How to mode-lock an atom laser

Peter D. Drummond and Karen V. Kheruntsyan
Department of Physics, University of Queensland
Brisbane, Qld 4072, Australia
Kerson Huang
Massachusetts Institute of Technology,
Cambridge, MA 02138

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Gross-Pitaevskii equation

We suggest making use of atom-atom repulsion, with a periodic output coupler, to form a stable dark soliton in a toroidal trap.

- This forms a modelocked, pulsed atom laser.
- **The atom laser output pulses are in all phase with each other.**

In the limit of a deep trap, we suppose that any mode-structure transverse to the trap circumference can be neglected. This gives a dimensionless G-P equation on the torus, where α is the nonlinearity:

$$i\frac{\partial\psi}{\partial\tau} = -\frac{\partial^2\psi}{\partial\theta^2} + \alpha|\psi|^2\psi\,. \tag{1}$$

The periodicity condition is:

$$\psi(\theta + 2\pi, \tau) = \psi(\theta, \tau)\,. \tag{2}$$

Uniform vortex

A uniform vortex state with angular momentum l has the condensate wave function

$$\psi_l(\theta, \tau) = e^{-i\nu_l \tau + il\theta} \tag{3}$$

where l must be an integer, in order to satisfy the boundary condition. The equation of motion gives

$$\nu_l = \alpha + l \,. \tag{4}$$

The momentum per particle of the uniform vortex state is

$$J = l \qquad (l = 1, 2, 3, \cdots) \,. \tag{5}$$

Dark Solitons

For a more general soliton solution, put

$$\psi(\theta, \tau) = f(\theta - v\tau)e^{i\eta(\theta,\tau)}\,. \tag{6}$$

The boundary condition demands

$$f(\bar{\theta} + 2\pi) = (-1)^{2l} f(\bar{\theta})\,, \tag{7}$$

$$\eta(\theta + 2\pi, \tau) = \eta(\theta, \tau) + 2\pi l\,, \tag{8}$$

where $\bar{\theta} \equiv \theta - v\tau$, $v = 2l$ and $l = 0, \pm\frac{1}{2}, \pm 1, \pm\frac{3}{2}, \pm 2, \cdots$. Assume the form

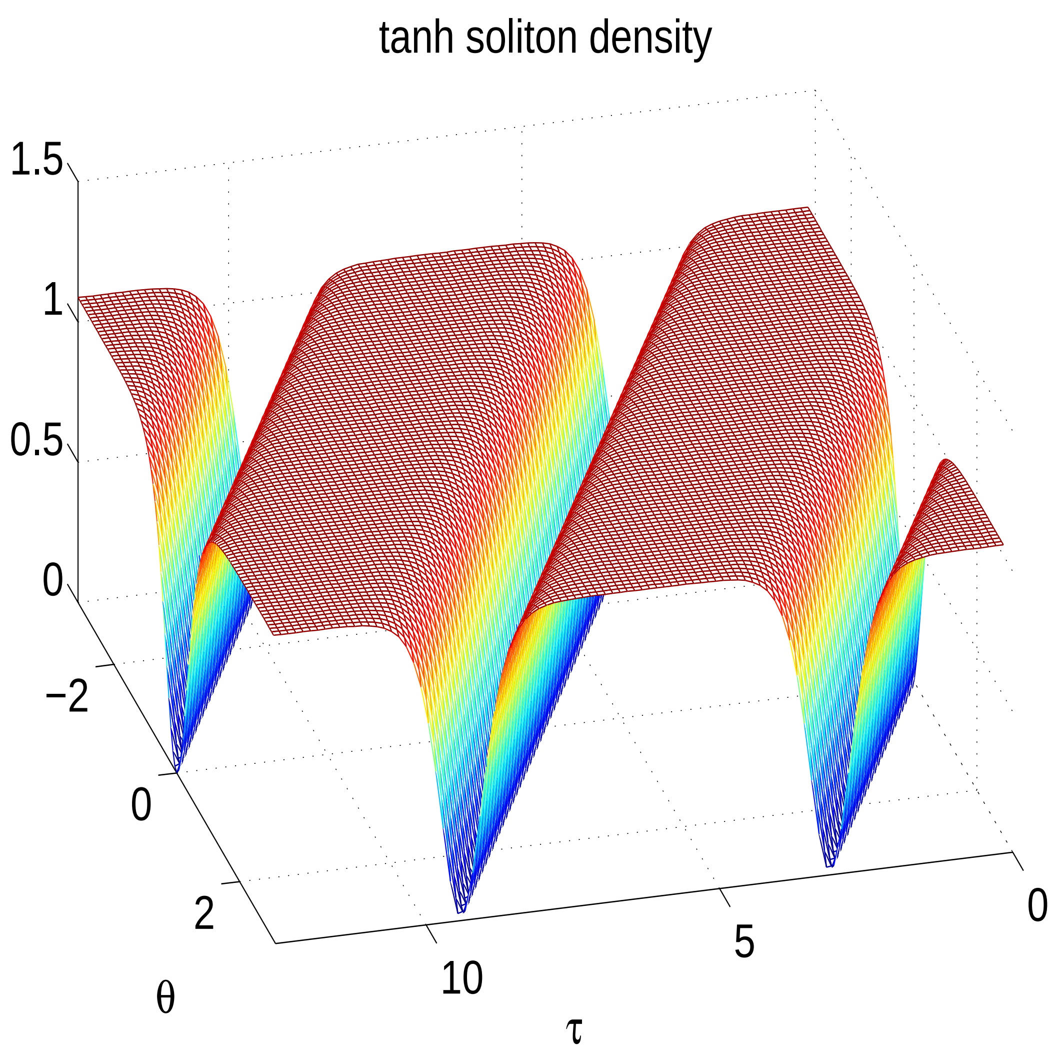
$$\eta(\theta, \tau) = l(\theta - v\tau) - \omega\tau\,, \tag{9}$$

