Lecture 1

Experimental BEC: Back of the Envelope Calculations

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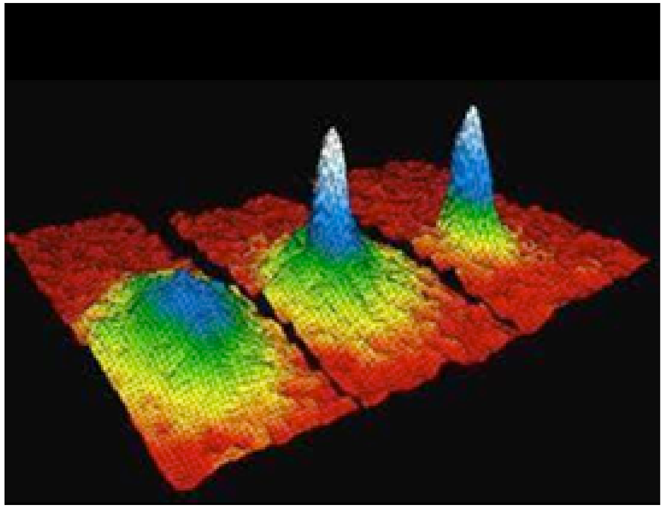
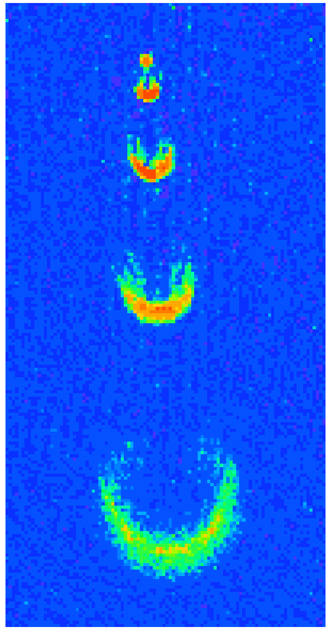
First BEC:JILA



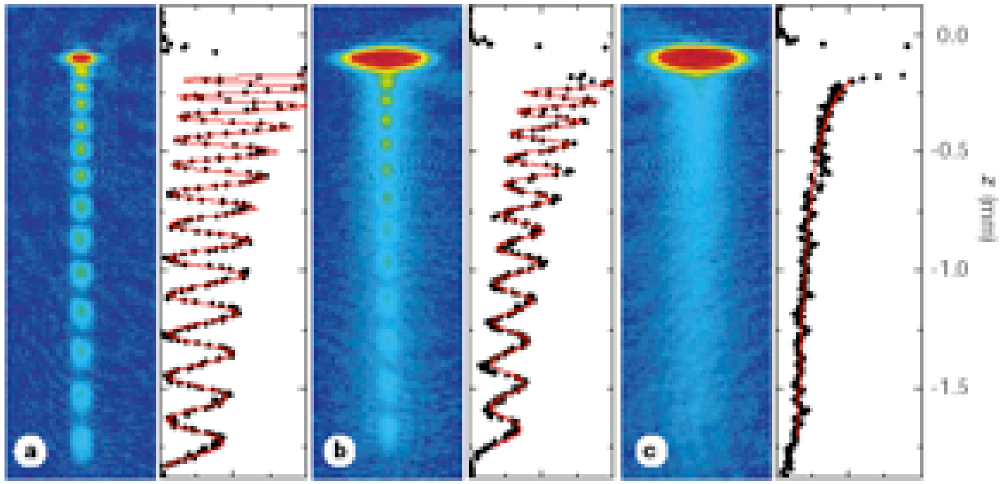
M.H. Anderson et al, Science, 269, 198 (1995).

First Atom Laser:MIT



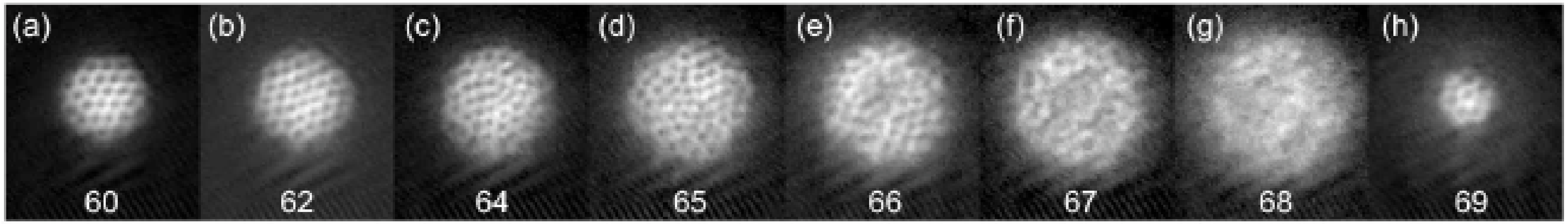
M. O. Mewes et al. ,PRL,**78**, 582 (1997).

Coherence: MPI Munich



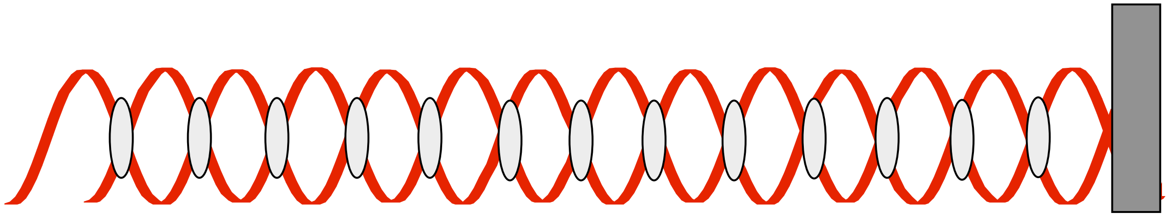
I.Bloch et al.,Nature 403,166, (2000).

