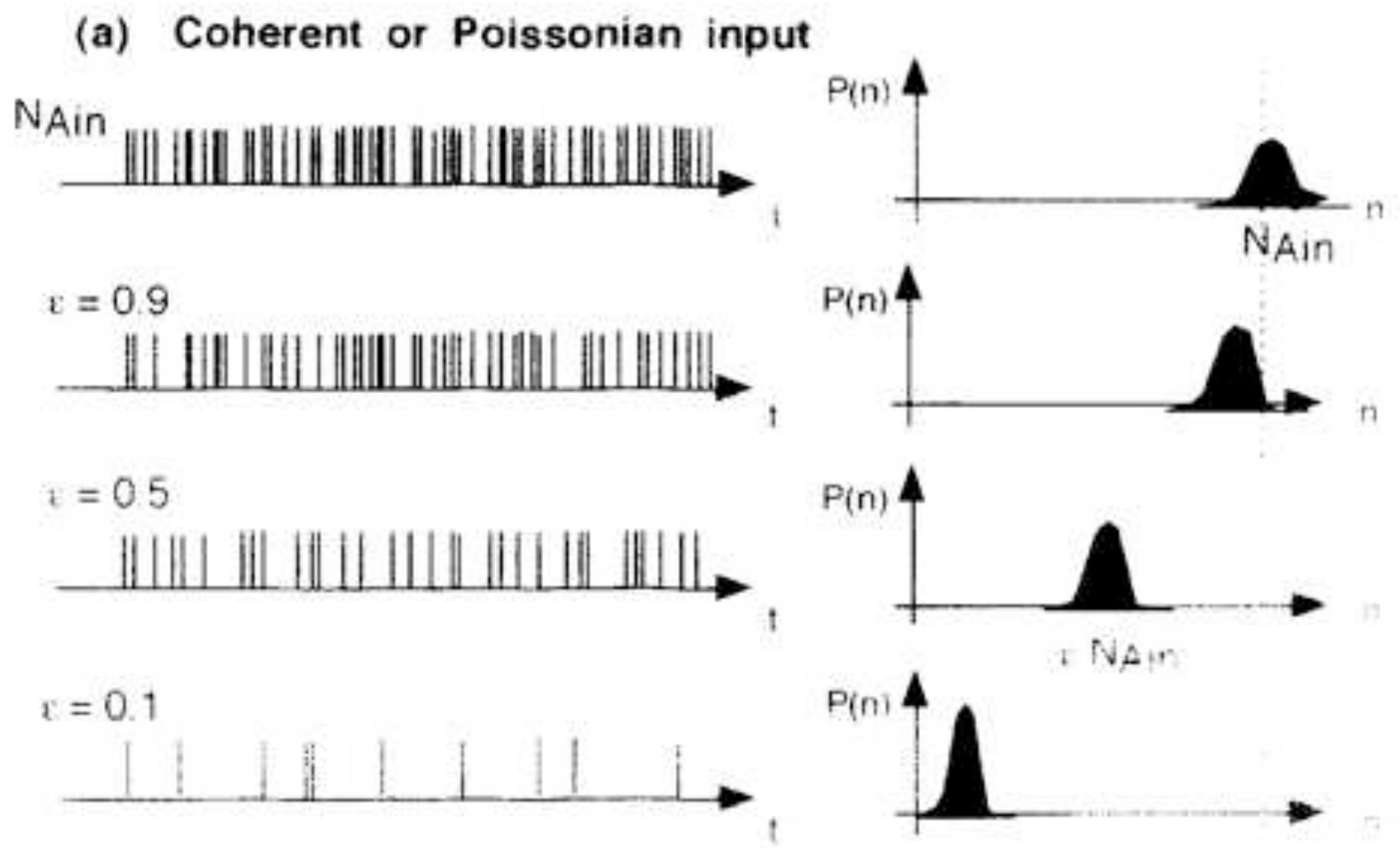
Lecture 3

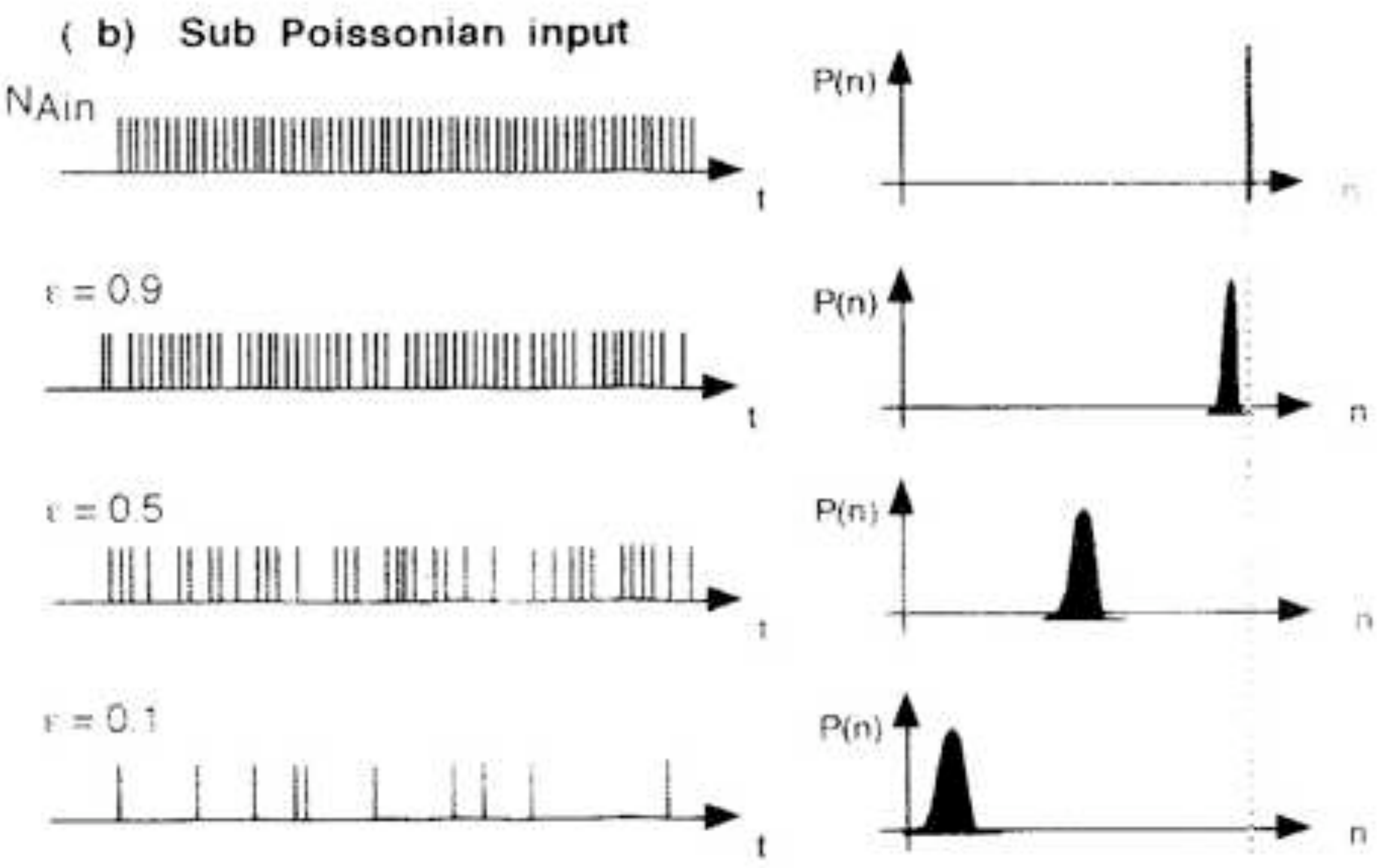
- <u>Example of transfer functions</u>
- <u>Laser intensity noise</u>
- <u>Photo detection</u>

Beamsplitter in the Photon picture :