where ω is a parameter corresponding to the energy. This satisfies the boundary condition.

Tanh with a twist

For large α, we find the solution **with** $l = 1/2$:

$$\psi(\bar{\theta}) \simeq \tanh\left(\sqrt{\frac{\alpha}{2}}\,(\bar{\theta} - \pi/s)\right) e^{i(\bar{\theta}/2 - \omega\tau)} . \qquad (10)$$



Mode-locked atom laser

We now include a stirring potential, gain and loss terms into the G-P equation:

$$i\frac{\partial\psi}{\partial\tau} = -\frac{\partial^2\psi}{\partial\theta^2} + \alpha|\psi|^2\psi + V\psi + i(g-\gamma)\psi\,, \qquad (11)$$

where V is the 'stirring' potential.

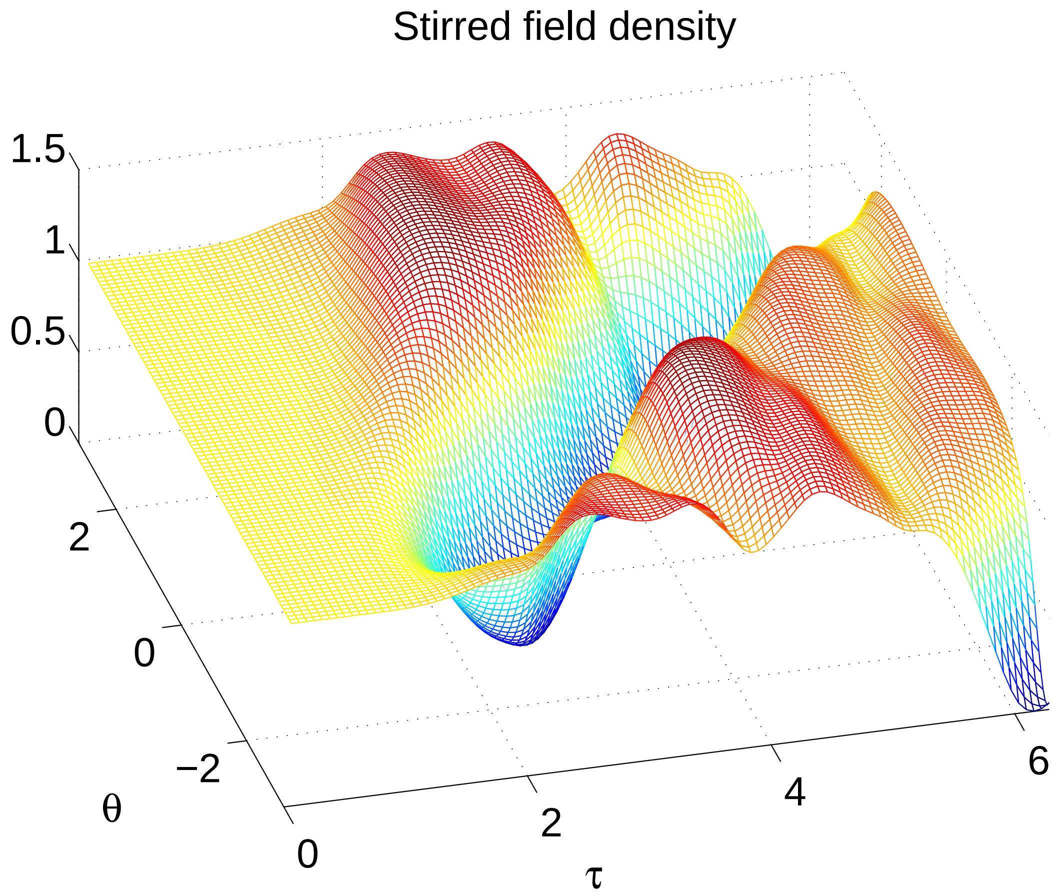
- **The gain mechanism g can be provided via stimulated emission from non-condensed atoms.**

- **The loss mechanism $\gamma(\tau,\theta)$ is due to an output coupler: a local Raman-tuned transition to a non-trapped state.**

Stirred, not shaken

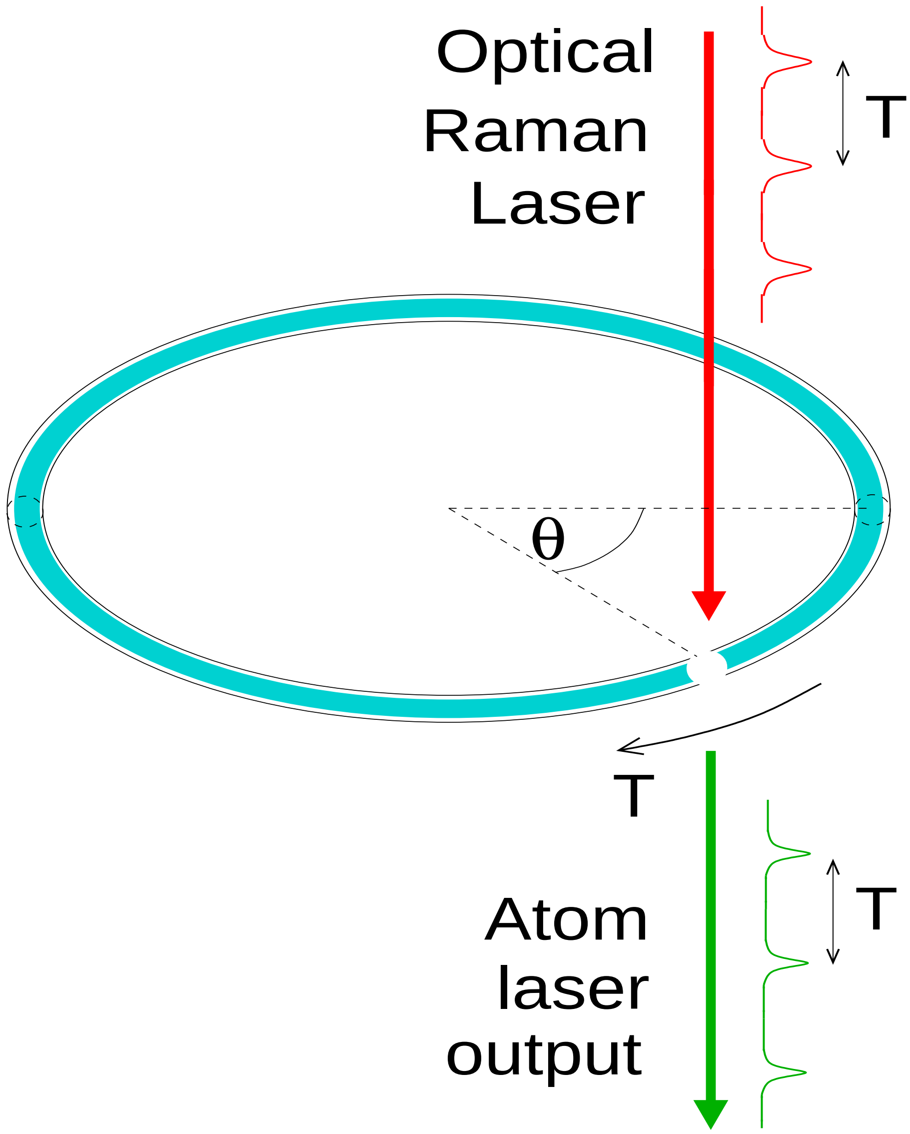
- We start by using a blue-detuned ‘stirring’ laser to form a moving potential hill.