Vortices:ENS



Bretin et al.,PRL, **92**, 050403-1 (2004)

Quantum Fields and Coherence

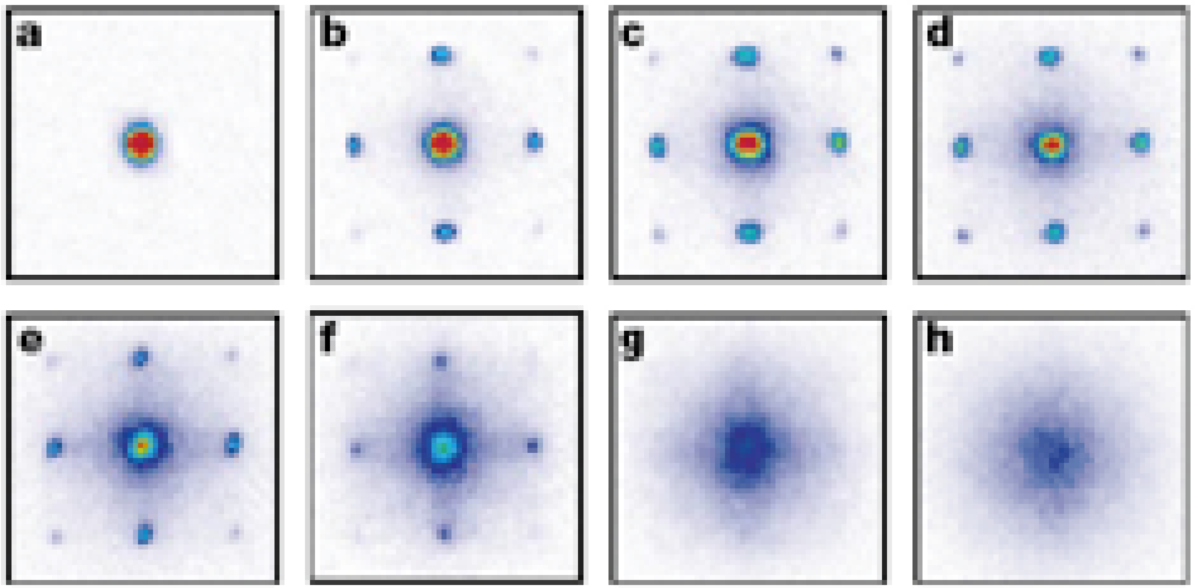


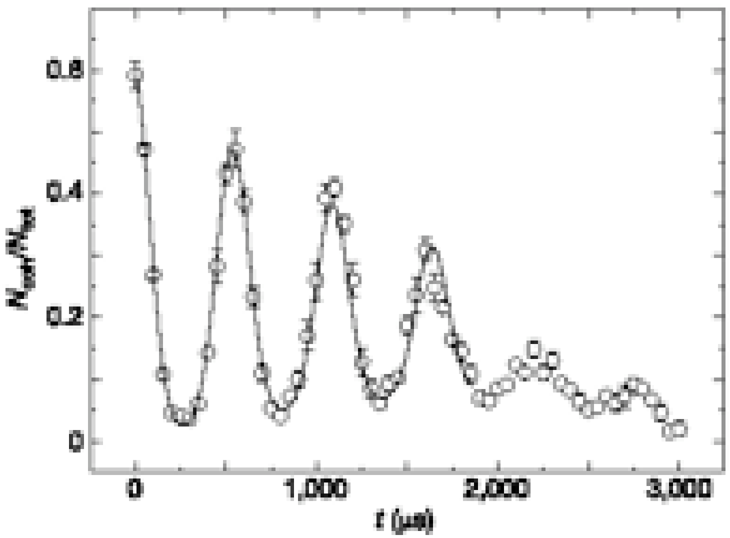
$$H = -J\sum_{i,j}\hat{a}_i^{+}\hat{a}_j + \sum_i \varepsilon_i a_i^{+} a_i + \frac{1}{2}U\sum_i a_i^{+} a_i\left(a_i^{+} a_i - 1\right)$$

$$\left|\Psi\right\rangle = \left|N\right\rangle \; \textit{Fock state}$$

$$\left|\Psi\right\rangle = \exp-\left(\left|\alpha\right|^2\right)\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}\left|n\right\rangle \; \textit{coherent state}$$

MPI Garching:Greiner et al,



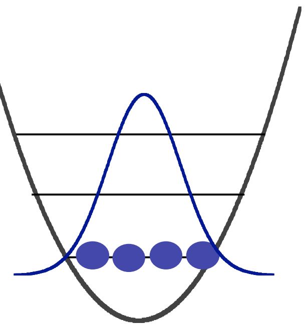


Nature 415, 40 (2002).
Nature 419, 51 (2002).

$$\hat{\Psi}(\vec{r})$$

What is a BEC ? Statistical Mechanics Definition

BEC is the macroscopic occupation of the ground state.



$$p(\varepsilon) = \frac{\exp(-\varepsilon / kT)}{\sum_{all\ states} \exp(-\varepsilon / kT)} \quad Boltzmann$$

