^2 + \sqrt{-i\nu} \psi_2 \eta_2.$$

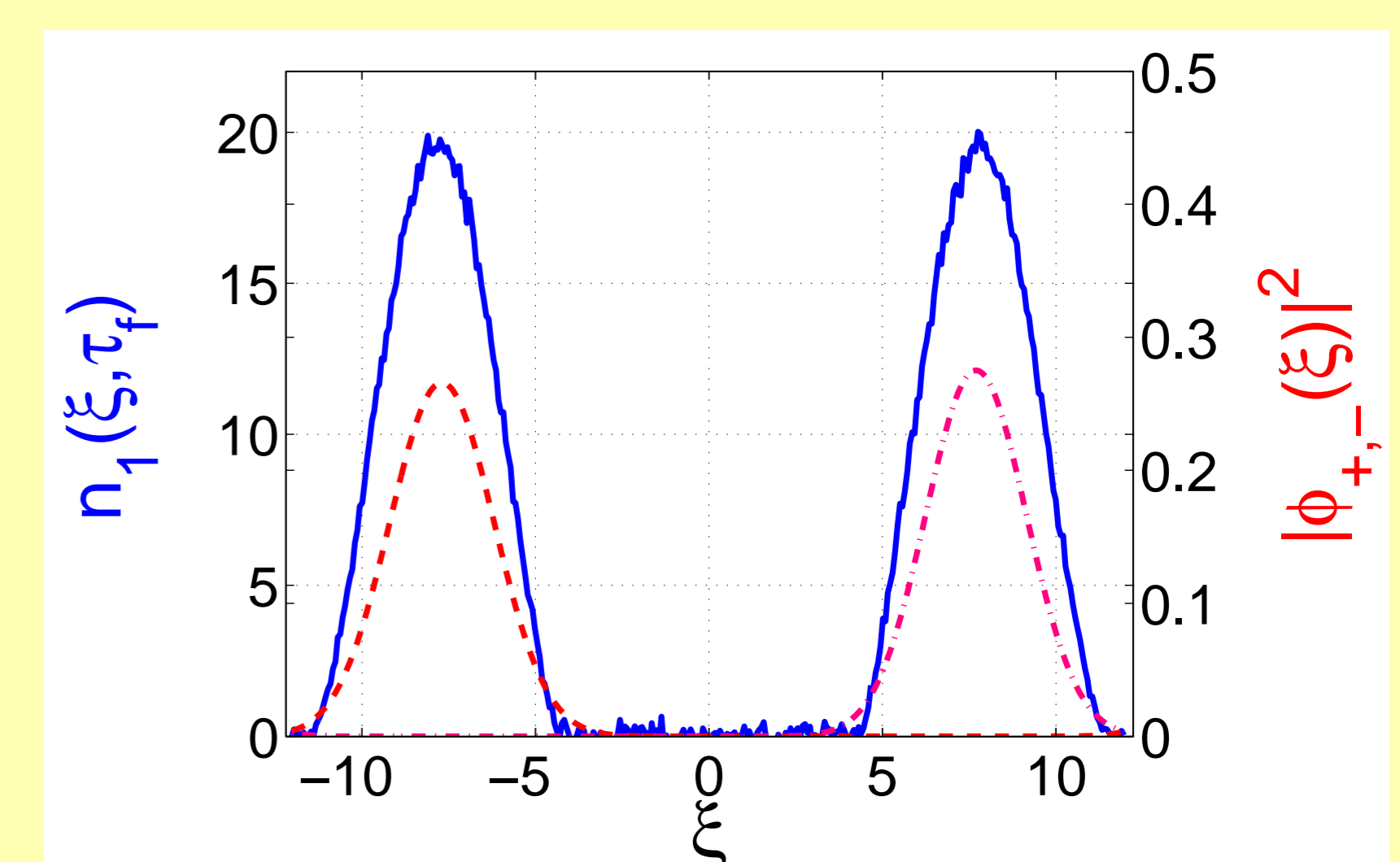
- Plus equations for $\psi_1^\dagger, \psi_2^\dagger$
- Four independent variables: can model QM with c-number fields
- Use stochastic integration; take trajectory averages
- Stochastic averages give normally-ordered operator moments

Mode matching

- Naive calculation as in single-mode case shows no EPR correlations
- Different momentum modes are mixed
- Solution: mode-matched local oscillators
- Four mode-matched quadrature operators:

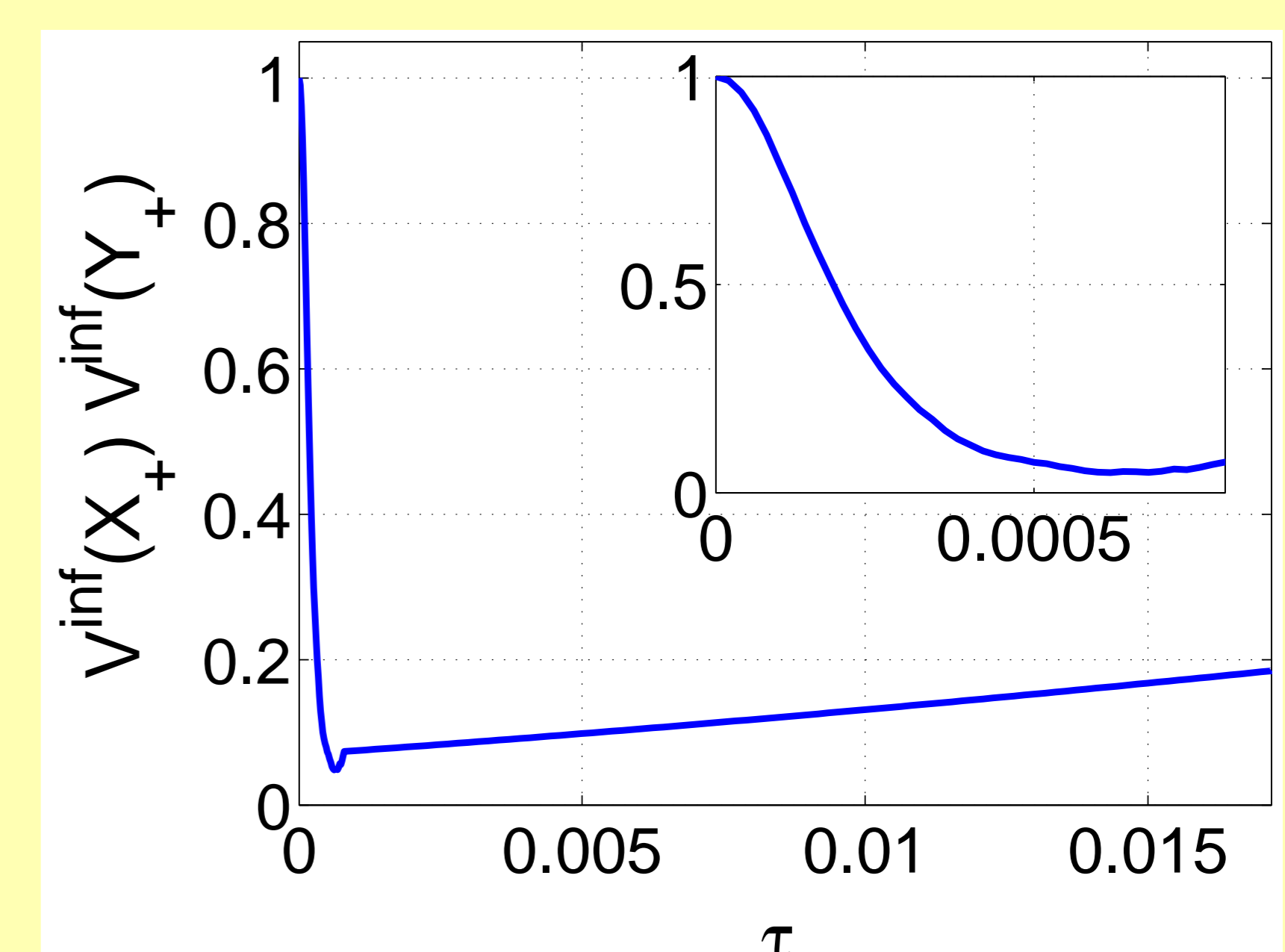
$$\hat{X}_\pm(\tau) = \int d\xi \left[\phi_\pm^*(\xi) \hat{\Psi}_1(\xi, \tau) + h.c. \right], \\ \hat{Y}_\pm(\tau) = -i \int d\xi \left[\phi_\pm^*(\xi) \hat{\Psi}_1(\xi, \tau) - h.c. \right].$$

- $\phi_\pm(\xi) = |\phi_\pm(\xi)| \exp(\mp i q_0 \xi)$ are Gaussian local oscillator fields



$$n_1(\xi, \tau_f) = \overline{\Psi_1^\dagger(\xi, \tau_f) \Psi_1(\xi, \tau_f)}$$

RESULTS



- Inset: dissociation coupling on
- Clear demonstration of EPR paradox

Summary

- Homodyne measurements of BEC could open new lines of investigation
- Macroscopic test of quantum mechanics with massive particles
- **Important to use realistic condensates**

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