- This repels the condensate from a localized region, which forms the phase singularity, and hence a soliton.



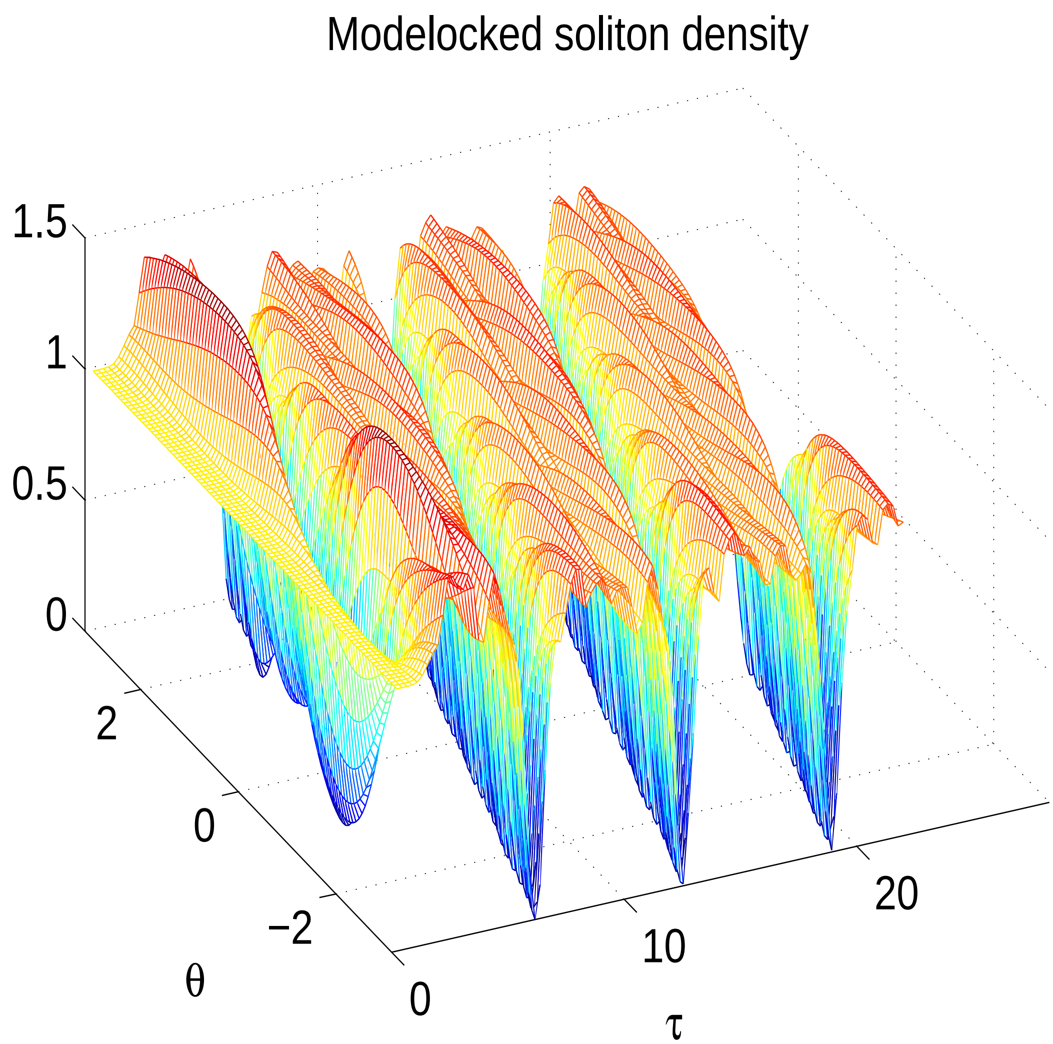
Dark Soliton Modelocking

Next, a periodic sequence of localized Raman laser pulses can be used to 'clean-up' or stabilize the dark soliton, while also producing an output pulse-train.