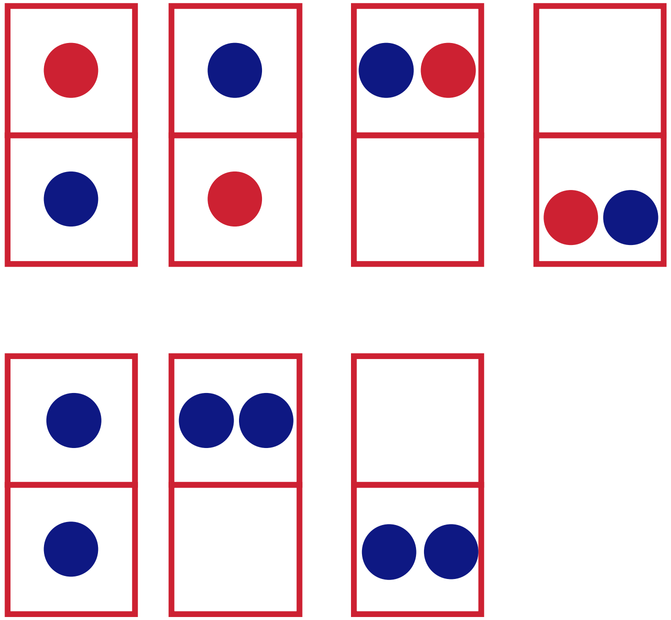
<u>Bose Einstein</u>

$$f(\varepsilon) = \frac{1}{\exp\left(\frac{\varepsilon - \mu}{kT}\right) - 1}$$

$$kT_c = \frac{2\pi\hbar^2}{m}\left(\frac{N}{2.612V}\right)^{\frac{2}{3}}$$

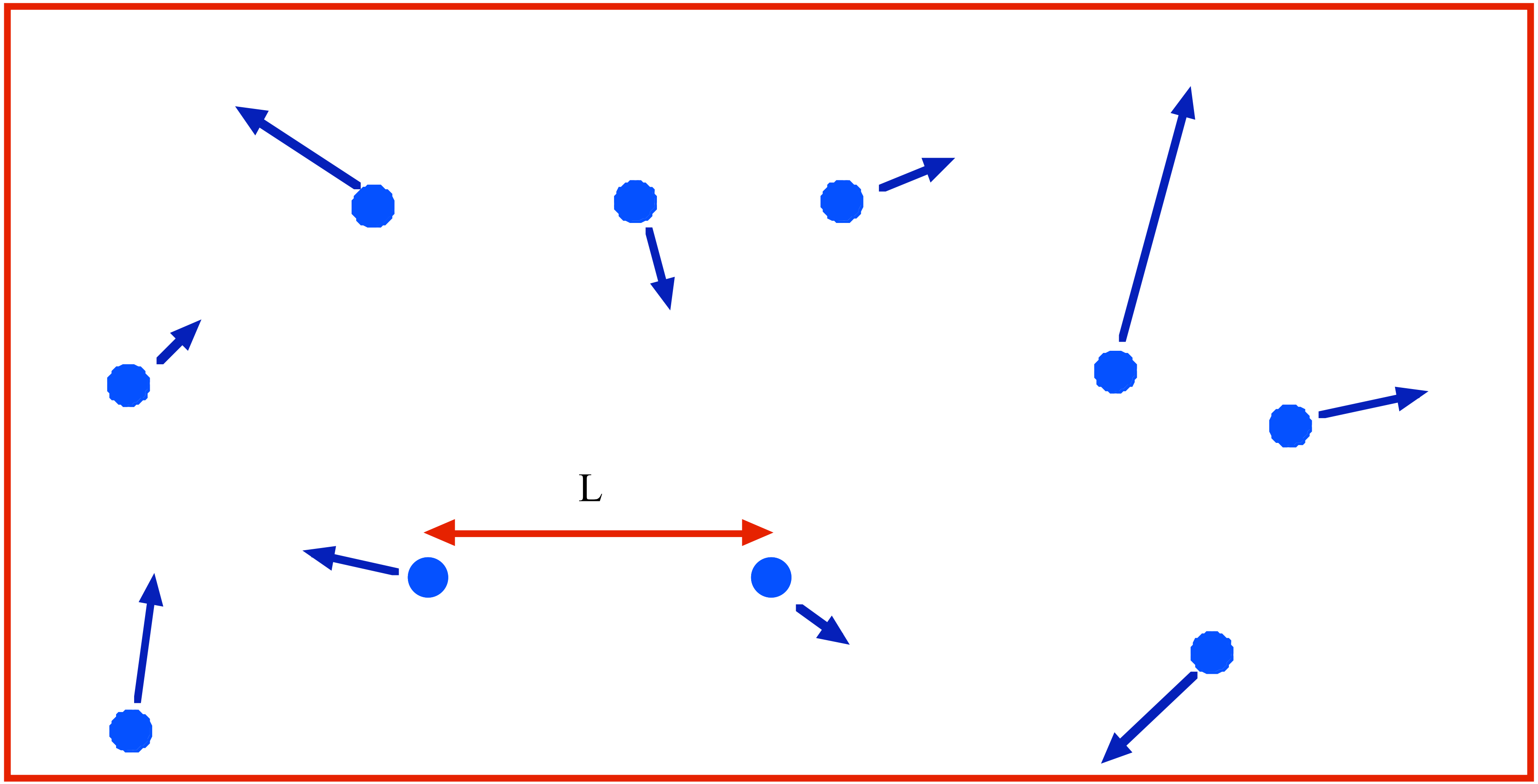
What Drives BEC ?