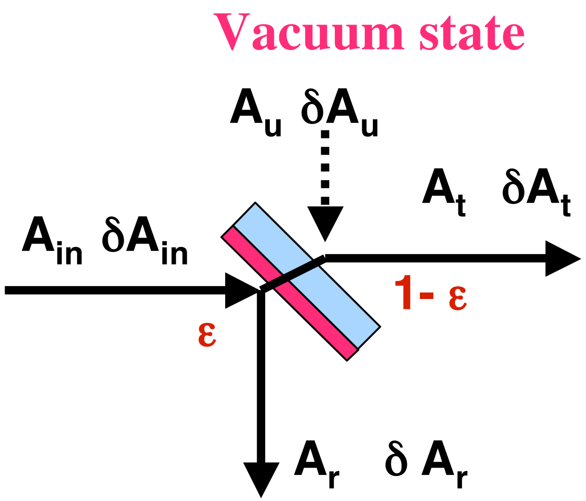
Remains Poissonian

Beamsplitter in the Photon picture



Turns into Poissonian

Beam splitter: Quantum calculation I



$$\delta A_r = \sqrt{\varepsilon}\, \delta A_{in} + \sqrt{(1-\varepsilon)}\, \delta A_u$$
$$\delta A_t = \sqrt{(1-\varepsilon)}\, \delta A_{in} - \sqrt{\varepsilon}\, \delta A_u$$

Input and output operators have to obey the commutation relationship for Bosons $[\delta A_i\, \delta A_j^\dagger] = \delta_{ij},\quad [\delta A_i\, \delta A_j] = 0$

We find that for the fluctuations we cannot ignore the mode δA_u.

If we ignored it the commutators for δA_r and $\delta A_t^\dagger$ would no longer work.
For example, we want $[\delta A_r\, \delta A_t^\dagger] = 0$. If we set $\delta A_r = \sqrt{\varepsilon}\, \delta A_{in}$ and $\delta A_t = \sqrt{1-\varepsilon}\, \delta A_{in}$, we would get $[\delta A_r\, \delta A_t^\dagger] = \sqrt{(\varepsilon (1-\varepsilon))}\, [\delta A_{in}\, \delta A_{in}^\dagger] = \sqrt{(\varepsilon (1-\varepsilon))}$ which is not correct.

We have to include δA_u, the vacuum state makes a contribution to the noise.

Beam splitter: Quantum calculation II

Recipe: determine $V1(\Omega) = < \delta X1(\Omega)\ \delta X1^*(\Omega) >$

$$\delta X1_r(\Omega) = \sqrt{\varepsilon}\,(\delta A_{in}^\dagger(\Omega) + \delta A_{in}(\Omega)) + \sqrt{1-\varepsilon}\,(\delta A_u^\dagger(\Omega) + \delta A_u(\Omega))$$

$$\begin{aligned} < \delta X1_r(\Omega)\, \delta X1_r^*(\Omega) > = \\ & \varepsilon < (\delta A_{in}^\dagger(\Omega) + \delta A_{in}(\Omega))^2 > + (1-\varepsilon) < (\delta A_u^\dagger(\Omega) + \delta A_u(\Omega))^2 > \\ & + \sqrt{(\varepsilon(1-\varepsilon))}\ \{< \delta A_{in}^\dagger(\Omega)\ \delta A_u(\Omega) > + < \delta A_{in}(\Omega)\ \delta A_u(\Omega) > + \ldots \} \end{aligned}$$

where the terms in the { } bracket all cancel .

Result: $V1_r = \varepsilon\ V1_{in} + (1-\varepsilon)\ V1_u$ **and** $V1_t = (1-\varepsilon)\ V1_{in} + \varepsilon\ V1_u$

since $V1_u = 1$ we get $(V1_r - 1) = \varepsilon\ (V1_{in} - 1)$.
that means the difference between $V1_{in}$ and the quantum noise limit gets reduced.
$V1_r$ is closer to the QNL than $V1_{in}$. We say: A vacuum beam gets mixed in.

We can interpret the effect of a beam splitter as adding a part $(1-\varepsilon)$ of the vacuum beam.

Effect on signal to noise ratio

The signal to noise ratio is given by

$$\text{SNR} = \left(V1_{signal}(\Omega) - V1_{quantum\ noise}(\Omega)\right) / V1_{quantum\ noise}(\Omega)$$

and for $V1_{quantum\ noise}(\Omega) = 1$ and for signals sufficiently large we simply have:

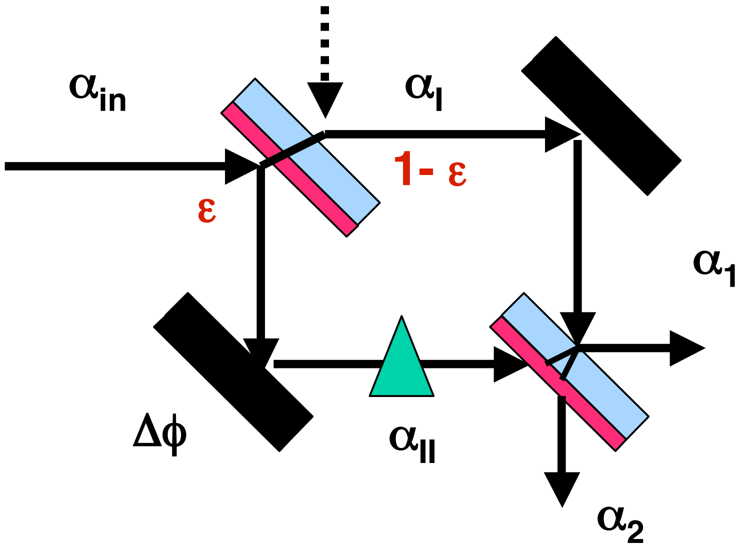
$$\boxed{\text{SNR}_{out} = \varepsilon\ \text{SNR}_{in}}$$

for a quantum noise limited system with large signal

This applies to all situations where we have attenuation, such as absorption losses, diffraction, quantum inefficiencies, mode matching errors, …….

This makes the experiments so challenging.

Interferometer: Classical calculation



$\alpha_I = \sqrt{1/2}\,\alpha_{in}$
$\alpha_{II} = \sqrt{1/2}\,\alpha_{in}$

$\alpha_1 = \sqrt{1/2}\ \alpha_I\ \exp(i\,\Delta\phi) - \sqrt{1/2}\ \alpha_{II}$
$\alpha_1 = \sqrt{1/2}\ \alpha_I\ \exp(i\,\Delta\phi) + \sqrt{1/2}\ \alpha_{II}$

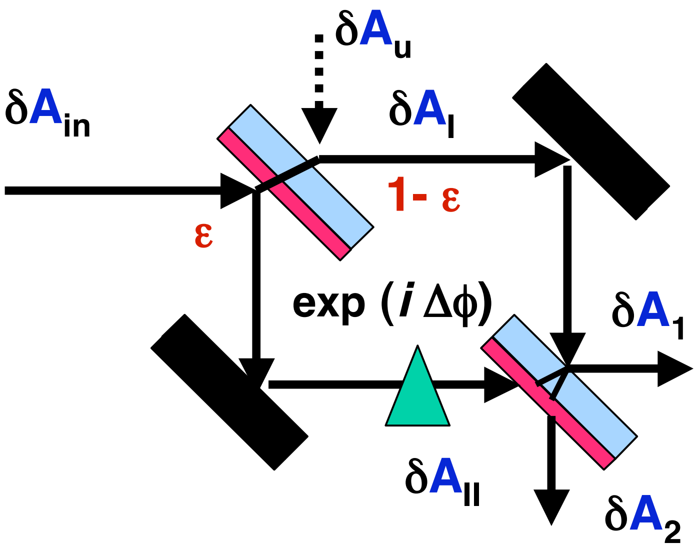
Intensity at the output Is the standard fringe pattern

$I_1(\Delta\phi) = I_{in}\cos^2(\Delta\phi/2)$
$I_2(\Delta\phi) = I_{in}\sin^2(\Delta\phi/2)$