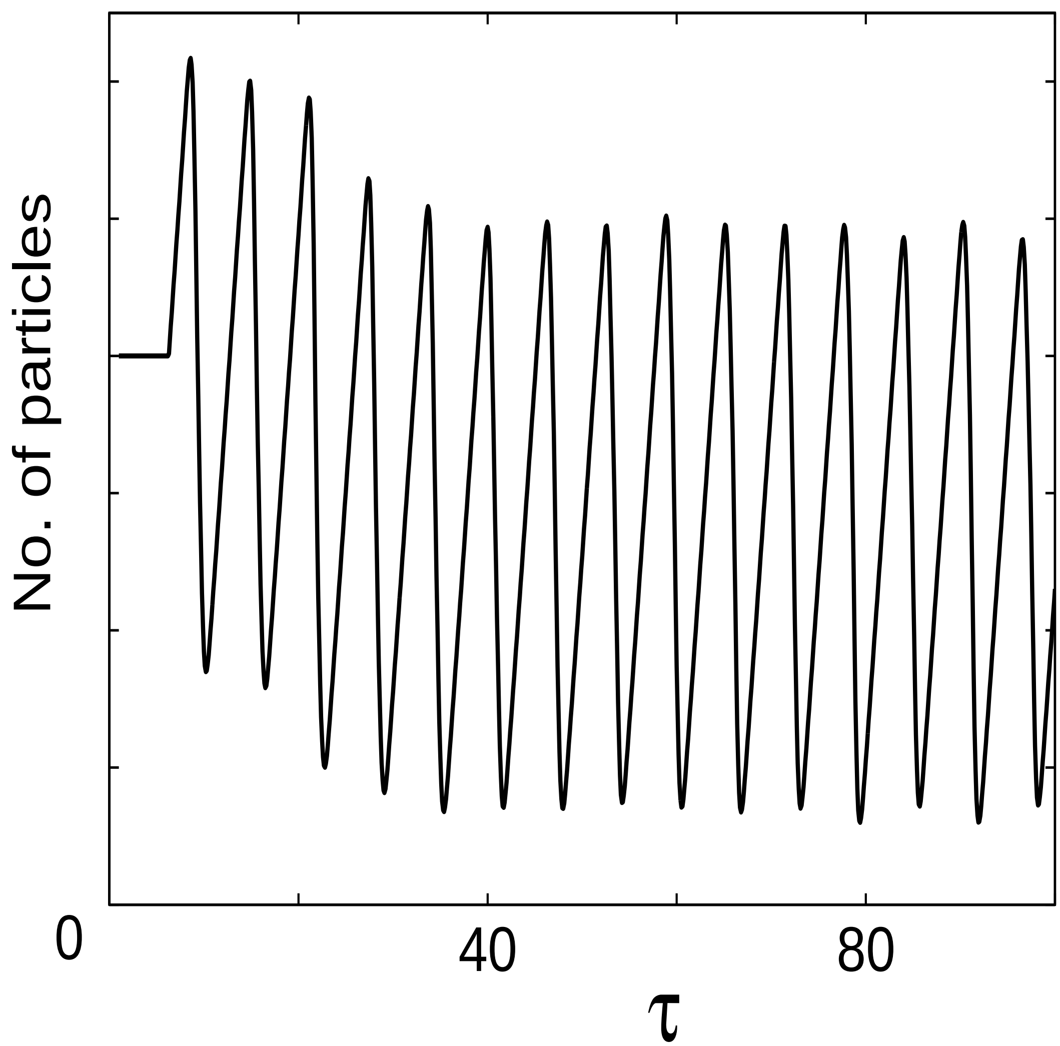
Evolution of the condensate

- The condensate is cleaned up, as phonons are removed by the mode-locker, to give a mode-locked output.