The Statistics of Indistinguishable Bosons



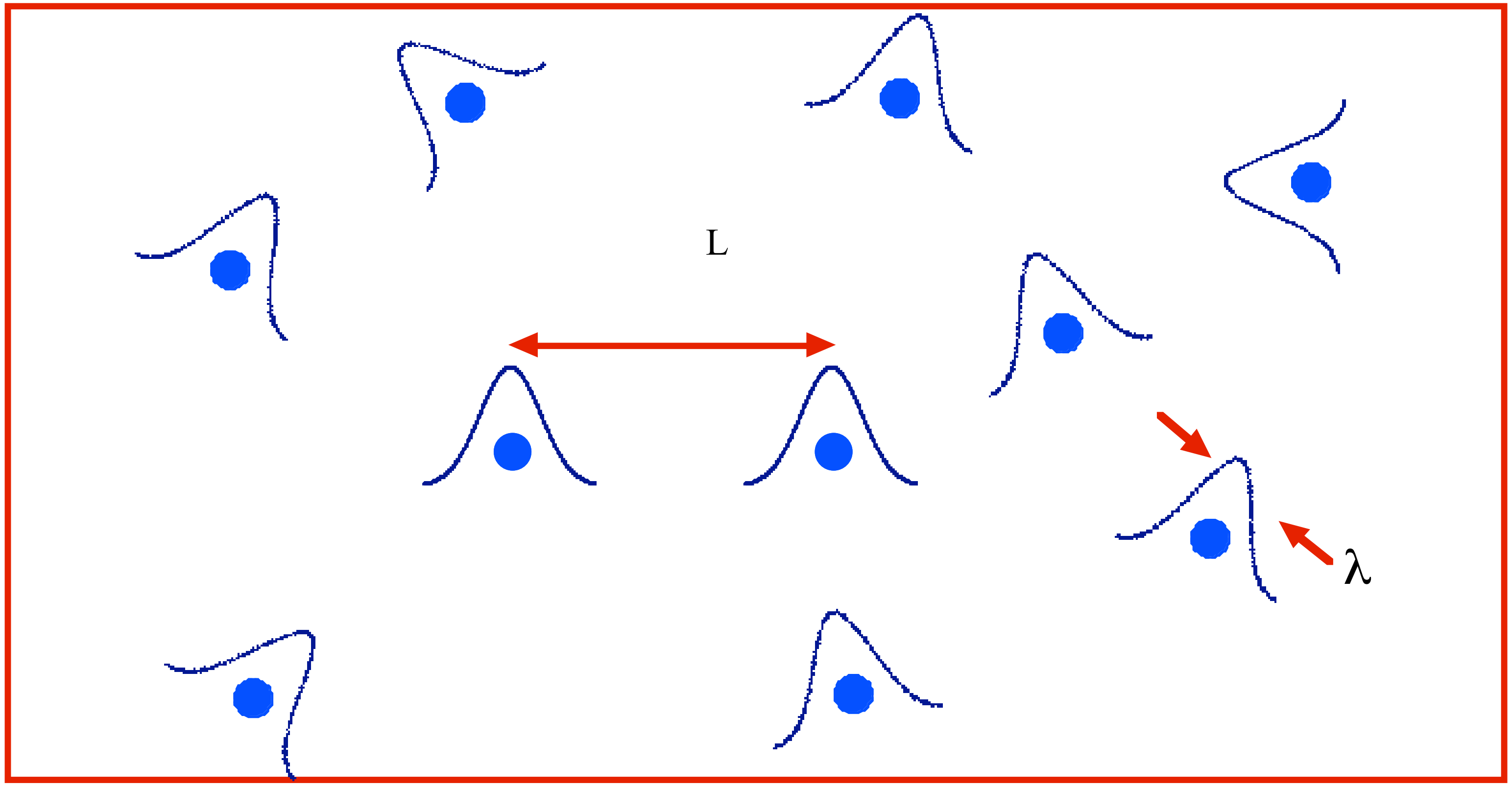
Ratio of the probability of ground state occupancy =4/3

A Classical Gas



$$L = \frac{1}{\sqrt[3]{n}} \qquad V = \sqrt{\frac{2kT}{m}}$$

Gas With Delocalised Atoms



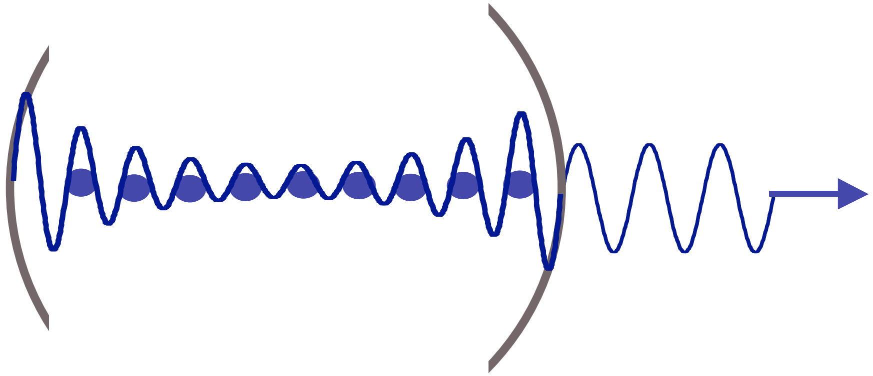
$$\lambda = \frac{\hbar}{p} = \hbar\sqrt{\frac{2\pi}{mkT}}$$

$$L = \frac{1}{\sqrt[3]{n}}$$

Phase space density

$$\rho = \left(\frac{\lambda}{L}\right)^3 = n\left(\frac{2\pi\hbar^2}{mkT}\right)^{\frac{3}{2}}$$