Visibility

$\text{Vis} = (I_{max} - I_{min}) / (I_{max} + I_{min})$

Interferometer: Quantum calculation



$$\delta A_I = \sqrt{1/2}\,\delta A_{in} - \sqrt{1/2}\,\delta A_u$$
$$\delta A_{II} = \sqrt{1/2}\,\delta A_{in} + \sqrt{1/2}\,\delta A_u$$

$$\delta A_1 = \sqrt{1/2}\;\delta A_I \;-\; \exp(i\,\Delta\phi)\,\sqrt{1/2}\;\delta A_{II}$$
$$\delta A_2 = \sqrt{1/2}\;\delta A_I \;+\; \exp(i\,\Delta\phi)\,\sqrt{1/2}\;\delta A_{II}$$

$$\delta A_1 = \sqrt{1-\varepsilon}\,\sqrt{\varepsilon}\;\;\delta A_{in} - \sqrt{\varepsilon}\,\sqrt{1-\varepsilon}\;\exp(i\,\Delta\phi)\,\delta A_{in}$$
$$+\;\sqrt{1-\varepsilon}\,\sqrt{\varepsilon}\;\;\delta A_u + \sqrt{\varepsilon}\,\sqrt{1-\varepsilon}\;\exp(i\,\Delta\phi)\,\delta A_u$$

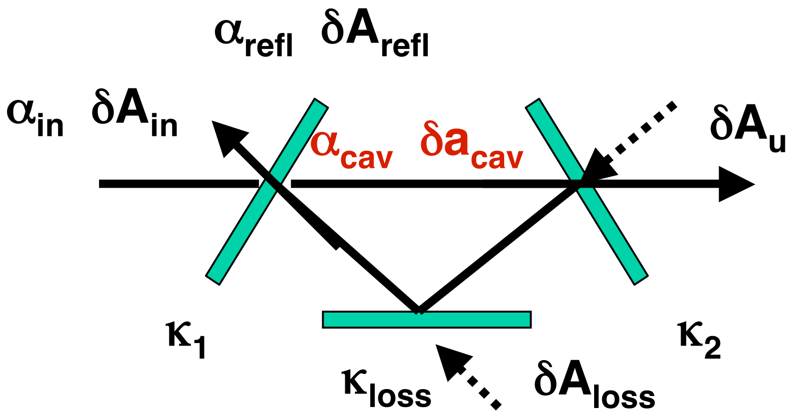
determine $V1_1(\Omega) = \langle\, \delta X1_1(\Omega)\, \delta X1_1^{*}(\Omega) \,\rangle$

For $\varepsilon = 1/2$

$$V1_1(\Omega) = \cos^2(\Delta\phi/2)\; V1_{in}(\Omega) + \sin^2(\Delta\phi/2)\; V2_u(\Omega)$$

This is like a variable attenuator .

Cavity: Quantum calculation



Cavity on resonance
$\Delta = 0$, that means cavity is locked to the laser frequency

Total loss $\kappa = \kappa_1 + \kappa_2 + \kappa_{loss}$ and small losses are important here

$$d\ \delta a_{cav}/dt = -\kappa\,\delta a_{cav} + \sqrt{2\kappa_1}\,\delta A_{in} + \sqrt{2\kappa_2}\,\delta A_u + \sqrt{2\kappa_{loss}}\,\delta A_{loss}$$

$$d\ \delta X1_{cav}/dt = -\kappa\,\delta X1_{cav} + \sqrt{2\kappa_1}\,\delta X1_{in} + \sqrt{2\kappa_2}\,\delta X1_u + \sqrt{2\kappa_{loss}}\,\delta X1_{loss}$$

Boundary conditions $\delta X1_{out} = \sqrt{2\kappa_2}\,\delta X1_{cav} - \delta X1_u$ **and** $\delta X1_{refl} = \sqrt{2\kappa_1}\,\delta X1_{cav} - \delta X1_{in}$

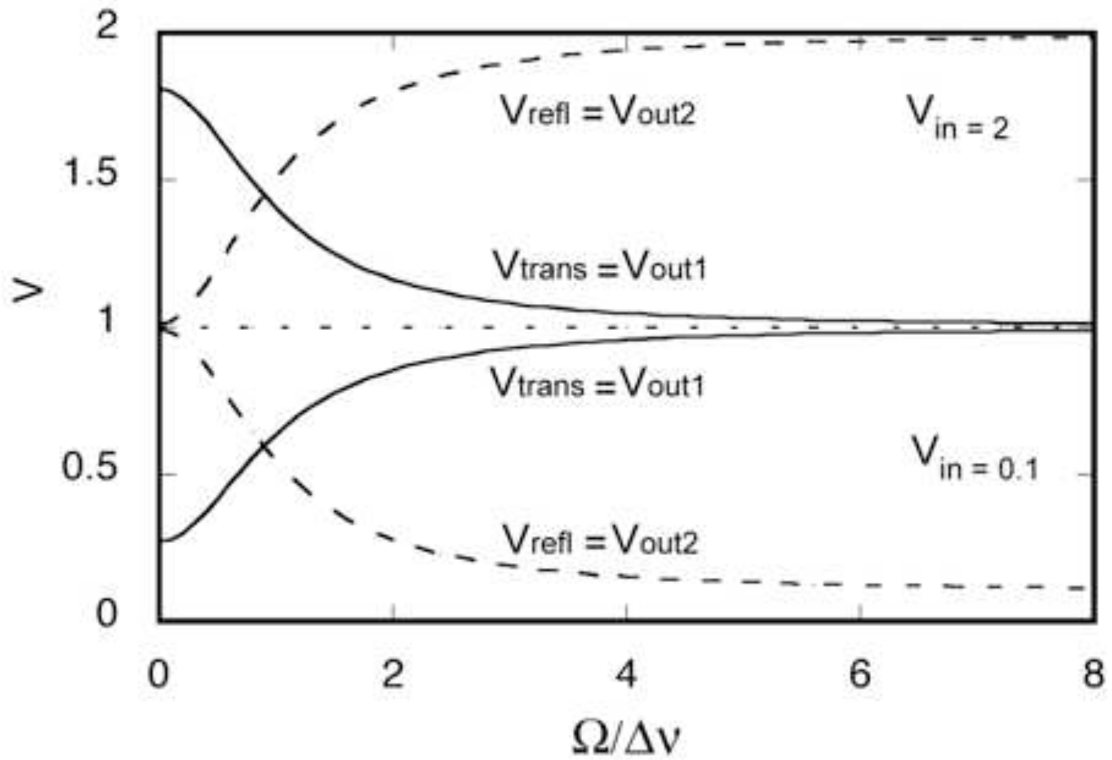
$$\delta X1_{out} = \sqrt{4\kappa_1\kappa_2}\,\delta X1_{in} + \sqrt{4\kappa_1\kappa_{loss}}\,\delta X1_{loss} + (2\kappa_1 - \kappa - i2\pi\Omega)\,\delta X1_u\ /(\kappa - i2\pi\Omega)$$

With $\mathcal{F}\{df(t)/dt\} = 2\pi\Omega\,\mathcal{F}\{f(t)\}$

$$V1_{out}(\Omega) = (\,4\kappa_1\kappa_2\,V1_{in}(\Omega) + 4\kappa_1\kappa_{loss}\,V1_{loss} + [(2\kappa_1-\kappa)^2 + (2\pi\Omega)^2]\,V1_u\,)\,/\,(\kappa^2 + (2\pi\Omega)^2)$$

Transfer function is frequency dependent. Cavity needs to be locked to the laser.
Different behaviour inside and outside the cavity linewidth.