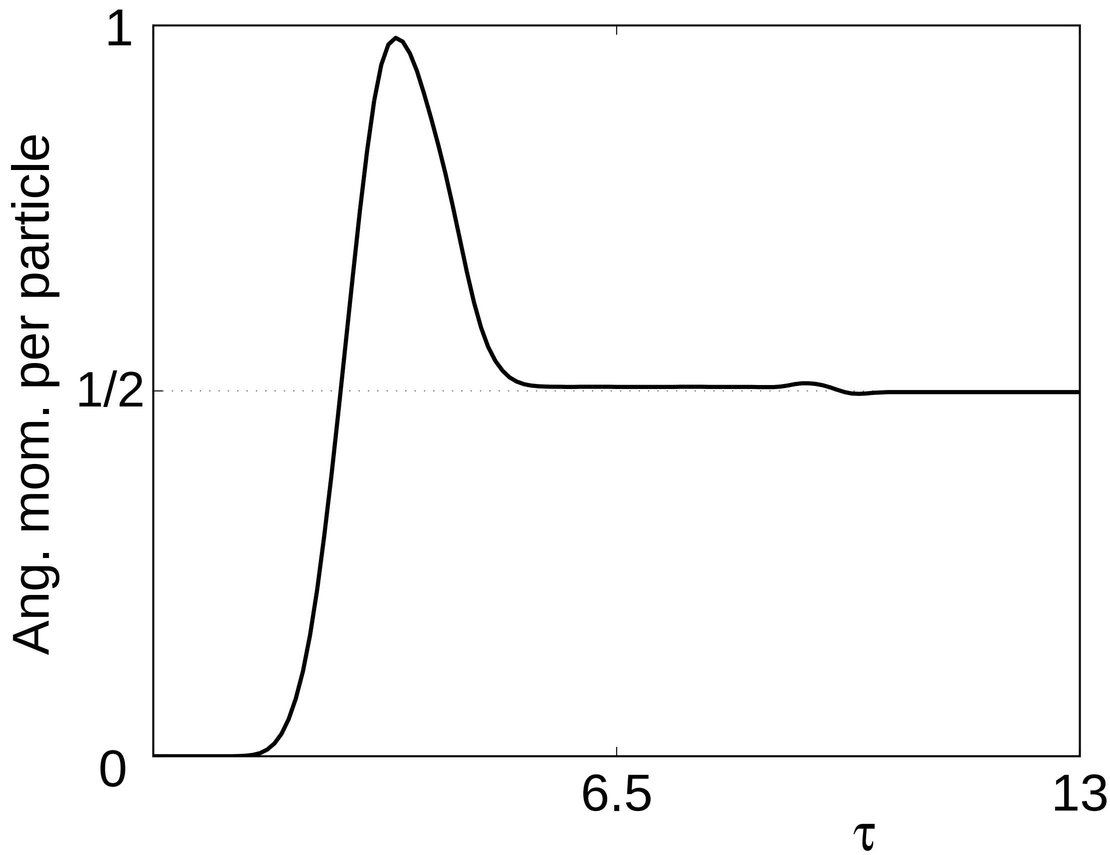
Stabilization

- The BEC is stabilized by the fact that the output-coupling efficiency is a nonlinear function of the hole diameter, and hence of the atom number.



