4 Lectures on

Quantum Optics
with photons and continuous laser beams

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More details can be found in:
A guide to experiments in quantum optics
H-A.Bachor & T.C.Ralph, VCH-Wiley 2004

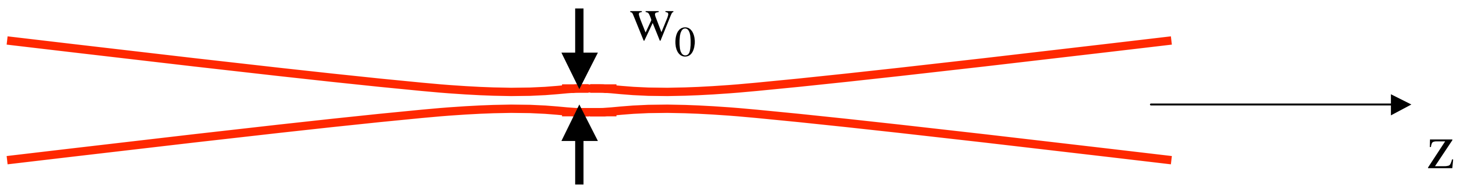
Lecture 1

- Overview of the concepts and ideas

- Classical model for
  laser beams & applications

- Define quantum optics

A mode of light



Intensity I

Direction z

Size w_0

Polarisation P

Frequency ν

Phase Φ (relative to second mode)

Information is sent in the form of modulation
of any one of these parameters

Classical waves

An electromagnetic wave can be described by the harmonic function at the Optical frequency ν and the dimensionless complex amplitudes $\alpha(\mathbf{r},t)$

$$\mathbf{E}(r,t) \sim [\alpha(\mathbf{r},t) \exp(i\, 2\pi\nu t) + \alpha^{*}(\mathbf{r},t) \exp(-i\, 2\pi\nu t)]\, \mathbf{p}\, (\mathbf{r},t)$$

Phase is an important concept expanding the complex amplitude into The magnitude $\alpha_0(\mathbf{r},t)$ and the phase $\phi(\mathbf{r},t)$

$$\alpha(\mathbf{r},t) = \alpha_0(\mathbf{r},t) \exp(\, i\, \phi(\mathbf{r},t)\,)$$

The spatial distribution of the phase $\phi(\mathbf{r},t)$, or wavefront, determines the shape of the wave; plane wave: $\phi(\mathbf{r},t) = \mathbf{k}\, \mathbf{r}$, $\alpha(z) = a_0 \exp(\, i\, kz)$
spherical wave: $\alpha(\mathbf{r},t) = \alpha_0/r \exp(\, i\, k\, r\,)$

Quadrature amplitudes

We can describe the same wave using quadrature amplitudes X1 and X2.

$$\mathbf{E}(r,t) \sim [X1(\mathbf{r},t)\cos(2\pi\nu t) + X2(\mathbf{r},t)\sin(2\pi\nu t)]\ \mathbf{p}(\mathbf{r},t)$$

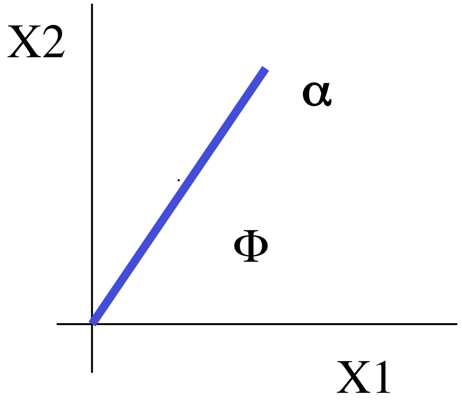
With the definition for X1 and X2:

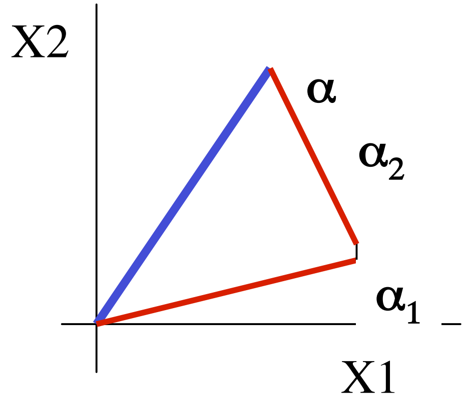
$$X1(\mathbf{r},t) = \alpha(\mathbf{r},t) + \alpha^{*}(\mathbf{r},t) \qquad X2(\mathbf{r},t) = I\,[\,\alpha(\mathbf{r},t) - \alpha^{*}(\mathbf{r},t)[$$

Each wave can be represented by a wave in a phasor diagram :