Transfer of signals



Signal
$V1_{in}$ = 2

Squeezing
$V1_{in}$ = 0.1

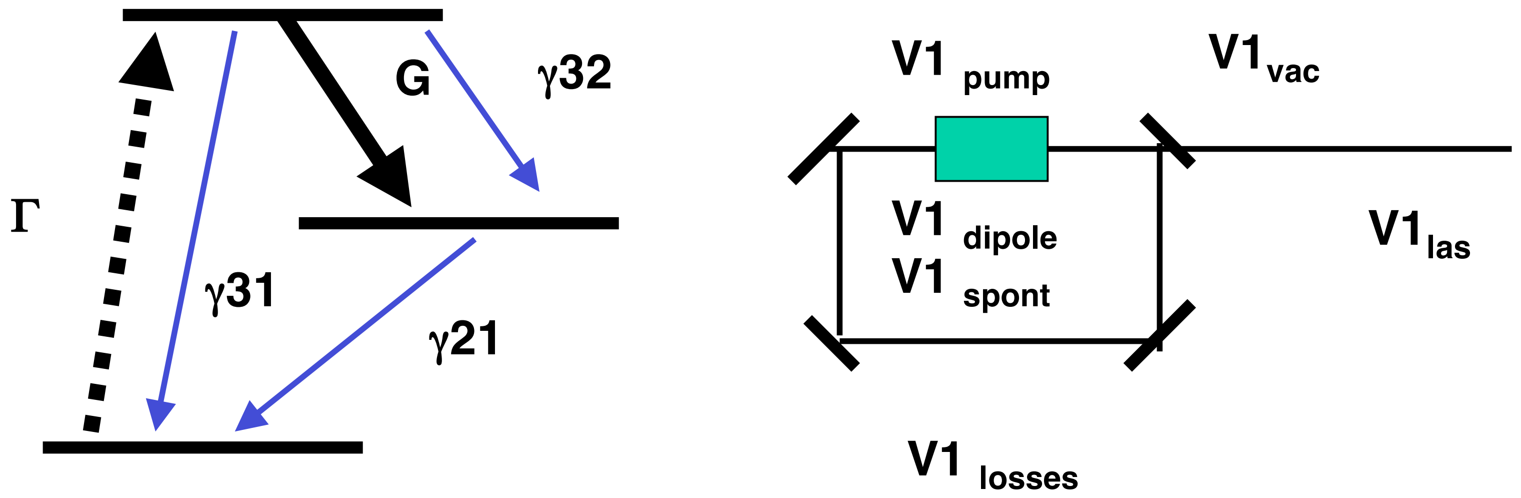
Δν = cavity linewidth

The cavity transmits the low frequencies but reflects the high frequencies.

This applies to signals (V1 >1) and to squeezing (V1 <1).

If all inputs are at the QNL all the outputs are at the QNL.This is a good test.

Quantitative effect of loss



The model for a laser: the energy level and processes and all the possible noise sources

The laser noise equations

$$V1_{las}(\Omega) = 1/\left[\left((2\pi\,\Omega_{RRO})^2 - (2\pi\,\Omega)^2\right)^2 + (2\pi\,\Omega)^2\,\gamma_L^{\,2}\right.$$

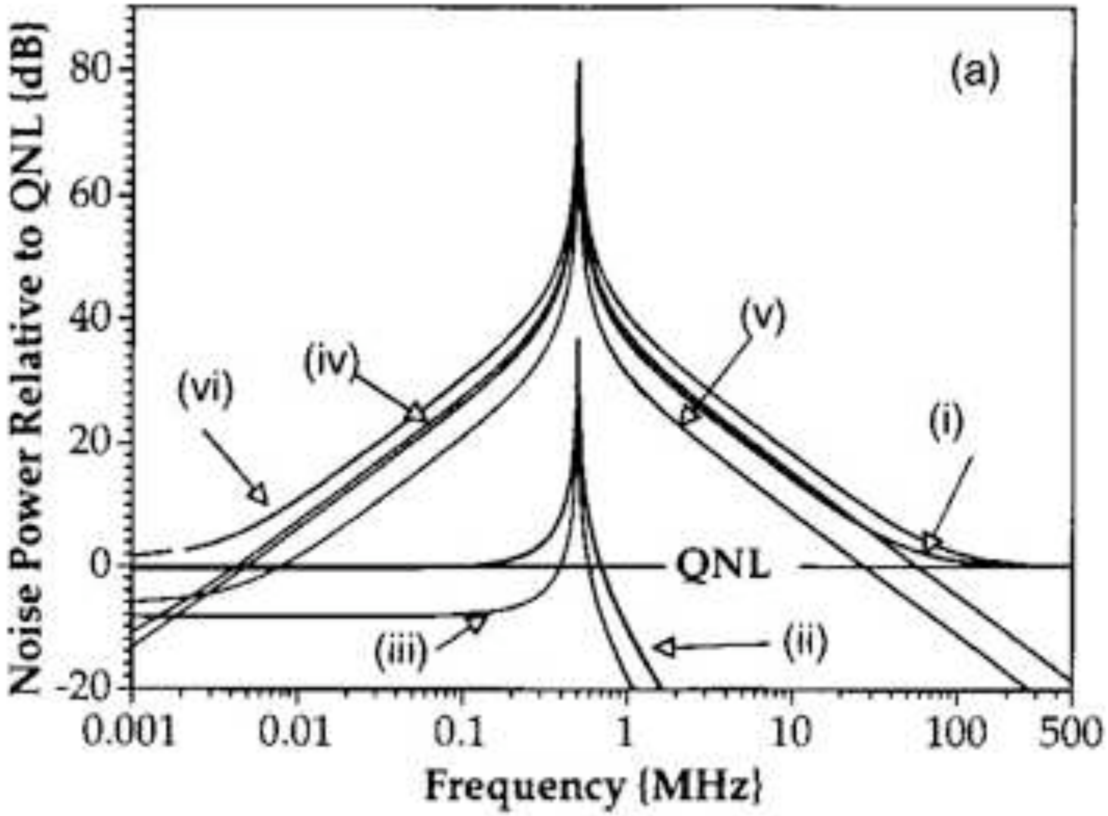
$\{\ 1 + 4\kappa_m^{\,2}\,(2\pi\,\Omega)^2 + \gamma_L^{\,2}) - 8\,\kappa_m\,G_{32}\,\alpha^2\,\gamma_L$	V_{vac}
$+\ \ 2\,\kappa_m\,G_{32}^{\,2}\,\alpha^2\,\Gamma$	V_{pump}
$+\ \ (2\,\kappa_m\,G_{32}^{\,2}\,\alpha^2\,\gamma_t\,J_3$	V_{spont}
$+\ \ 2\,\kappa_m\,G_{32}\,(\gamma_t + \Gamma)^2 + (2\pi\,\Omega)^2$	V_{dipole}
$+\ \ 4\kappa_m\,\kappa_l\,(2\pi\,\Omega)^2 + \gamma_L^{\,2})$	$V_{losses}\ \}$

with κ_m the resonator mirror, G_{32} rate constant , γ_L = combined rate , Γ= pump rate etc….. these are all parameters which can be determined independently for each laser.

The main point: the noise spectrum is a linear combination of the various input losses.

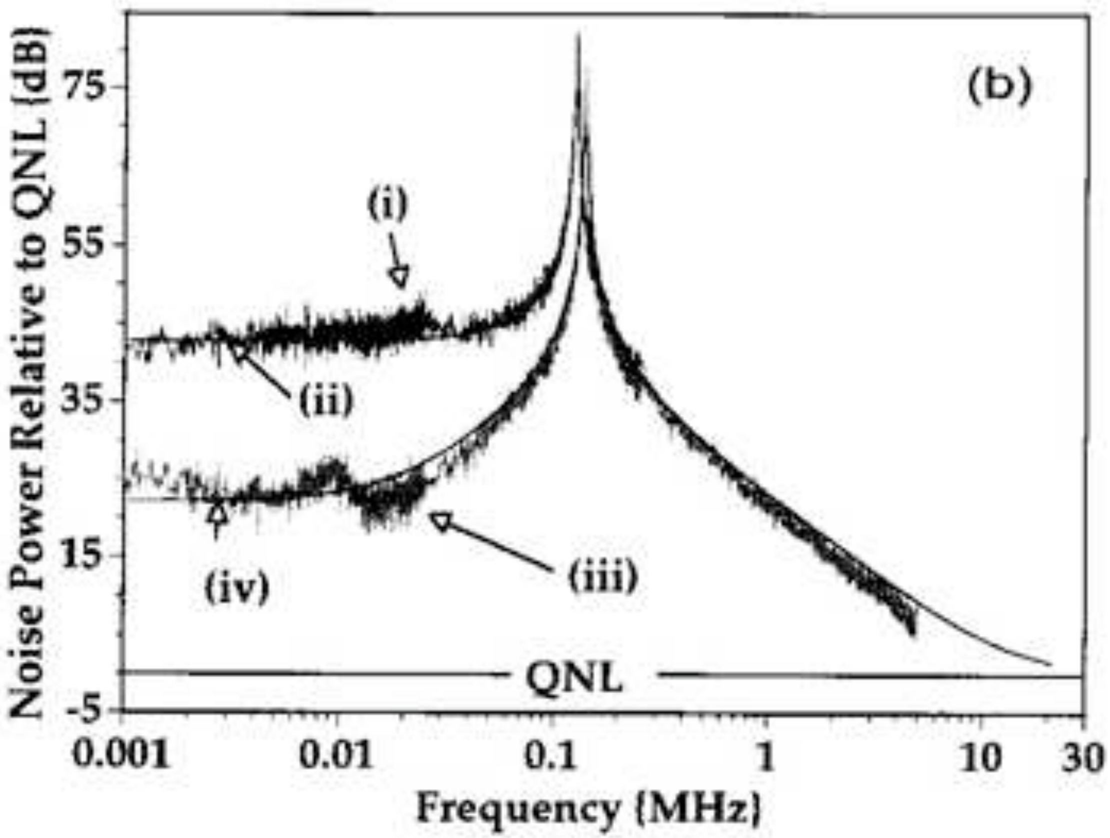
This model predicts a complex noise spectrum.

