ATOMIC FERMIONS IN 1D OPTICAL LATTICE
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**Motivation**
- The Mott metal-insulator transition (MMIT) is a fundamental concept in strongly correlated many-body systems. The MMIT with bosonic atoms has been demonstrated.
- How can we detect the emergence of fermionic Mott-insulator phases in real experiments?

**Phase Diagram**
- 1D Hubbard model:
  \[ \mathcal{H}_0/t_b = \sum_i n_i a_i a_i + \sum_i (e_i c_{i+1} a_i + h.c.) \]
  \[ \mathcal{H} = \mathcal{H}_0 + \sum_i \frac{m \omega_0^2}{2} p_i^2 a_i. \]
- Method: exact solution.
- The different phases:
  A: pure metallic phase; B: Mott insulator domain in center; C: metallic phase at center surrounded with Mott insulator plateaus; D: band insulator at center; E: band insulator at center surrounded by metallic regions, in turn surrounded by Mott insulators.
- Characteristic parameters
  - Interaction strength: \( \kappa = w^2 / (16N\omega) \)
  - Filling factor: \( \nu = 2\sqrt{N\omega}/\pi \)
  - Central density for \( \kappa = 0.01 \) (a), 1 (b), and 100 (c):

**Collective Oscillations**
- Luttinger liquid model:
  \[ \mathcal{H}_{LL} = \sum_i \int dx \left[ \frac{u_1 K_v}{2} \left( \frac{\partial \phi_i}{\partial x} \right)^2 \right]. \]
  - \( \partial \phi_i(x) \): density or spin density fluctuations;
  - \( \Pi_i(x) \): density or spin density currents;
  - \( u_1 \): density or spin velocities;
  - \( K_v \): Luttinger exponents.
- Hydrodynamic equation:
  \[ \frac{\partial^2 \delta n_i}{\partial x^2} = \beta \left[ \frac{1}{\nu} \frac{\partial}{\partial x} \left( \frac{u_1(x) K_v(x) \partial}{\partial x} \right) \right]. \]
- Focus on the density modes
- Boundary conditions: current vanishes
- Breathing mode:

**Experimental proposal**
- Optical lattice

**Summary**
- Present a phase diagram in 1D optical lattice.
- The collective modes provides a signature of the Mott metal-insulator transition.
- A detailed experimental implementation is proposed.
- How about the damping rates of collective modes?

**References**