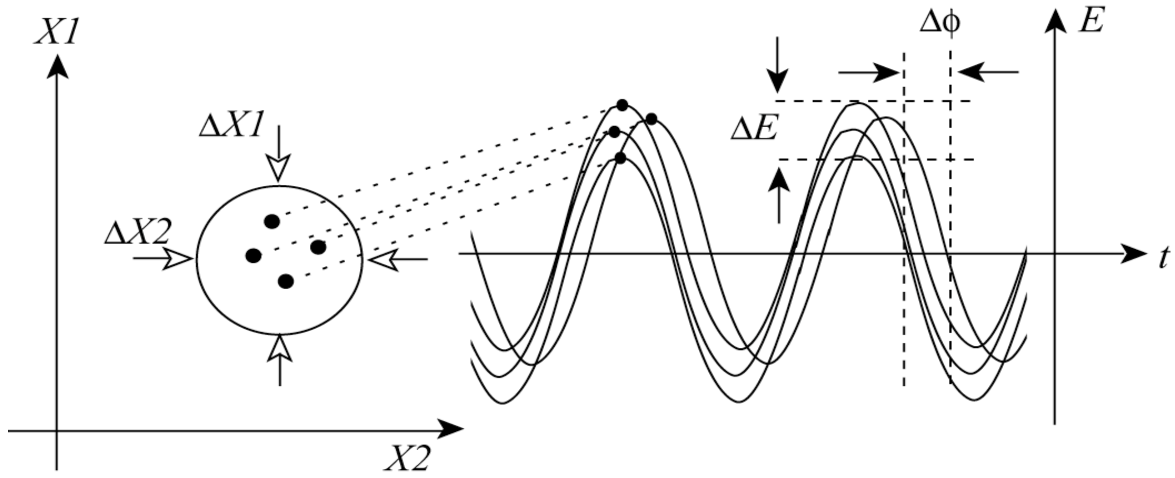
Phasor diagrams

In a phasor diagram each complex amplitude is represented by a vector. (phase space representation)

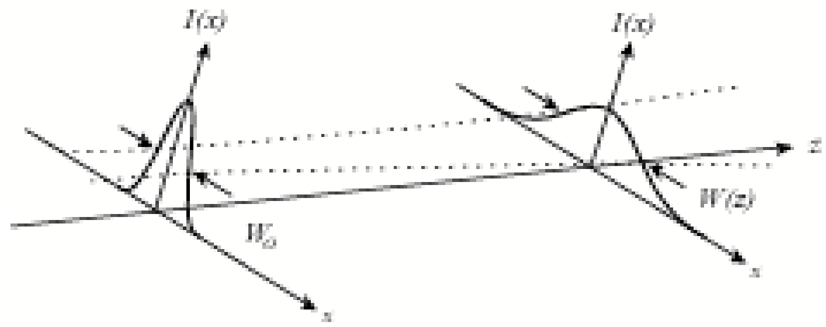




A beam with fluctuating magnitude and phase will provide quadratures that lie within an uncertainty area.



Gaussian beam



The shape and the total energy of a Gaussian beam remains fixed , but the beam broadens. The shape is preserved. Lenses and mirrors transform the Gaussian size and wavefront. (paraxial approximation)

This is the ideal TEMoo output mode from a laser or the mode created inside a cavity. In reality a beam has imperfections. These can be expressed as higher order modes. TEM ij.

Mode-matching refers to overlapping two beams with the same mode, That means the same size and mode curvature. Interference with high fringe visibility requires mode-matching.

Modulation

Amplitude modulation AM

$$\alpha(t) = \alpha_0 \left(1 - M/2 \ (1- \cos (2\pi\, \Omega_{mod}\, t) \right) \ \exp (i\, 2\pi\, \nu_L\, t)$$

$$= \alpha_0 (\ 1 - M/2) \ \exp (i\, 2\pi\, \nu_L\, t)$$

$$+ \alpha_0 M/4 \ [\exp (i\, 2\pi \ (\nu_L + \Omega_{mod}) \ t) \ + \exp (i\, 2\pi \ (\nu_L - \Omega_{mod}) \, t) \,]$$

Phase or frequency modulation FM

$$\alpha(t) = \alpha_0 \exp (i\, M \ \cos (2\pi\, \Omega_{mod}\, t) \ \exp (i\, 2\pi\, \nu_L\, t)$$