Noise of a Nd:YAG laser



Laser noise spectrum from the quantum transfer functions previous page. The spectrum peaks around Ω_{RRO}, the resonance frequency for the relaxation oscillation. The pump noise (ii) dominates at very low frequencies. Total noise, for a QNL pump is given by line (vi). At high frequencies the laser is QNL. This is the best a conventional laser can achieve.

Noise of a Nd:YAG laser



below RRO: laser noise comes from the pump

Above RRO laser can be quantum noise limited

Experimental test of laser noise:
(I) experiment, large pump noise (ii) theory large pump noise
(iii) experiment , low pump noise (iv) theory , low pump noise

Summary

We have a technique for predicting the noise spectrum for our experiment.

This can include passive components (mirrors, lenses,…) as well as active components (lasers, amplifiers,…..)

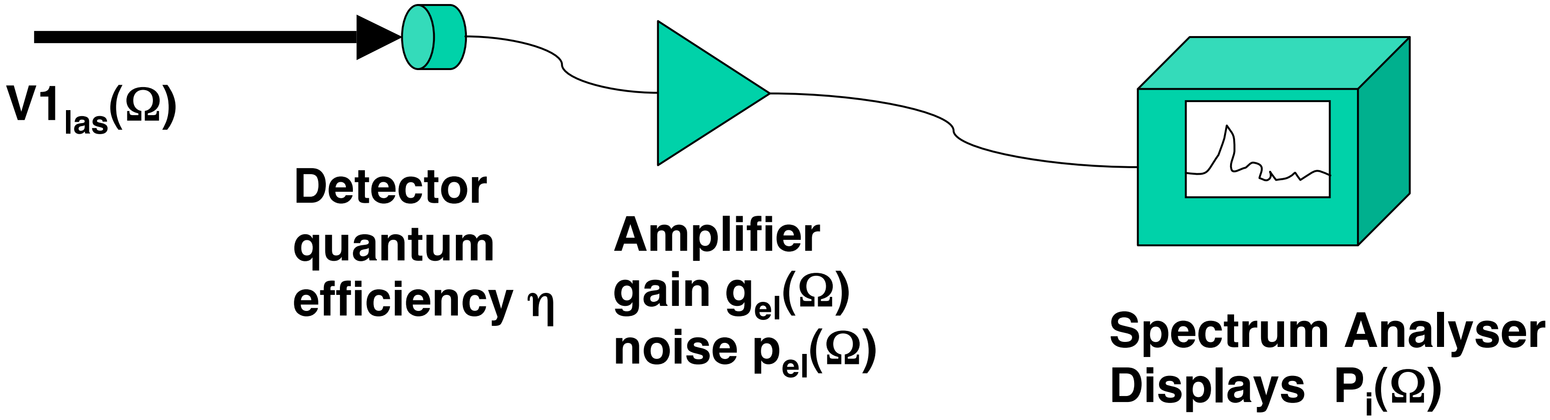
Once resonant systems (cavities) are involved the quadratures X1 and X2 will mix, that is we are getting a rotation is phase space.

Many lasers can be QNL, at least a high detection frequencies.

The coherent state is a perfect description of the idealised laser.

Real lasers can reach the pure coherent state for some detection frequencies.

