Multi-partite qudit entanglement, steering and Bell's nonlocality

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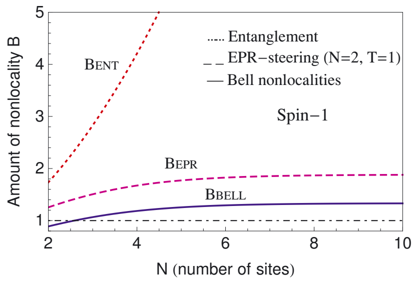
Three forms of quantum nonlocality are entanglement, steering or the Einstein-Podolsky-Rosen paradox, and failure of local hidden variable theories, which we refer to as Bell's nonlocality [1]. We have examined these nonlocalities in situations of largeness, of many sites and many particles (higher dimension). Surprisingly, nonlocality is predicted to persist in all cases, even for mesoscopic quantum systems. Earlier work includes Bell's nonlocality for multisite qubits [2], for bipartite qudits [3], and to a lesser extent on multi-site qudits [4].

We generalise the inequalities of [2] so that they apply to all three nonlocalities, but with a different threshold for each [5]. Our approach can be applied to higher spins. We follow Mermin [2] and Cavalcanti et al [6,7] and construct non-hermitian operators $F_k^{\pm} = (X_k \pm iP_k)/2$ from noncommuting observables X_k and P_k defined for spatially separated sites $k = 1, ..., N$. For any separable or local hidden variable (LHV) model, the inequality $|\langle\prod_{k=1}^{N} F_k^{\pm}\rangle|^2 \leq \int_\lambda d\lambda P(\lambda)\prod_{k=1}^{N}|\langle F_k^{\pm}\rangle_\lambda|^2$ holds. By restricting to local quantum states we can also derive criteria for entanglement. This enables us to obtain a hierarchy of constraints for entanglement, steering and Bell's nonlocality on $|\langle F_k^{\pm}\rangle_\lambda|^2$. We have also shown that entanglement itself is a physical quantity that satisfies conservation laws under certain conditions [8].



Entanglement ($T = N$), steering ($T = 1$) and Bell's nonlocality are predicted when the ratio B of left to right sides of the appropriate inequality (1) is greater than one for N spin-1 systems. Steering relates to the hybrid case introduced by Wiseman et al [1,9], in which some sites are quantum and others not. Such nonlocality is applicable in principle to a spinor BEC with $F = 1$ atoms trapped in an optical lattice.

We derive spin criteria to show entanglement, steering or failure of local hidden variables where $T = N, 1, 0$ respectively [10].

$$|\langle\prod_{k=1}^{N} J_{\pm}^{k}\rangle|^2 > \langle\prod_{i=1}^{T}[(J^i)^2 - (J_z^i)^2 \pm J_Z^A]\prod_{j=T+1}^{N}[(J^j)^2 - (J_z^j)^2]....\rangle \quad (1)$$

Here the $J_{\pm}^{k}$ are the spin raising and lowering operators for the kth site. The spin j case is applicable to a BEC with many atoms per site. This is also studied for nonmaximally entangled states and shown to allow violation of the Bell and steering nonlocalities for high dimension and large number of sites. Lastly we show how the extent of the violations of the nonlocality inequalities gives information about the number of particles sharing the nonlocality [3].

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