$$= \alpha_0 \ \{ 1 - M^2/4 + \ldots) \ \exp (i\, 2\pi\, \nu_L\, t)$$

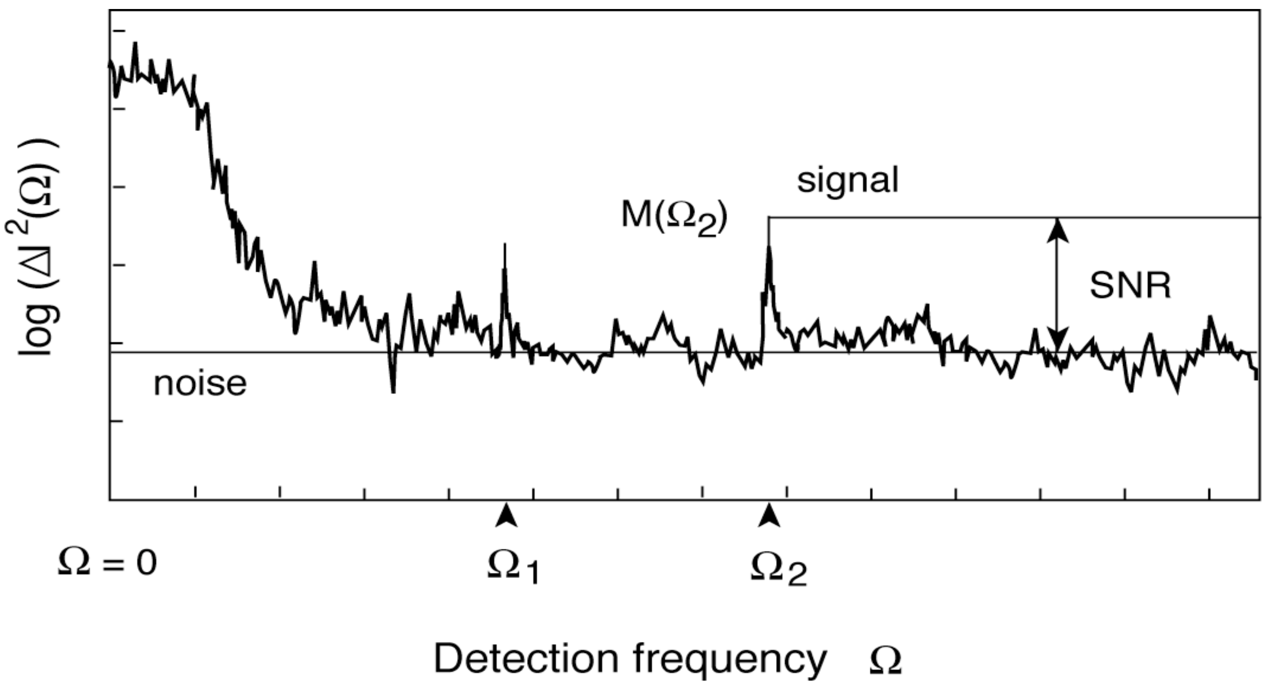
$$+ i \ (M/2 + \ldots .) \ [\exp (i\, 2\pi \ (\nu_L + \Omega_{mod}) \, t) + \ \exp (i\, 2\pi\, (\nu_L - \Omega_{mod}) \, t \,]$$

$$- \ (M^2/8 + \ldots .) \ [\exp (i\, 2\pi \ (\nu_L + \ 2\, \Omega_{mod}) \, t) \ - \exp (i\, 2\pi \ (\nu_L - \ 2\, \Omega_{mod}) \ t)] + \ldots\}$$

Graphical presentation of the sidebands

Both types of modulation (AM and FM) produces sidebands. For a laser at optical frequency ν_L and a modulation frequency Ω_{mod} these are at ν_L +/- Ω_{mod}

Noise spectrum



Example of a noise spectrum showing noise at many frequencies and two modulations at Ω_1 and Ω_2. This plot has a logarithmic y scale and the signal to noise ratio (SNR)can be read of directly if the modulation depth M(Ω) >> Var (I(Ω).

Quantum Optics 0. order

Processes: spontaneous & stimulated emission and absorption

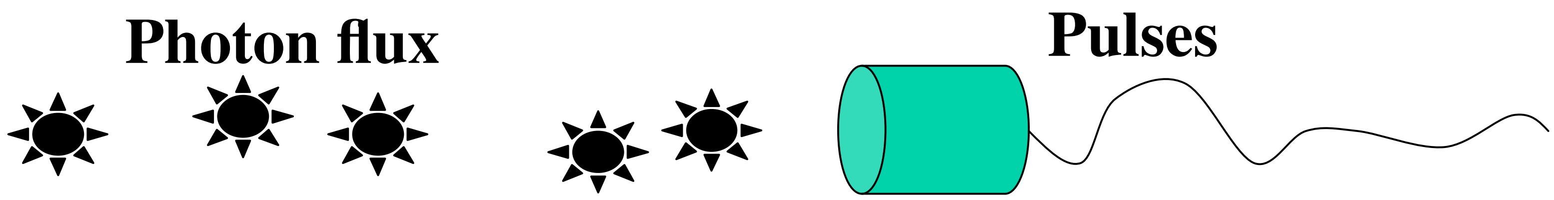
Light as an electromagnetic wave
and atoms are quantised

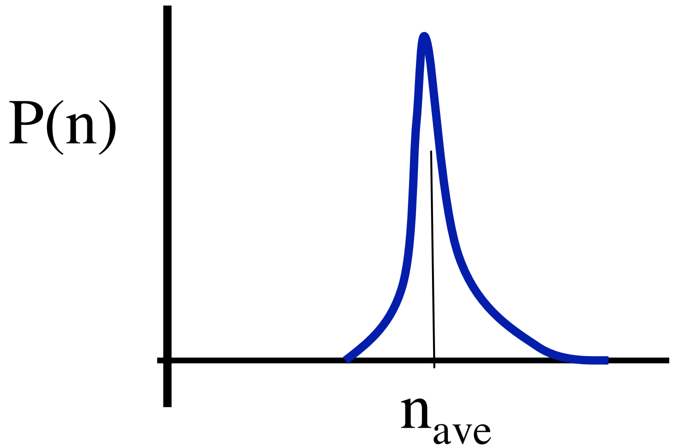
$$E_2 - E_1 = h\nu \qquad \Delta E_1 + \Delta E_2 = h\,\Delta\nu$$