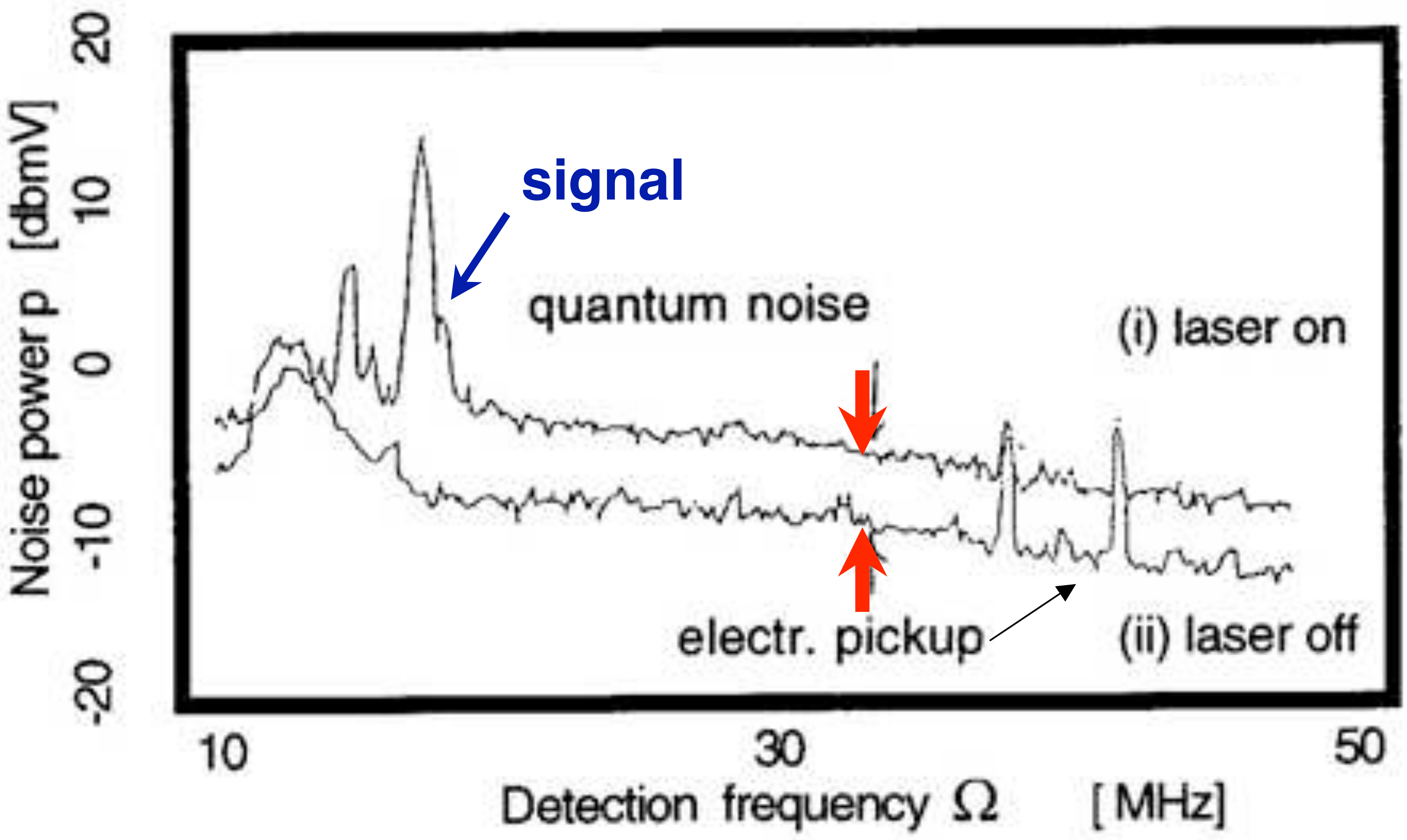
Direct detection



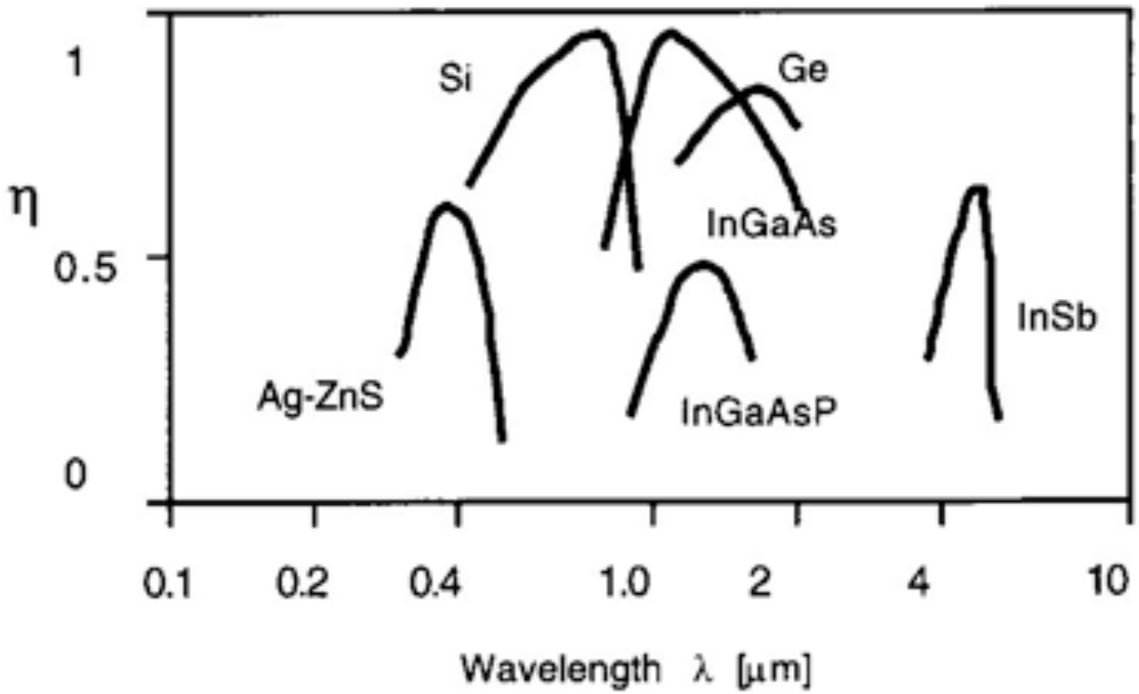
$$\boxed{V1_{las}(\Omega) = P_i(\Omega) \ / \ P_{i,\ QNL}(\Omega)}$$