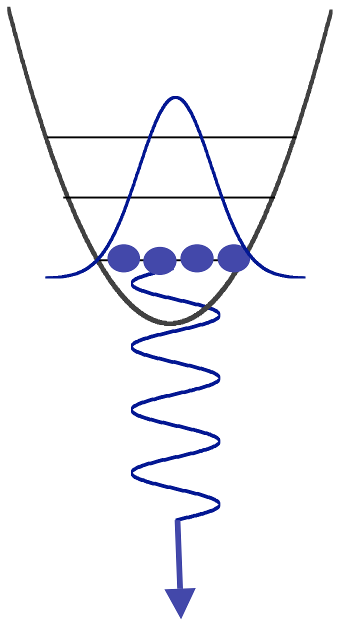
The Optical Laser



- Macroscopic occupation of a cavity mode.

- Bright, coherent,polarised opticalbeam.

The Atom Laser



- Macroscopic population of the ground state trap mode.

- Bright, coherent, polarised deBroglie matter beam.

Atoms and Fields: How it all Works

The simple story

$$\langle T \rangle = -\frac{1}{2}\langle V \rangle$$

$$a_0 = \frac{4\pi\varepsilon_0\hbar^2}{me^2} = 0.5\,\mathring{A}$$ Size of an atom

$$\left\langle \frac{p^2}{2m} \right\rangle = \frac{e^2}{8\pi\varepsilon_0}\left\langle \frac{1}{r} \right\rangle \frac{1}{a_0}$$

$$\Delta E = \frac{\hbar^2}{8ma_0^2} \approx 3eV$$

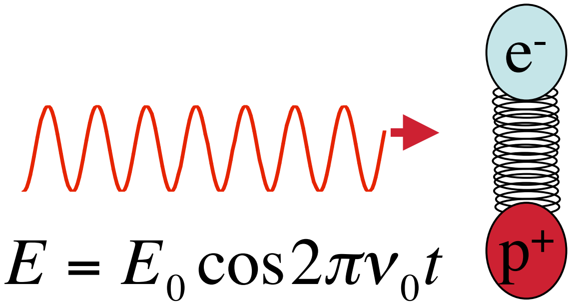
Atoms absorb
In the visible range !

$$\Delta\nu \approx 5 \times 10^{14}\ Hz$$

$$pa_0 \approx \hbar$$

Simple models that will get us a long way

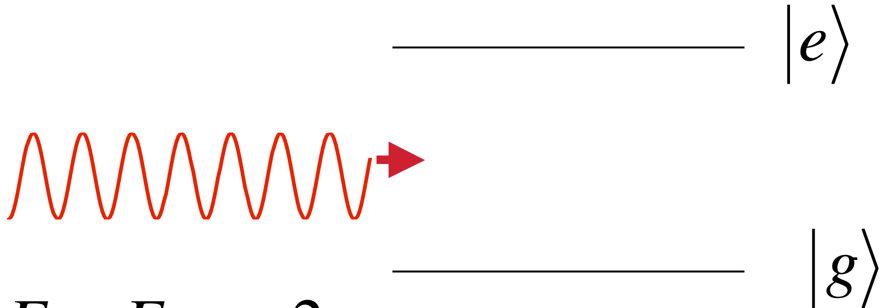
Classical



$$E = E_0 \cos 2\pi\nu_0 t$$

$$\upsilon_0 \approx 10^{15}\,Hz$$

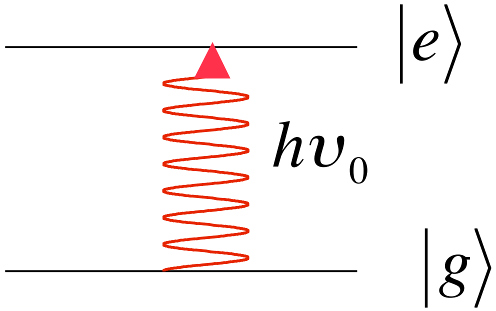
Semi-classical



$$E = E_0 \cos 2\pi\nu_0 t$$

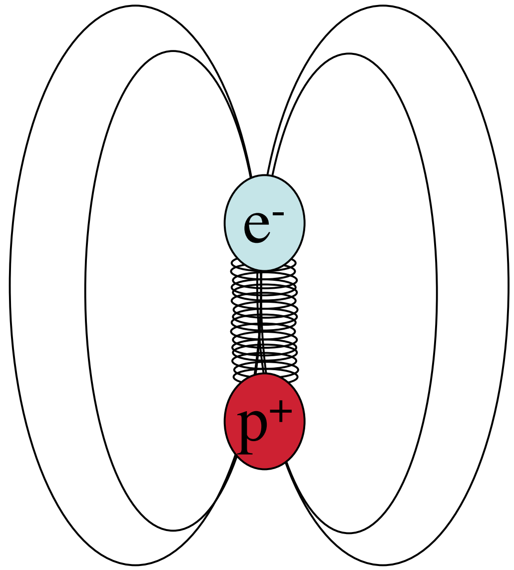
$$\upsilon_0 \approx 10^{15}\,Hz$$

Quantum



$$\upsilon_0 \approx 10^{15}\,Hz$$

How Long Does an Atom Stay in the Excited State ?



