Theory of decoherence in Bose-Einstein condensate interferometry

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A full treatment of decoherence, dephasing and trap fluctuation effects for double-well BEC interferometry using condensates with large boson numbers N has been developed [1]. This extends a simple theory of double-well BEC interferometry [2] based on a two mode approximation, which allows for possible fragmentations of the condensate into two modes [3] (these may be localized in each well), but is restricted to small condensates and only allows for transitions within the condensate modes and certain dephasing processes. The bosonic field operator is the sum of condensate and non-condensate mode contributions, the Hamiltonian being expanded in decreasing powers of $\sqrt{N}$, correct to the Bogoliubov approximation [4]. The density operator is mapped onto a phase space distribution functional, with the highly occupied condensate modes described via a generalized Wigner representation and the mainly unoccupied non-condensate modes described via a positive P representation. A similar hybrid approach has been applied to treat two coupled anharmonic oscillators [5]. An interferometry regime with macroscopic occupancy in only one condensate mode is assumed - the conditions to be found using the two-mode theory [2]. A functional Fokker-Planck equation (FFPE) for the distribution functional based on the truncated Wigner approximation is obtained, from which coupled Ito stochastic equations for condensate and non-condensate field functions are found. These equations contain deterministic and random noise terms - identifiable from the FFPE. Stochastic averages of the field functions give the quantum correlation functions that are used to describe interferometry experiments, and which exhibit decoherence and dephasing effects.

The Ito stochastic equations for the condensate field $\psi_C(\mathbf{s},t)$ are

$$\begin{aligned}
i\hbar\frac{\partial}{\partial t}\psi_C(\mathbf{s},t) &= -\frac{\hbar^2}{2m}\nabla^2\psi_C + V\psi_C + \frac{g_N}{N}\{\psi_C^+\psi_C - |\phi_1|^2\}\psi_C \\
&+\frac{2g_N}{N}\{\psi_C^+\psi_C - \frac{1}{2}N\,|\phi_1|^2\}\psi_{NC} + \frac{g_N}{N}\{\psi_C\psi_C\}\psi_{NC}^+ \\
&+\frac{2g_N}{N}\{\psi_{NC}^+\psi_{NC}\}\psi_C + \frac{g_N}{N}\{\psi_{NC}\psi_{NC}\}\psi_C^+ \\
&+d_{C,C}\Gamma_C + d_{C,C+}\Gamma_{C+} + d_{C,NC}\Gamma_{NC} + d_{C,NC+}\Gamma_{NC+},
\end{aligned}$$

the terms being displayed in decreasing powers of $\sqrt{N}$. In these equations V is the trap potential, the boson-boson interaction strength is g_N/N, the condensate wave function is $\phi_1(\mathbf{s},t)$ and satisfies a time-independent Gross-Pitaevskii equation. The $\Gamma_C,..,\Gamma_{NC+}$ are stochastic noise fields, and the matrix elements $d_{C,C},..,d_{C,NC+}$ are related to the diffusion matrix in the functional Fokker-Planck equation. A similar equation applies for the non-condensate field $\psi_{NC}(\mathbf{s},t)$. The first line of the condensate equation is the time-dependent Gross-Pitaevskii equation, the mean field being depleted by one boson. The stochastic condensate and non-condensate fields are coupled together, each being affected by stochastic noise fields.

References

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