We need experimental calibration techniques to verify $P_{i,\ QNL}(\Omega)$

Quantum Noise: Real spectrum of laser

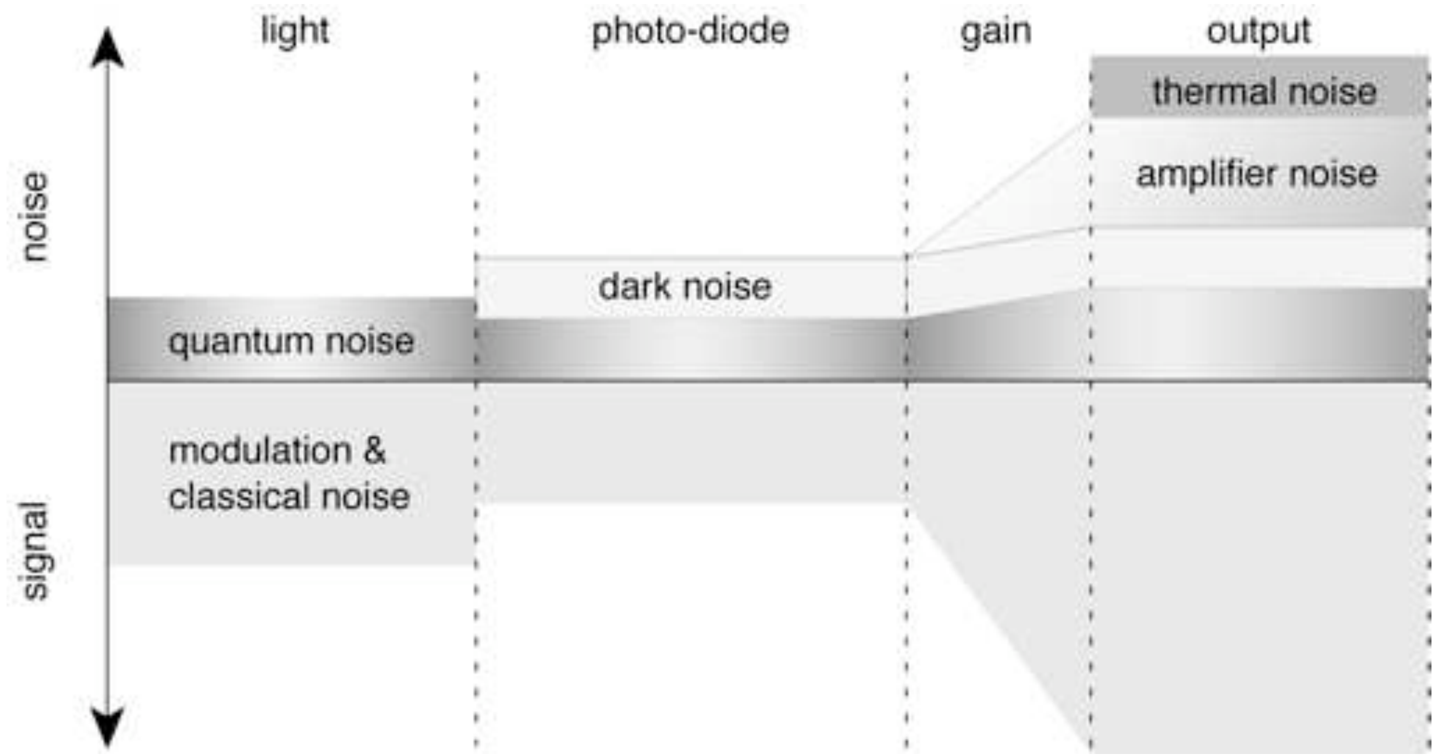


Detection efficiency



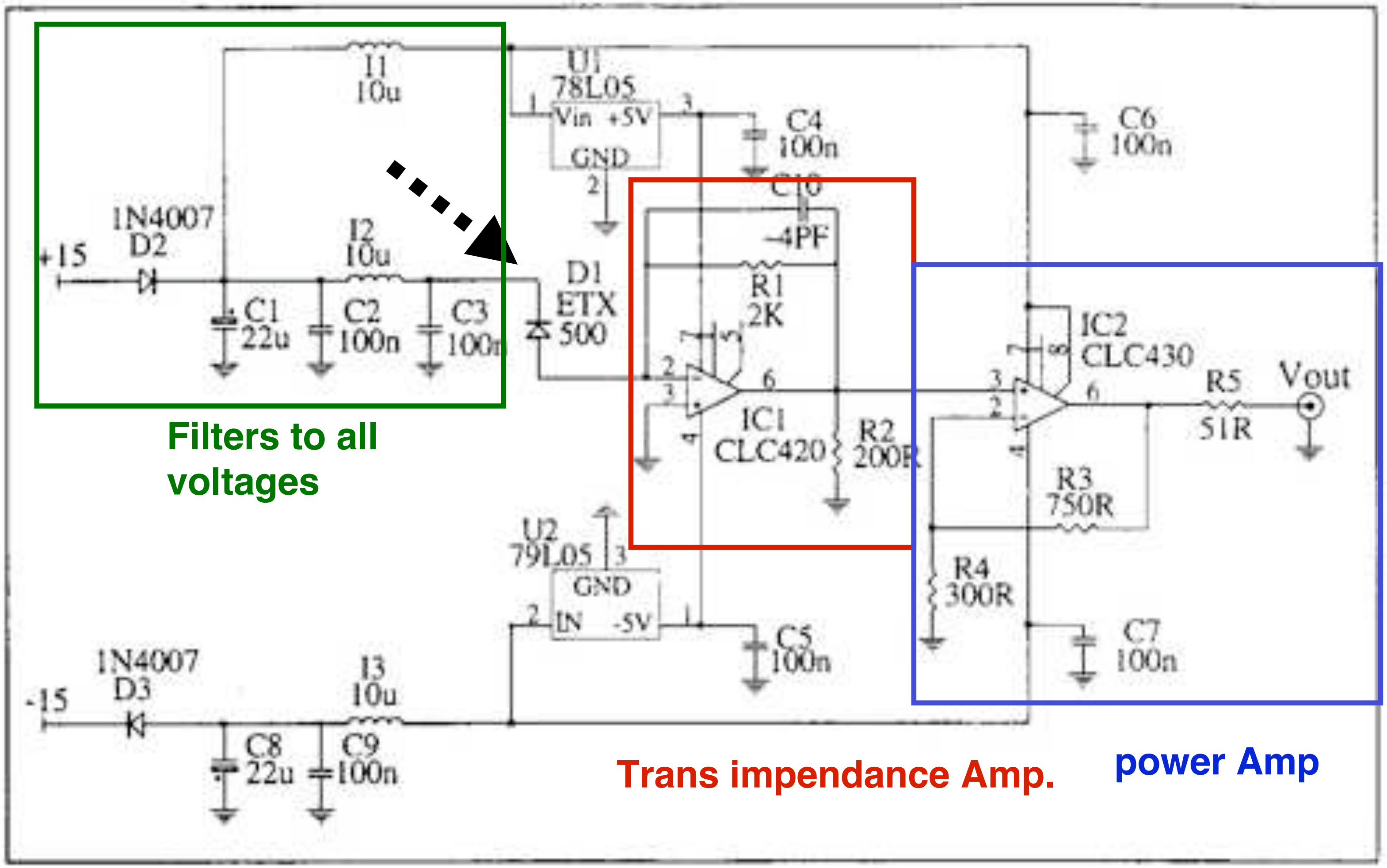
The quantum efficiency varies with the material of the detector. The main materials are Si for visible light and InGaAs for light between 1000-2000 nm. The only fast photon counting detectors are so far made from Si. For fast CW detection we can use both Si and InGaAs. Practical efficiencies are 85% to 90% at 550-750 nm and at 1060 nm.

Noise contributions:



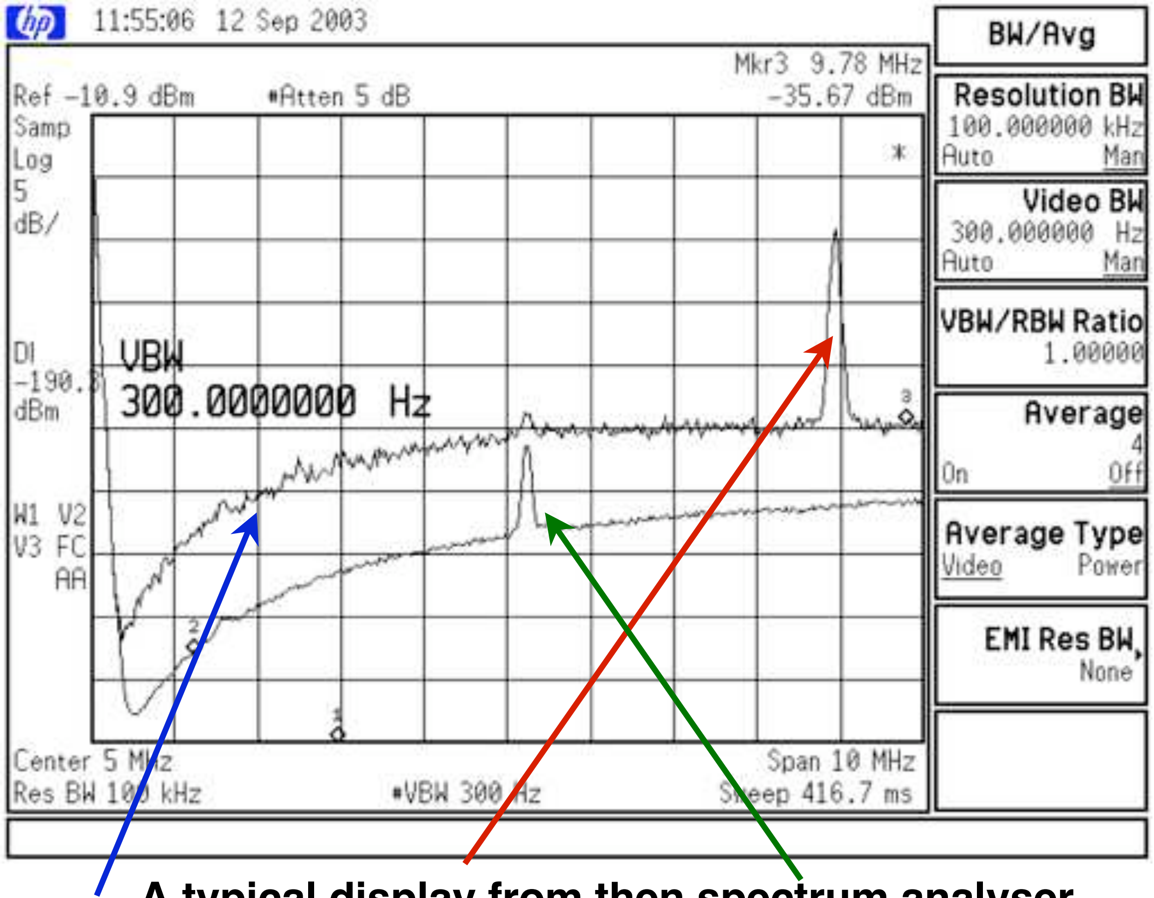
The noise contributions at various stages of the detection and amplification process. The amplifier generates gain noise, and the electronics adds thermal (or Johnson) noise. This illustrates the decline in the ratio of signal (downwards) to noise (upwards).