Lifetimes τ of atoms are given by dipole moments

Find these as the solutions of the Schrödinger equation of the atom

Beams of photons



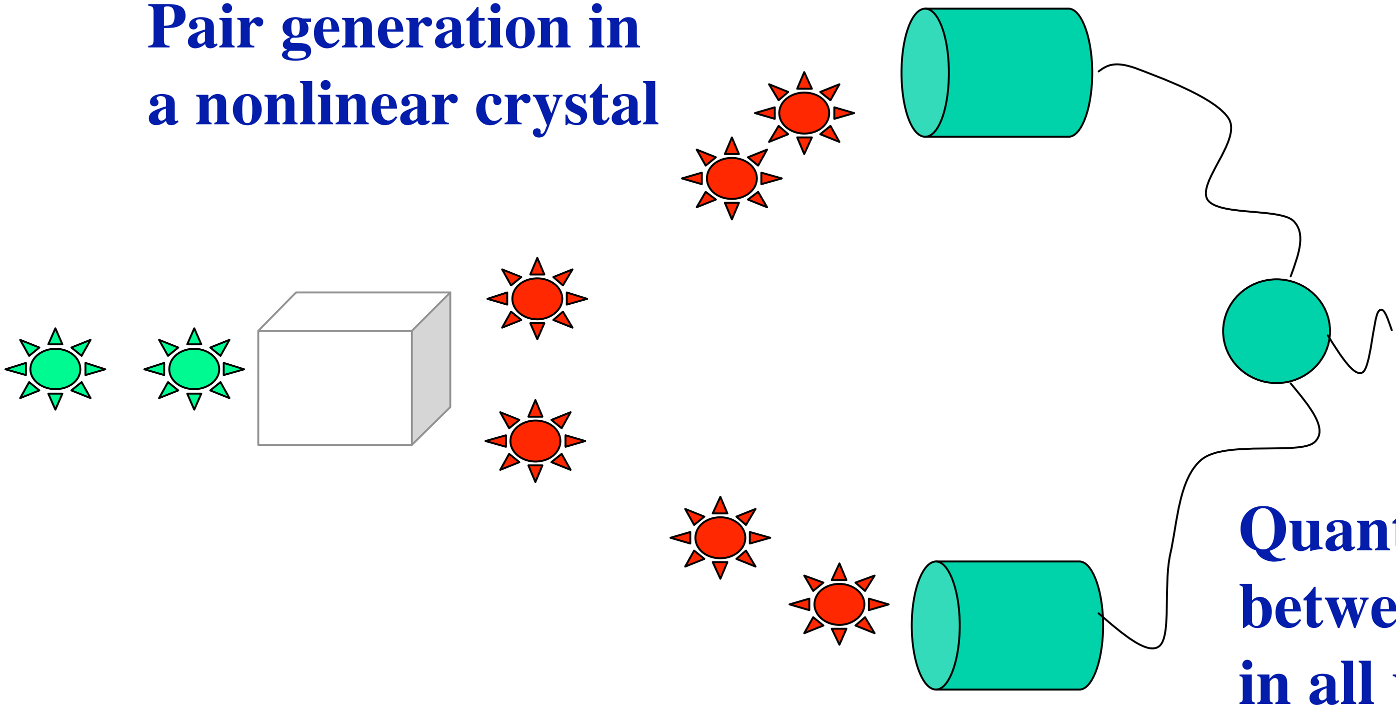


Poisson distribution

$$V(n) = n_{ave}$$

Entangled photons

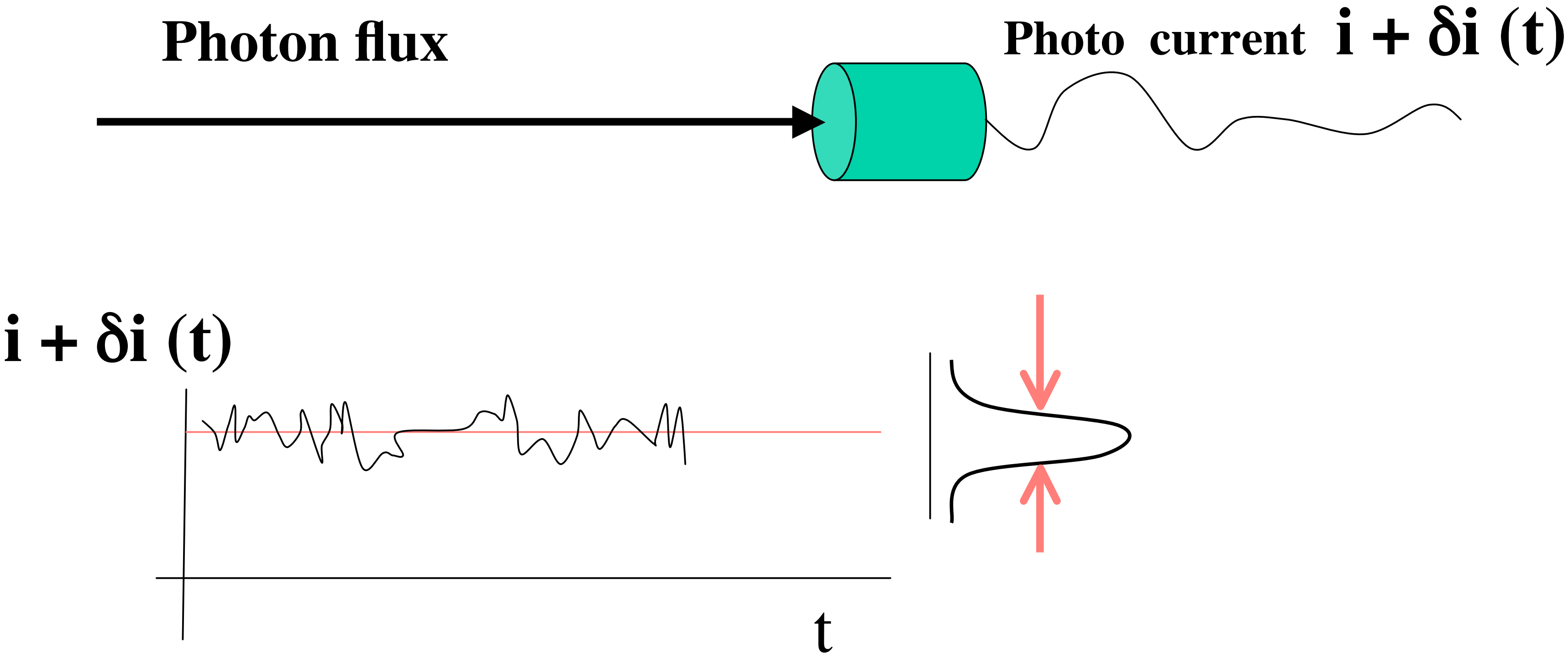
Pair generation in a nonlinear crystal



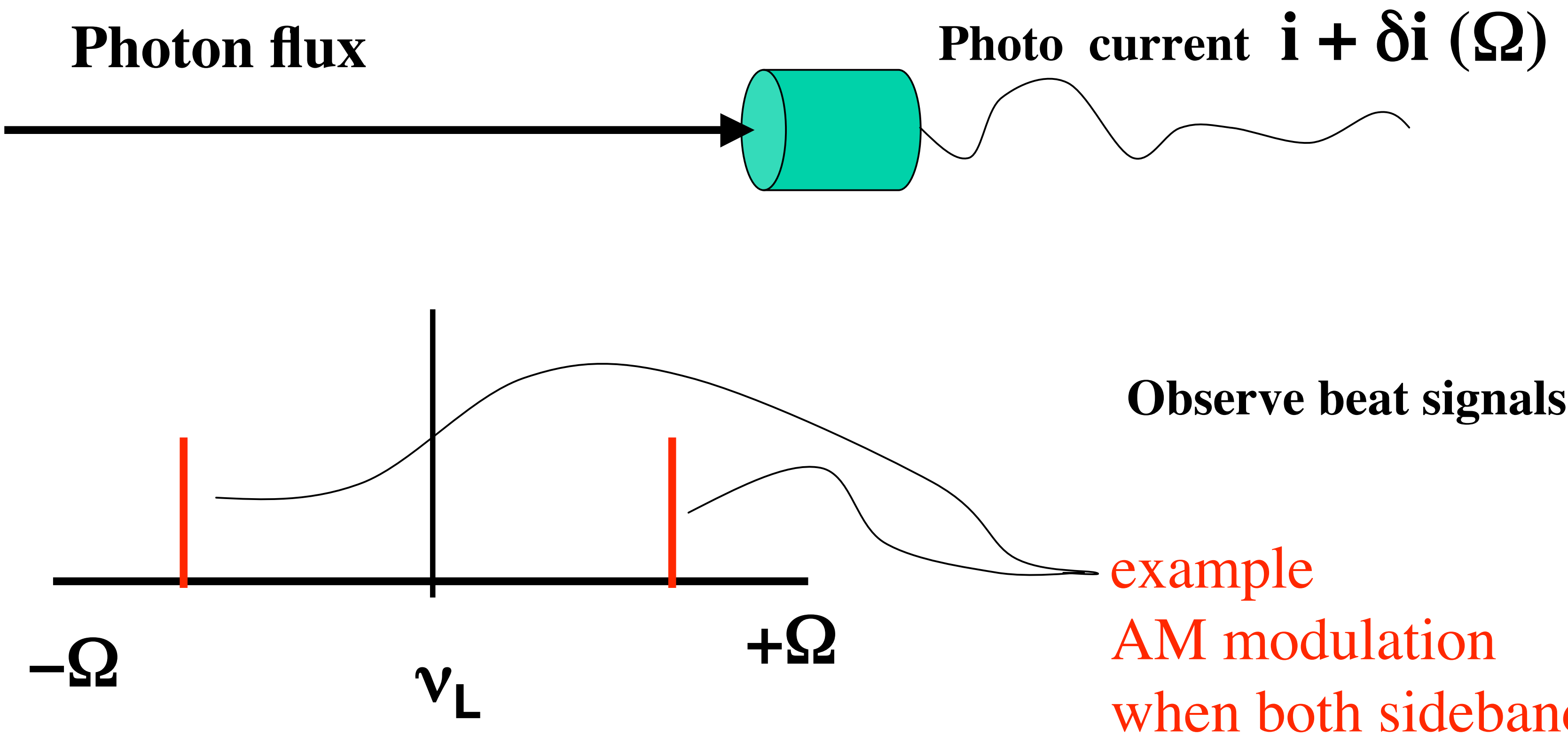
Correlated clicks

Quantum correlations between the photons in all properties: time, polarisation, …

Quantum noise in communication



Sending information



example
AM modulation
when both sidebands
are in phase

Photons & laserbeams: what we observe

Single Photon	**Laserbeams**
Clicks	**Photocurrent**
Information: **yes/ no within Δt**	**Information:** **Modulation & Noise**
Correlations between two detectors	**Correlations between two photocurrents**