$$\textit{power radiated by a classical dipole} = \frac{\wp^2\omega^4}{12\pi\varepsilon_0 c^3}$$

$$\textit{spontaneous emission rate} = \Gamma = \frac{\wp^2\omega^4}{12\pi\varepsilon_0 c^3}\frac{1}{\hbar\omega}$$

$$\Gamma = \frac{\wp^2\omega^3}{12\pi\hbar\varepsilon_0 c} \qquad \Gamma = \frac{\wp^2\omega^3}{3\pi\hbar\varepsilon_0 c^3}$$

So how fast do atoms scatter photons ?

$$\wp \approx ea_0 \approx 8 \times 10^{-29}\, Cm$$

$$\omega \approx 2\pi \times 5 \times 10^{14}\, Hz$$

$$\Gamma \approx 10\, MHz$$

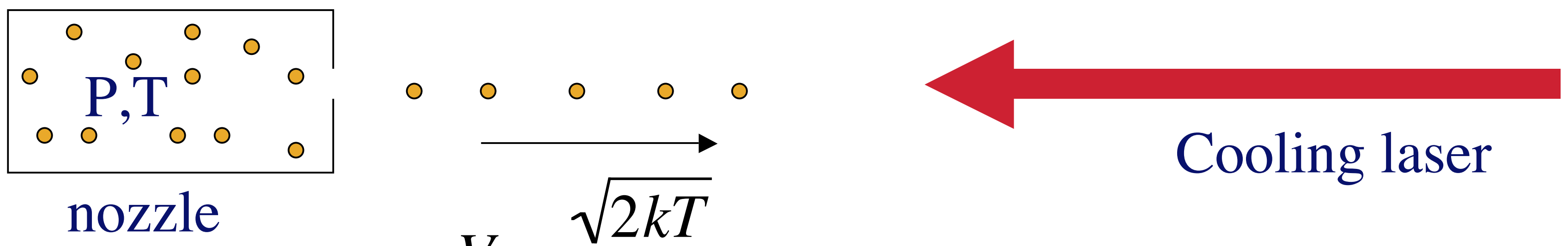
What have we found out so far ?

1) Atoms are roughly $5 x 10^{-11}$ metres in size.

2) Atoms absorb light in the visible spectrum $\left(\nu \approx 10^{15}\, Hz\right)$

3) Atoms can scatter roughly 10^7 photons per second.

How long does it take to stop a room temperature atom ?



$$V \approx \frac{\sqrt{2kT}}{m}$$

$$a = \frac{F}{m} = \frac{\Gamma \hbar k}{m} = \frac{\Gamma h \nu}{mc} \approx 2 \times 10^{5}\, ms^{-2}$$

$$t_{stop} = \frac{V}{a} = \frac{1}{a} \underbrace{\sqrt{\frac{2kT}{m}}}_{300 m/s} \approx 10^{-3}\, s$$

$$x_{stop} \approx 1m$$

What Intensity do we need in our laser to achieve this cooling ?

$$I_{saturation} = 10^7 \; photons / \underbrace{photon\ scattering\ cross\ section}_{\sigma} / second$$

$$\sigma \approx \pi a_0^2 \approx 10^{-21} m^2 \, ?$$

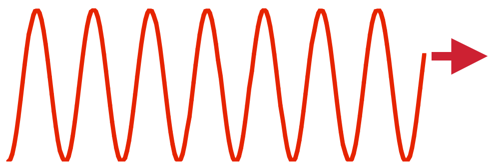
$$I_{saturation} = 10^{28} \, photons / m^2 / s$$

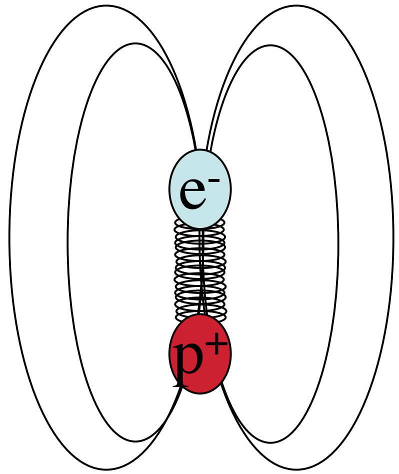
$$1 \, visible\ photon \approx 3 \times 10^{-19} J \approx 2eV$$

$$1 \, Watt\ of\ visible\ photons \approx 3 \times 10^{18} \, photons/second$$

$$I_{saturation} = 10^{28} \, photons / m^2 / s = 300 kW / cm^2$$

Calculate the scattering cross section from a driven dipole





$$m\ddot{x} + \alpha\dot{x} + \frac{1}{2}m\omega^2 x^2 = E_0 e^{i\omega t}$$

Solve the equation of motion above and calculate the power dissipated by the dipole in terms of α. Set this equal to the power radiated by the dipole to determine α.Then use:

$$\sigma = \frac{P_{radiated}}{I_{incident}} = \frac{2}{c\varepsilon_0 E_0^2} P_{radiated} = \frac{3\lambda^2}{2\pi} \approx 10^{-13} m^2$$

What Intensity do we need from our laser to achieve this cooling ?