A real photo detector circuit



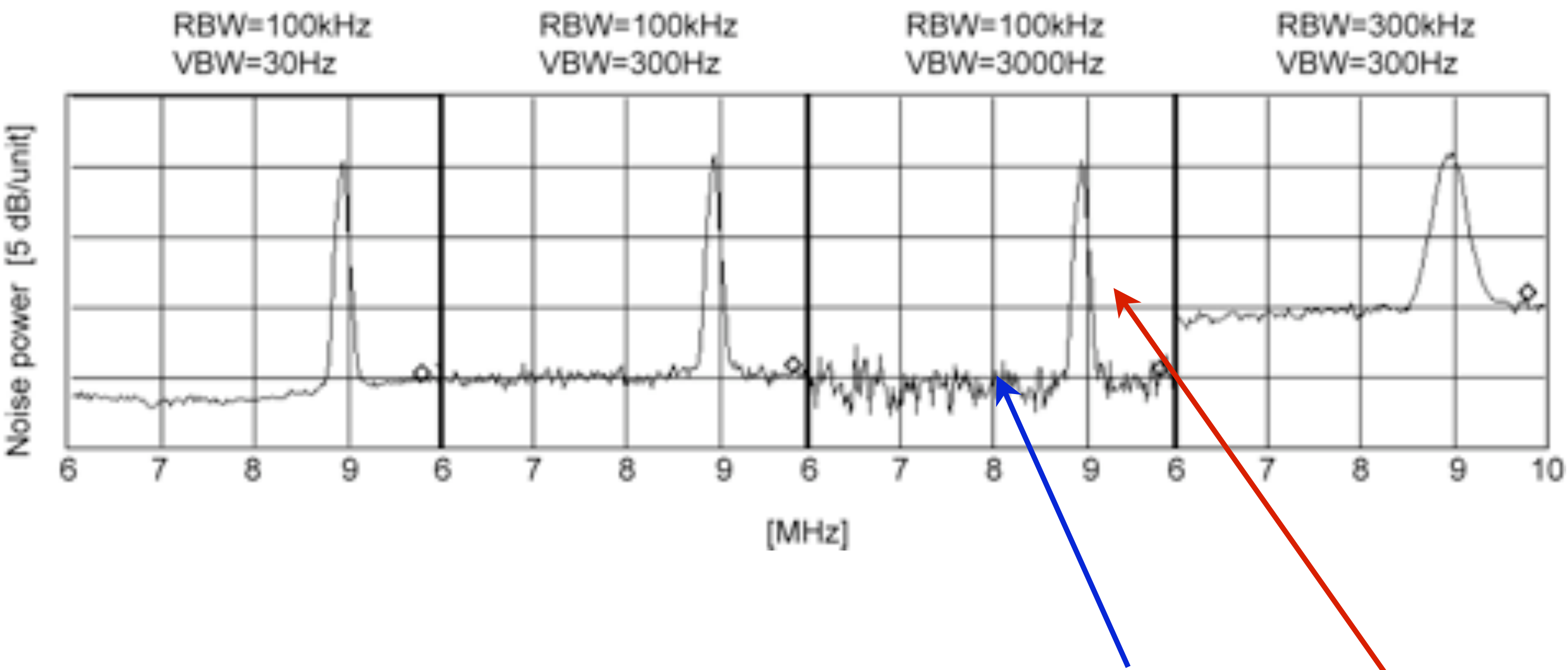
An example of an actual photo detector. For more details see
M.B. Gray, D.A.Shaddock, C.C.Harb, H-A.Bachor, "Photodetector designs for experiments in quantum optics", Rev. of Sci. Instruments 69, 3755 -3762 (1998)

The effect of a spectrum analyser



A typical display from then spectrum analyser
fluctuations and a signal and also pickup of stray radiation.
The apparatus stores all settings

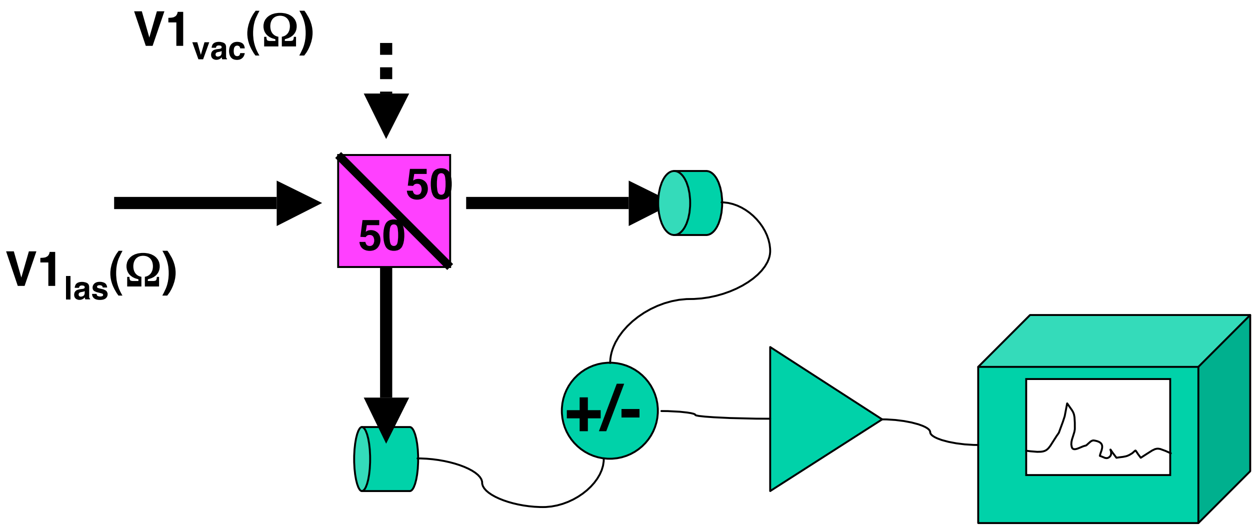
The effect of a spectrum analyser



The spectrum analyser displays the variance of the fluctuations and a signal.

The actual noise level depends on the Resolution Bandwidth (*RBW*) . We cannot make it too large because we are averaging over many spectral components of the light. The display can be made smoother by using a lower Video Bandwidth (*VBW*). However, this averages in time and the experiment has to be very stable.

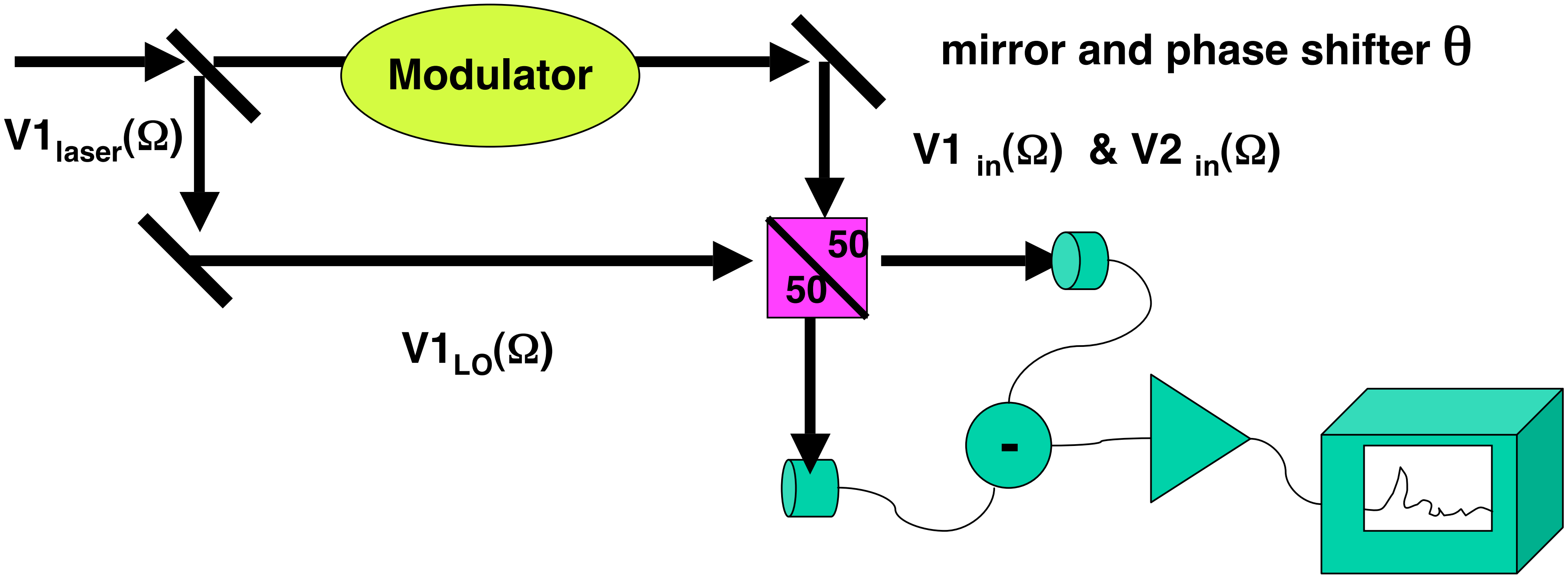
Calibration: Balanced detector



Vi(+) signal and q-noise
Vi(-) only q-noise

$V1_{las}(\Omega)$ = Vi (+) / Vi (-)

Balanced homodyne detector



$$V\theta\ (\Omega) = \sin^2(\theta)\ V1_{in}\ (\Omega) + \cos^2(\theta)\ V2_{in}\ (\Omega)$$

For $\alpha_{lo} >> \alpha_{in}$

we can observe both quadratures, one at a time.
Calibrate by blocking V1in.