Grassmann Phase Space Theory in Quantum-Atom Optics

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In particle and condensed matter physics the use of anti-commuting Grassmann variables in treating fermion systems via path integral methods is a well-established approach [1]. Quantum-atom optics deals with systems such as atoms, quantised electromagnetic fields, and ultra-cold atomic gases - both fermionic and bosonic. Phase space methods (where the quantum density operator is represented by a quasi-distribution function of variables that replace the annihilation and creation operators), constitute one of the major approaches [2]. However, in spite of the seminal work by Cahill and Glauber [3] and a few applications [4, 5, 6], the use of phase space methods in quantum-atom optics to treat fermionic systems by representing (anti-commuting) fermionic annihilation and creation operators by Grassmann variables [3] is rather rare. This is particularly the case for multi-atom bosonic and fermionic systems, where many quantum modes are often involved. Generalisations of phase space distribution functions of phase space variables for a few modes to phase space distribution functionals of field functions (which represent the field operators, c-number fields for bosons, Grassmann fields for fermions) are now being developed for large systems [7].

To illustrate the applicability of the Grassmann variable approach to quantum-atom optics, it is shown that one of the most fundamental models in quantum optics and quantum physics can be treated via a Grassmann distribution function approach. The Jaynes-Cummings model of a two-level atom (TLA) in a single mode cavity involves the interaction of two simple quantum systems - one fermionic (the TLA), the other bosonic (the cavity mode). Phase space methods using a distribution function involving c-number variables (for the cavity mode) and Grassmann variables (for the two level atom) have been used to treat this model [8]. The Grassmann distribution function is equivalent to six c-number functions of the bosonic variables. Bosonic phase space integrals involving these functions determine the experimental quantities. A Fokker-Planck equation involving both left and right Grassmann differentiation has been obtained for the Grassmann distribution function. Equivalent coupled equations for the six c-number functions have been found. This feature that the final equations only involve c-numbers will also apply to more complex fermion systems.

References

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