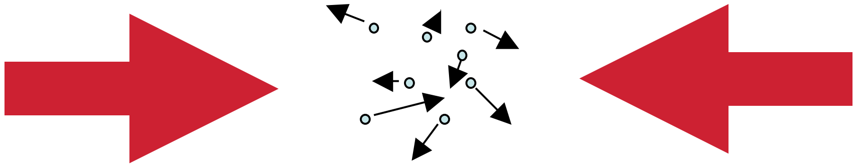
$$I_{saturation} = 10^7 \, photons/\underbrace{photon\ scattering\ cross\ section}_{\sigma}/second$$

$$\sigma = \frac{3\lambda^2}{2\pi} \approx 10^{-13} m^2 \, ?$$

$$I_{saturation} = 10^{20} \, photons/m^2/s$$

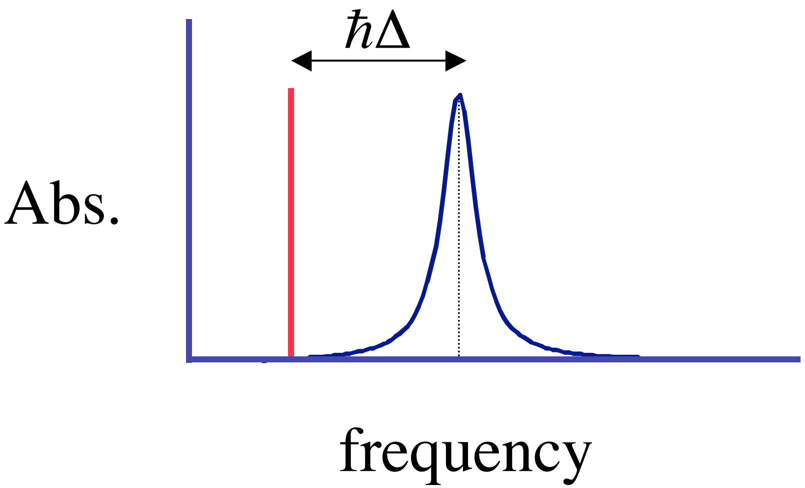
$$I_{saturation} = 10^{20} \, photons/m^2/s = 3mW/cm^2$$

What is the limit to this kind of cooling ?



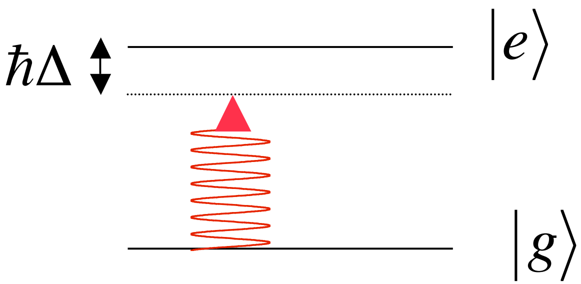
$$Force = F(V,k,\Delta)$$

$$F = \hbar\frac{\Delta}{\Gamma}k^2V$$



$$\left(\frac{dE}{dt}\right)_{cool} = FV = \frac{2\hbar k^2}{M}E$$

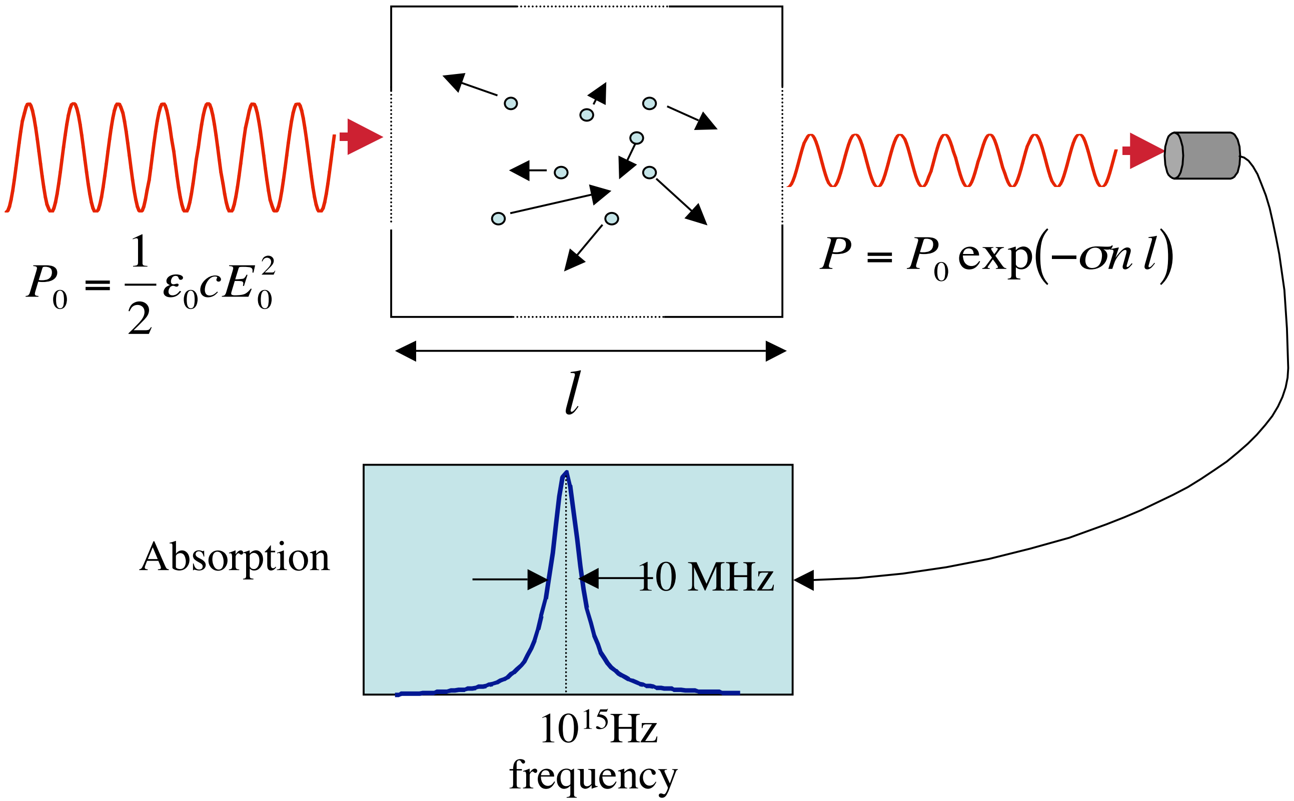
$$\left(\frac{dE}{dt}\right)_{heat} = \frac{1}{2M}\frac{d\langle p^2\rangle}{dt} = \frac{\Gamma(\hbar k)^2}{2M}$$