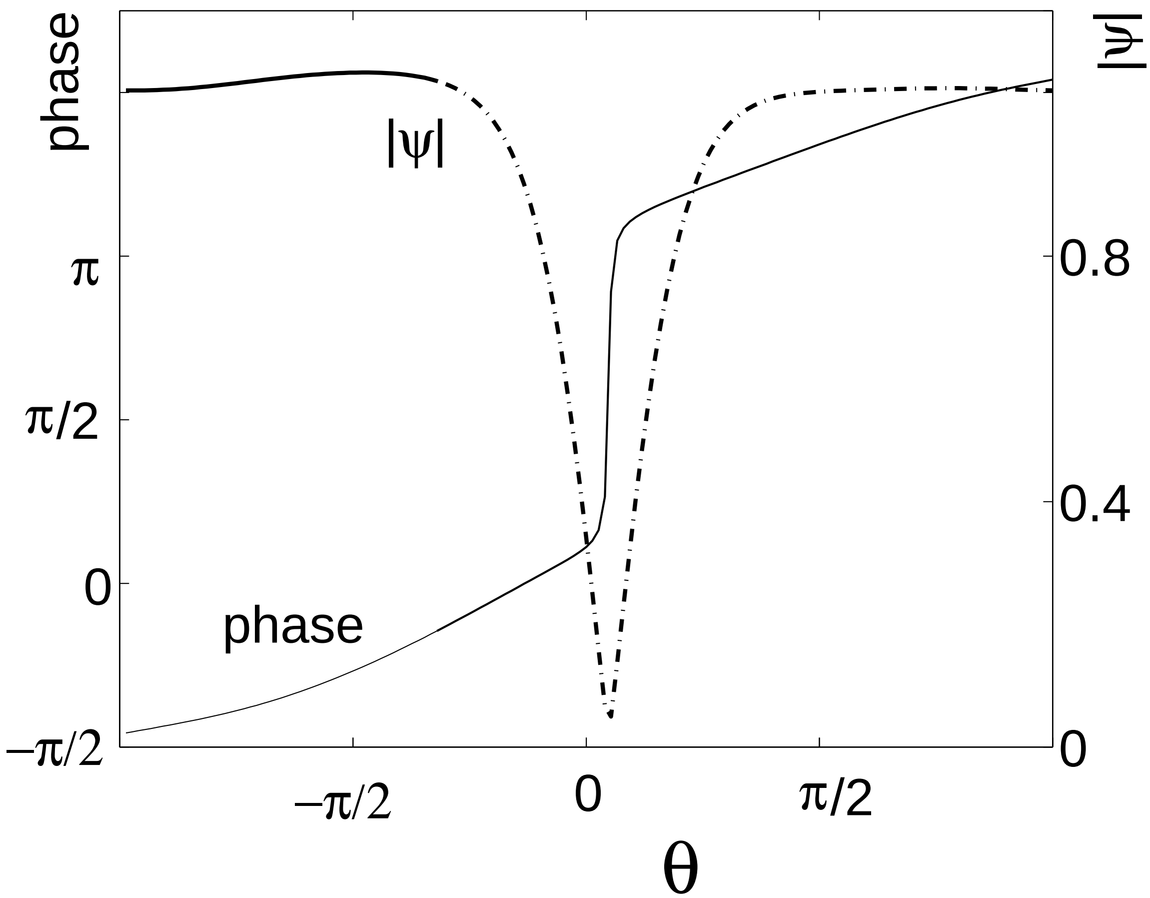
Angular momentum per particle

- The angular momentum per particle stabilizes to $\hbar/2$!



Stable mode-locked dark soliton

- The final result has an intensity 'hole', and stable phase from pulse to pulse.



Conclusion

- We analyze stable, circulating dark solitons, in a toroidal atom laser .
- Lasing is achieved through super-saturated gain
- A periodic output coupler based on Raman transitions is used.
- The result is a dark soliton with angular momentum of $\hbar/2$ per atom.
- We have given a numerical treatment of soliton generation.
- **A modelocked atom laser is obtained.**