Quantum Optics 1. level

Photon statistics

Arrival times

Poissonian
Bunching
Anti-bunching

Application :
Q. - cryptography

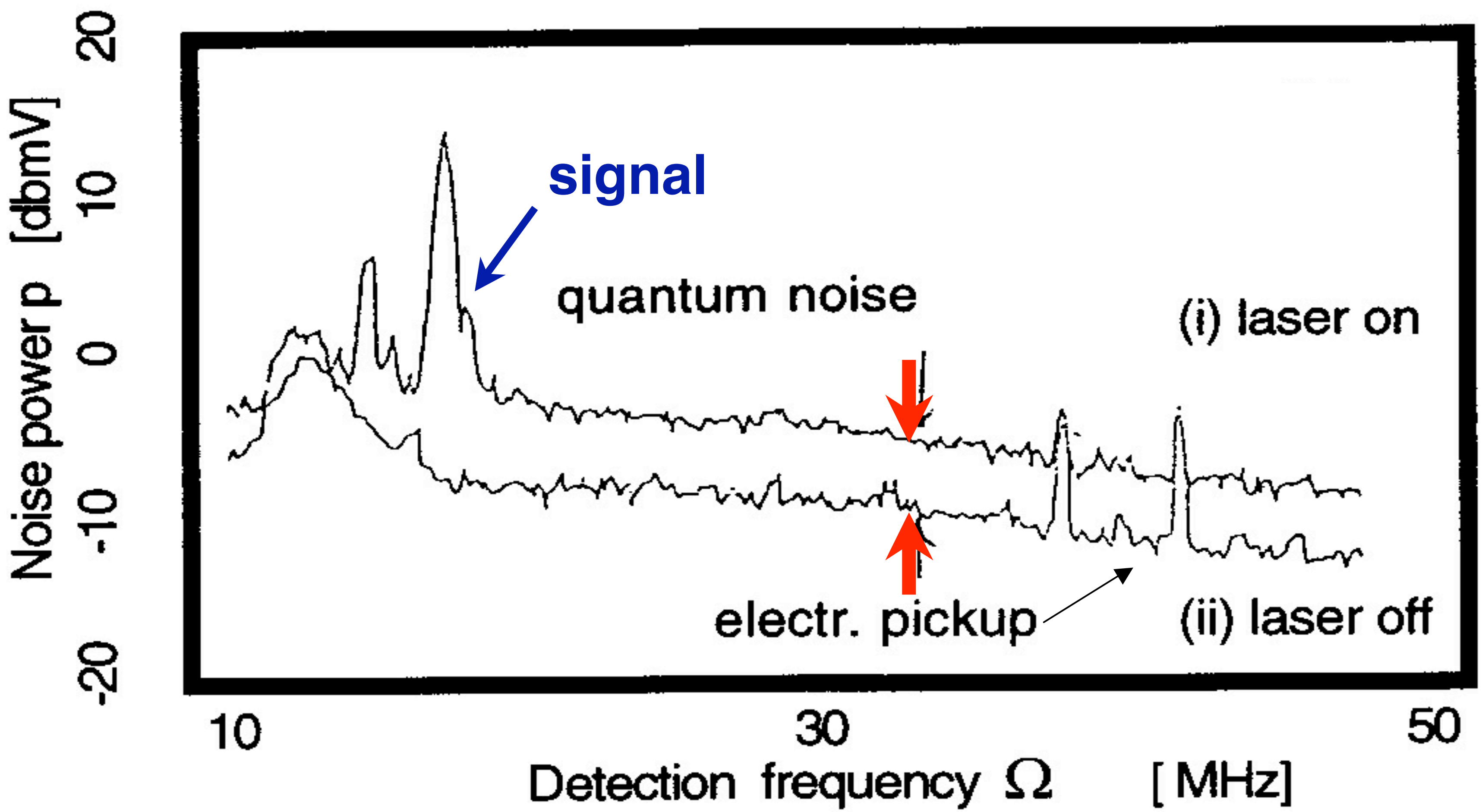
Quantum Noise limit
QNL

Quantum noise
in photo current

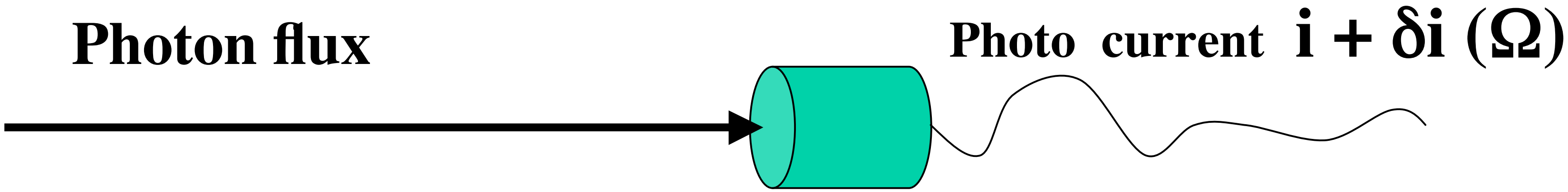
Limit to
signal to noise ratio
SNR reduces with power