$$E_{\min} = \frac{\hbar\Gamma}{4} \qquad T_{\min} = \frac{\hbar\Gamma}{4k_b}$$

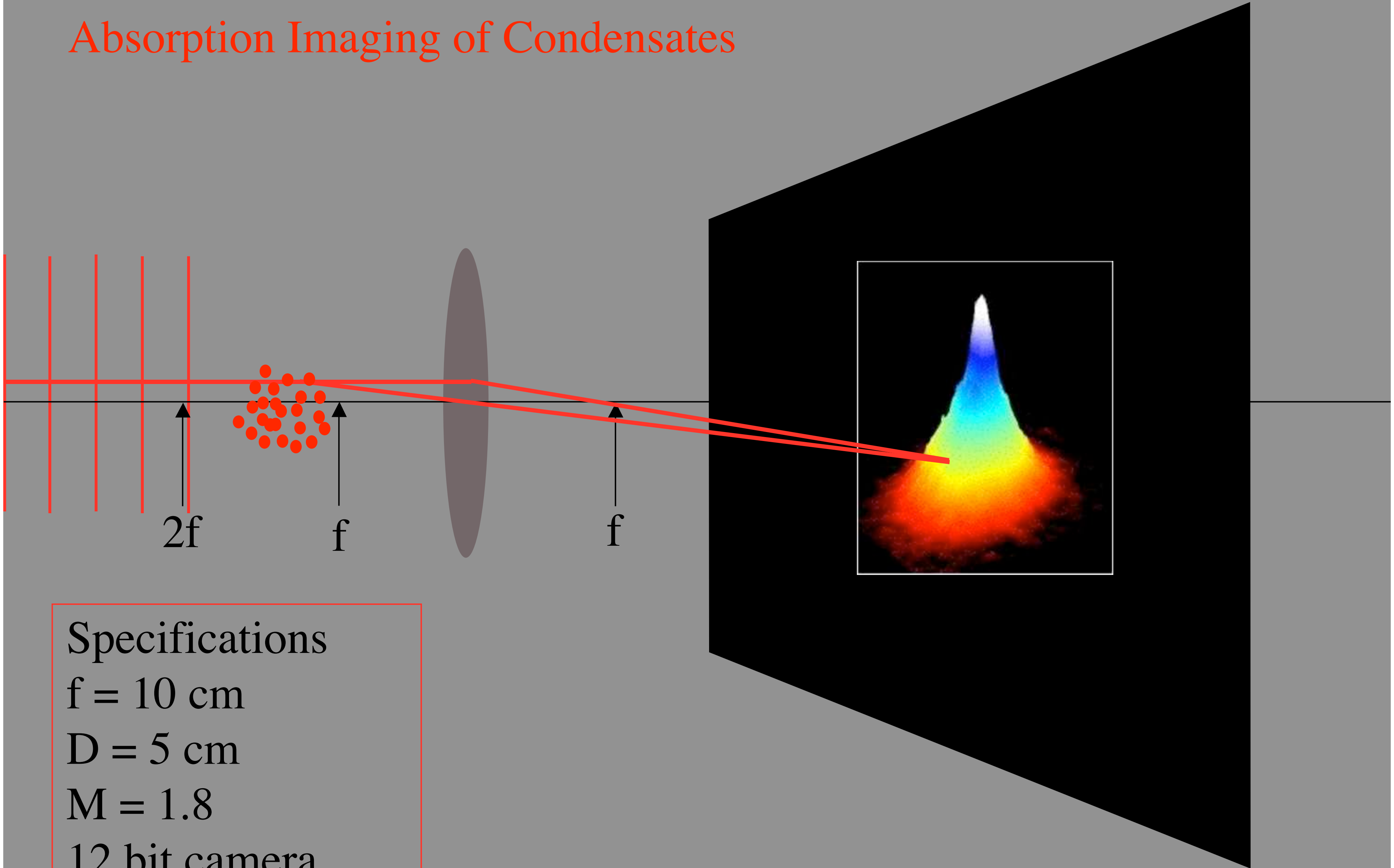
$$T_{\min} = \frac{\hbar\Gamma}{2k_b} \approx 100\mu K \;\; for\ \Gamma = 10\,MHz$$

Absorption Imaging



$$n = \frac{\ln P - \ln P_0}{\sigma l}$$

Absorption Imaging of Condensates



Specifications
f = 10 cm
D = 5 cm
M = 1.8
12 bit camera
9µm x 9µm pixels

We have established the following

Atoms:

-are on the order of 10^{-10} m in size.

-absorb in the visible range.

-can scatter roughly 10^{7} photons per second.

-have a photon scattering cross section of roughly $10^{-13} m^2$

-can be laser cooled to a temperature on the order of 100 μK in a time on the order of milliseconds by laser beams with intensities on the order of mW/cm^2. The cooling laser linewidth should be 1MHz or less.