Relative Phase States

B. J. Dalton
ACQAO, Swinburne University of Technology, Melbourne

Studies of phase dependent phenomena in both Bose-Einstein condensates and quantum optics are hindered because phase has at least three different meanings [1]. The introduction of phase as eigenvalues of a linear Hermitian phase operator is the most objective approach [1], and such an operator can be defined for BEC following the method of Pegg and Barnett [2] for EM fields.

For the case of a two mode BEC with mode annihilation operators $\widehat{a}$, $\widehat{b}$ and spatial mode functions $\phi_a(\mathbf{r})$, $\phi_b(\mathbf{r})$ basis states $|n_a\rangle$, $|n_b\rangle$ involving n_a, n_b bosons in the modes can be used to define relative phase eigenstates $|\theta_p\rangle$ for the $N = n_a + n_b$ boson system, where $\theta_p = p(2\pi/(N+1))$, $p = -N/2, -N/2+1, .., +N/2$ is a quasi-continuum of $N+1$ equispaced phase eigenvalues, and from which the Hermitian relative phase operator $\widehat{\Theta}$ is then defined. We have

$$|\theta_p\rangle = \frac{1}{\sqrt{N+1}} \sum_{k=-N/2}^{N/2} \exp(ik\theta_p) \, |N/2-k\rangle_a \, |N/2+k\rangle_b \qquad \widehat{\Theta} = \sum_p \theta_p \, |\theta_p\rangle \, \langle\theta_p| \tag{1}$$

The relative phase eigenstate has several interesting properties. Firstly, it is a state with maximal mode *entanglement* [3] for the a, b sub-systems, so is of interest in quantum information Secondly, it is a *fragmented* state [4], since there are two natural orbitals with macroscopic occupancy. For large N the natural orbitals obtained from the first order quantum correlation function are $\chi_{\pm}(\mathbf{r}) = (\exp(i\theta_p/2)\phi_a^*(\mathbf{r}) \pm \exp(-i\theta_p/2)\phi_b^*(\mathbf{r}))/\sqrt{2}$, with occupancies $(\frac{1}{2} \pm \frac{\pi}{8})N$. For fragmented states generalized mean field theories [5] are required . Thirdly, the relative phase eigenstate is a *spin squeezed* state. Spin operators along $(\widehat{J}_z)$ and perpendicular $(\widehat{J}_x, \widehat{J}_y)$ to the Bloch vector may be defined by $\widehat{J}_x = \widehat{S}_z$, $\widehat{J}_y = \widehat{S}_x \sin\theta_p + \widehat{S}_y \cos\theta_p$, $\widehat{J}_z = \widehat{S}_x \cos\theta_p - \widehat{S}_y \sin\theta_p$, where $\widehat{S}_x = (\widehat{b}^\dagger\widehat{a} + \widehat{a}^\dagger\widehat{b})/2$, $\widehat{S}_y = (\widehat{b}^\dagger\widehat{a} - \widehat{a}^\dagger\widehat{b})/2i$, $\widehat{S}_z = (\widehat{b}^\dagger\widehat{b} - \widehat{a}^\dagger\widehat{a})/2$ are the usual Schwinger operators. For large N the Bloch vector is $\left\langle \widehat{J}_x \right\rangle = 0$, $\left\langle \widehat{J}_y \right\rangle = 0$, $\left\langle \widehat{J}_z \right\rangle = \frac{\pi}{8}N \approx 0.392N$, which is in the equatorial plane with azimuthal angle $\phi = 2\pi - \theta_p$, and inside the Bloch sphere of radius $N/2$ - another indicator of fragmentation. For large N the fluctuations $(\delta\widehat{\Omega}^2 \equiv \left\langle (\widehat{\Omega} - \left\langle \widehat{\Omega} \right\rangle)^2 \right\rangle)$ in the Bloch vector components are found to be $\delta\widehat{J}_x \approx \sqrt{1/12}N \approx 0.289N$, $\delta\widehat{J}_y \approx 1.30$, $\delta\widehat{J}_z \approx \sqrt{(1/6 - \pi^2/64)}N \approx 0.112N$. As $|\left\langle \widehat{J}_z \right\rangle|/2 \approx 0.196N$ we see that $\delta\widehat{J}_x \cdot \delta\widehat{J}_y \approx 0.375N$ is greater than $|\left\langle \widehat{J}_z \right\rangle|/2$, consistent with the Heisenberg uncertainty principle. However, although $\widehat{J}_x$ is unsqueezed, the other perpendicular component $\widehat{J}_y$ is highly squeezed, with a fractional fluctuation $\delta\widehat{J}_y / \left\langle \widehat{J}_z \right\rangle$ of order $1/N$. The relative phase state could be of interest in Heisenberg limited interferometry [6].

Finally, even though no proposal yet exists for preparing a BEC in a relative phase eigenstate, relative phase eigenstates are a valuable theoretical concept for describing behaviour in BEC interferometry experiments, such as the Dunningham and Burnett [7] proposal for Heisenberg limited interferometry in two mode BEC.

References

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