Applying the classical field method to experimental Bose condensed systems

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Overview

- Finite temperature Bose gases.
- Introduction to classical fields.
- Measuring condensate fractions.
- Shift in $T_c$ for interacting Bose gases.
- Summary.
The challenge for theorists

Can we come up with a *practical* non-equilibrium formalism for finite temperature Bose gases?

Desirable features:

- Can deal with inhomogeneous potentials.
- Can treat interactions non-perturbatively.
- Calculations can be performed on a reasonable time scale (say under one week).
Potential applications

Topics of interest include:

- Condensate formation.
- Vortex lattice formation, dynamics
- Low dimensional systems (fluctuations important)
- Correlation functions
- Atom lasers...
Classical field approximation

An example: the classical theory of electromagnetic radiation resulted in the Rayleigh-Jeans law.

Based on the equipartition theorem:

- Each oscillator mode has energy $k_B T$ in equilibrium.
The UV catastrophe

But we all know it doesn’t work . . .

So Planck says:
“Classical fields are no good”
However . . .

For the infra-red modes the RJ law is a good approximation.

Quantum and classical results are similar for

\[ E_{\text{photon}} \leq k_B T \]

Thus we require

- High occupancy per mode.
- An energy cutoff.
Classical fields for matter waves

The Projected Gross-Pitaevskii equation:

\[
i \frac{d\psi(x)}{dt} = H_{sp} \psi(x) + C_{n1} \mathcal{P} \{ |\psi(x)|^2 \psi(x) \}, \quad C_{n1} = \frac{8\pi a N}{L}.
\]
Classical fields for matter waves

The Projected Gross-Pitaevskii equation:

\[ i \frac{d\psi(x)}{d\tau} = H_{sp} \psi(x) + C_{n1} \mathcal{P} \left\{ |\psi(x)|^2 \psi(x) \right\}, \quad C_{n1} = \frac{8\pi a N}{L}. \]

All modes assumed to be highly occupied.
Projection prevents higher energy modes becoming occupied:

\[ \mathcal{P}\{F(x)\} = \sum_{k \in C} \phi_k(x) \int d^3x' \phi_k^*(x') F(x') \] — prevents UV catastrophe.
Classical fields for matter waves

The Projected Gross-Pitaevskii equation:

\[ i \frac{d\psi(x)}{dT} = H_{sp} \psi(x) + C_{nl} \mathcal{P} \{ |\psi(x)|^2 \psi(x) \}, \quad C_{nl} = \frac{8\pi a N}{L}. \]

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Advantages: 1. Relatively easy (i.e possible!) to simulate in 3D.
2. Method is non-perturbative.

However: Atoms above cutoff necessary for real calculations.
Ergodicity

Begin simulations with random initial conditions

\[ \Rightarrow \text{Result is thermal equilibrium} \]

System is \textbf{ergodic}: time average $\equiv$ ensemble average

Time-averaged column densities in momentum space, TOP trap.
Time-averaged column densities

Momentum space

Real space

(a) $f_c = 0.00$

(b) $f_c = 0.24$

(c) $f_c = 0.87$
Theorists’ criterion for BEC: Penrose-Onsager

⇒ Single-particle density matrix has a macroscopic eigenvalue.

Given \( \psi(x, t) = \sum_k c_k(t) \phi(x) \) we can calculate

\[
\rho_{ij} = \langle c_i^* c_j \rangle \approx \lim_{T \to \infty} \frac{1}{T} \int_0^T c_i^*(t) c_j(t) \, dt
\]

Typically have \( \sim 2000 \) states below cutoff

This can easily be diagonalized on a workstation

[Also have a microcanonical measure of temperature]
Experimentalists’ measure of BEC

⇒ Fit a bimodal distribution to column density.

Compare the two measures from an evaporative cooling calculation.
Shift in critical temperature with interactions

A difficult problem: perturbation theory doesn’t work near $T_c$.

Several competing phenomena:

- Finite size effects (downwards)
- Mean field effects (downwards)
- Critical fluctuations (upwards)

Homogeneous gas, thermodynamic limit: $\Delta T_c/T_{c0} = can^{1/3}$.

We find $c = 1.3 \pm 0.4$ — agrees with Monte Carlo calculations.

P. Arnold and G. Moore, PRL 87, 120401 (2001);
Critical temperature for trapped gas

Giorgini et al. estimate downwards shift in $T_c$ due to mean field.

$$\frac{T_c}{T_{c0}} \approx 1.33 N^{1/6} \frac{a}{a_{ho}}.$$ 

Are critical fluctuations important?

We compare PGPE calculations for a TOP trap to mean-field HFB-Popov calculations for the same basis set.

Answer: maybe!
Comparison with experiment


Analytic result is an estimate of mean field shift. We will calculate this numerically.
Other current topics

- Formation of vortices at the phase transition:
  - Kibble / Zurek scenario for BECs?
  - Only phenomenological time-dependent Landau-Ginzburg theory to date.

- Vortices in 2D
  - Pairing / Kosterlitz-Thouless transition?

- Trapped Bose gases with angular momentum.
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- Brief mention of the road ahead.
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That’s all, folks!