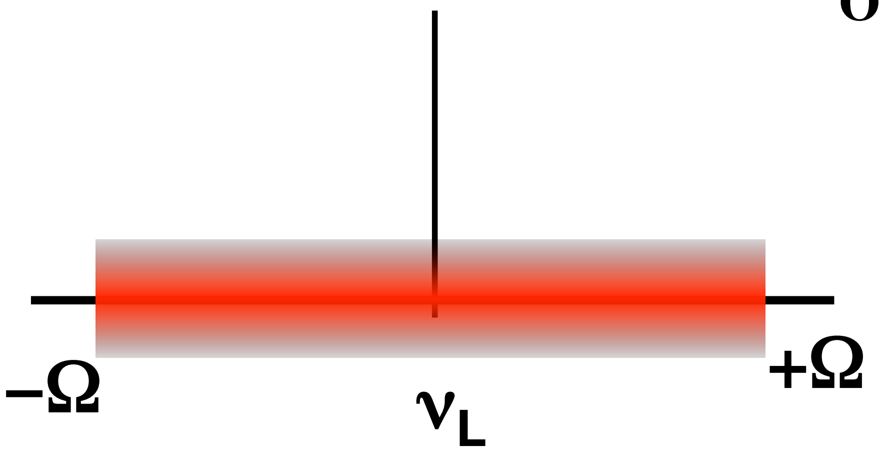
Limits to opt. Instruments
(shot noise limit)

Quantum Noise: Real spectrum of a laser



Quantum noise in communication



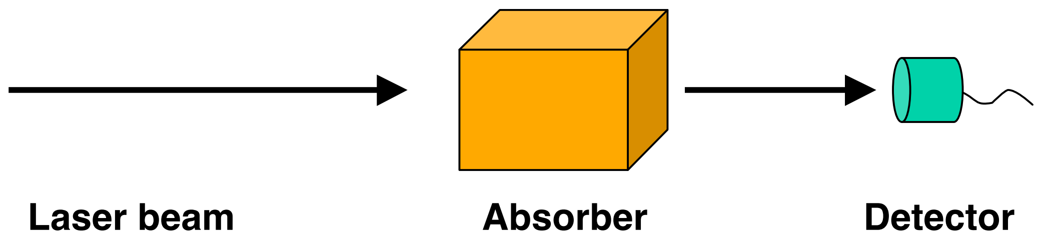


Observe beat signals

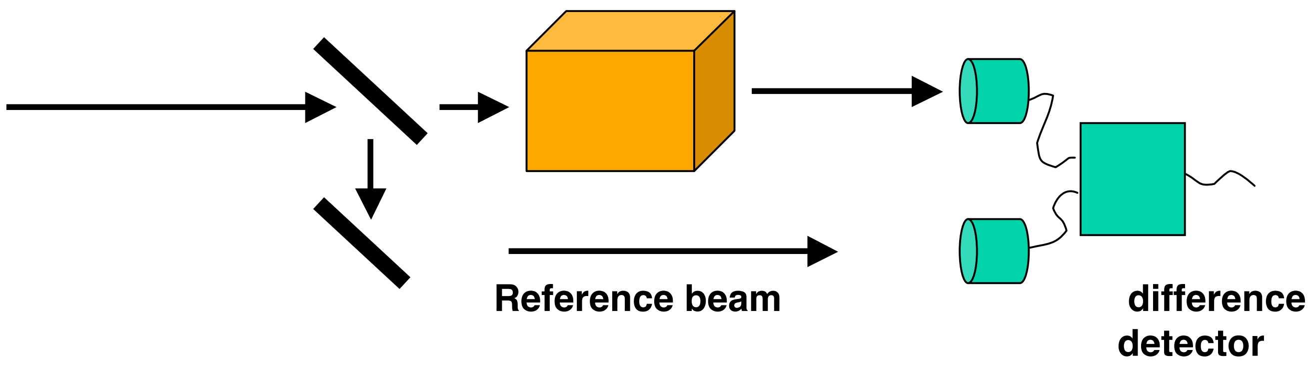
quantum noise

$= V\ (i) = i_{ave}$

Special properties of quantum noise



Absorption signal Is limited by quantum noise



Classical noise is cancelled

Quantum noise is added

<u>Measurements below the QNL</u>

noise < QNL <=> squeezed light

Measurements without noise penalty

Quantum non demolition experiments QND

Generate one photon at a time
(number or Fock states)
==> elusive single shoton source

Quantum Optics 3. level:

Entanglement

Two modes which allow information to be (perfectly) inferred

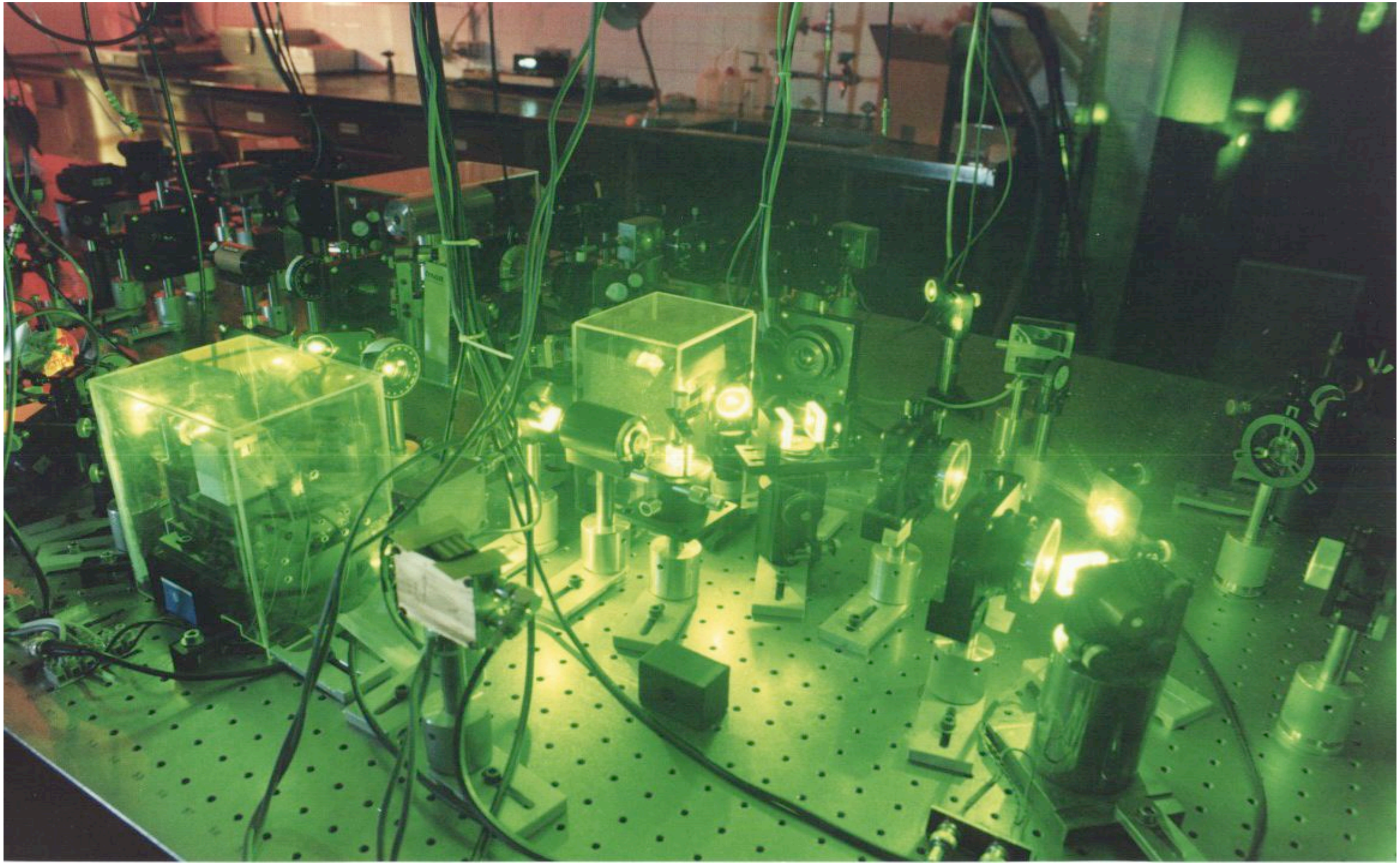
Pairs of photons

Two squeezed beams

Scientific goals:

Teleportation of information
Quantum logic ???
Transfer of entanglement light <=> atoms ???

A complete experiment … many losses



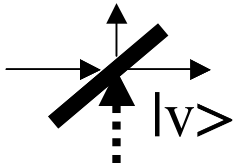
Experiment versus Theory

$|a_1>$, $|a_1>$

States

H

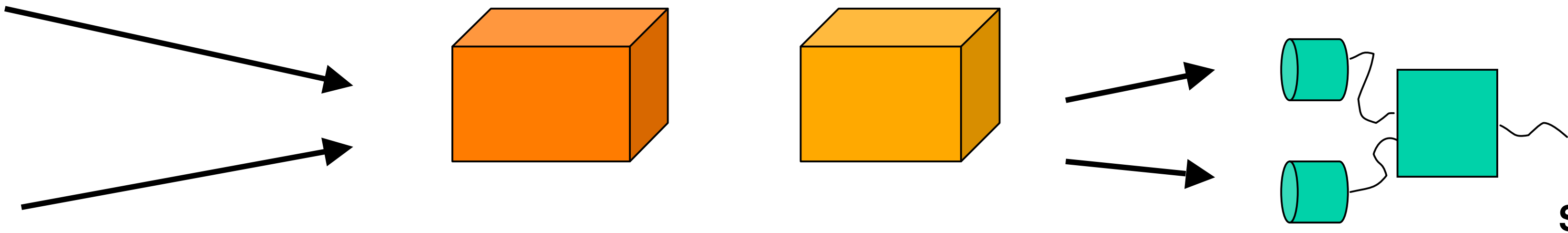
operators



Operator & vacuum state

$c|a_1>+c_2|a_1>+c_3|v>$

Complex state
Prob. of det & correlation



Info
SNR

Beams & modulation
intensity, phase
Signals & noise at Ω

Experiment
Linear & nonlinear components

Loss

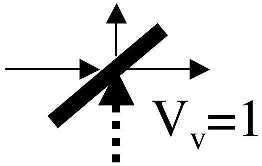
output beams

Detectors electronics

α^2, Φ, $\delta X1(\Omega)$, $\delta X2(\Omega)$
$V1_{in}(\Omega)$, $V2_{in}(\Omega)$

Coherent states
quadrature operators
Variances at Ω,

Quantum Transfer Function



Vacuum beam

α_{out}^2, Φ_{out}, $V1_{out}$, $V2_{out}$

Coherent states
Variances at Ω