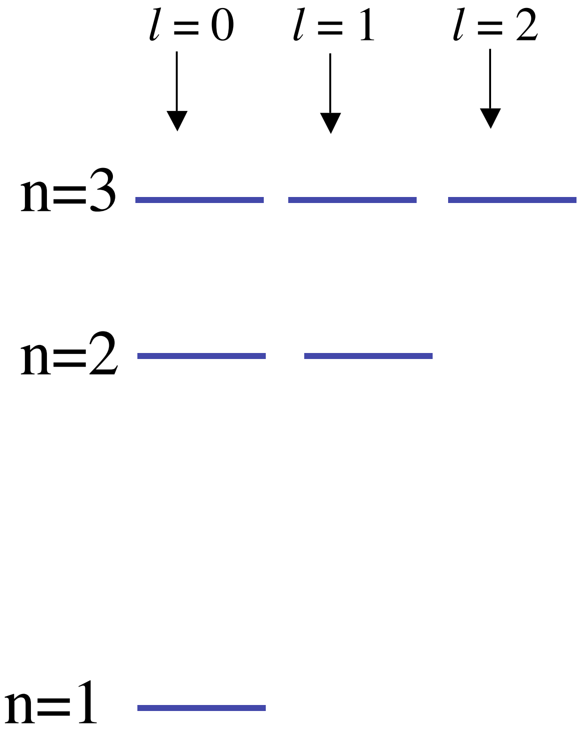
Real Atoms

$$H = \frac{\hat{p}^2}{2m_e} - \frac{e^2}{4\pi\varepsilon_0 r}$$



Good quantum numbers

n,l,m_l,s,m_s

Eigenstates

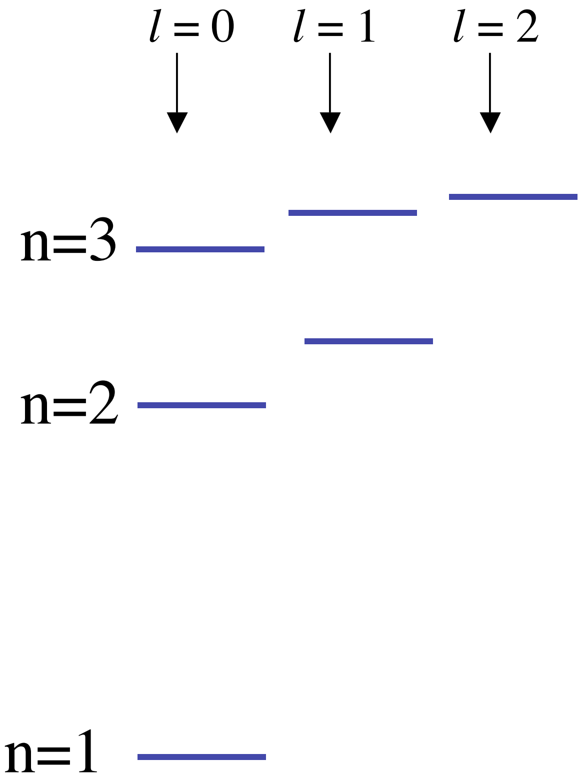
$$\Psi_{n,l,m}(\vec{r}) = R_{n,l}(r) Y_l^m(\theta,\phi)$$

Energy eigenvalues

$$E_n = -\frac{1}{2} mc^2 \frac{\alpha}{n^2} = \frac{-13.6eV}{n^2}$$

Alkali Atoms

$$H = \frac{\hat{p}^2}{2m_e} + U(r) - \frac{Ze^2}{4\pi\varepsilon_0 r}$$



Good quantum numbers

n,l,m_l,s,m_s

Eigenstates

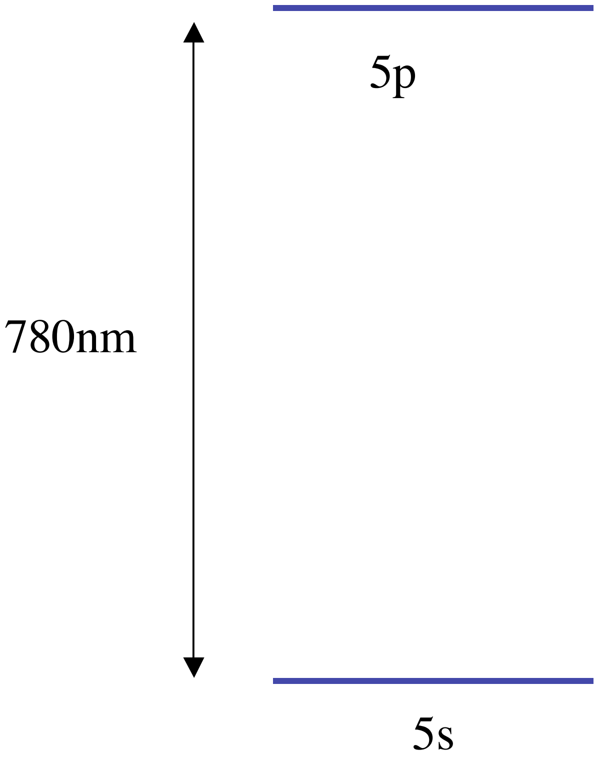
$$\Psi_{n,l,m}\left(\vec{r}\right) = R_{n,l}\left(r\right)Y_l^m\left(\theta,\phi\right)$$

Energy eigenvalues

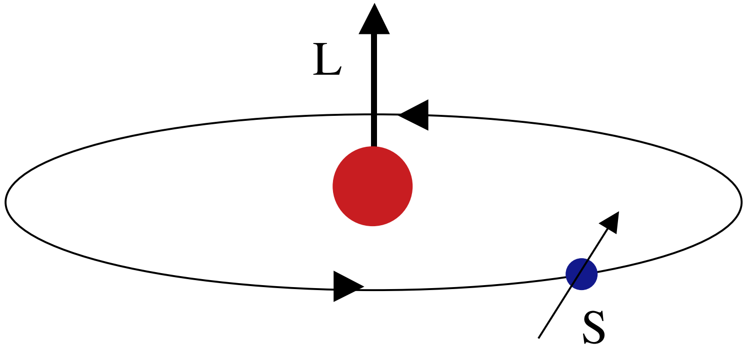
For quantum chemists to calculate

Rubidium

$$H = \frac{\hat{p}^2}{2m_e} + U(r) - \frac{Ze^2}{4\pi\varepsilon_0 r}$$



Spin Orbit Coupling



$$H = -\vec{\mu} \bullet \vec{B}$$

$$H_{so} \propto \vec{S} \bullet \vec{L}$$

$$\vec{J} = \vec{L} + \vec{S}$$

$$J^2 \left| J, M_J, L, S \right\rangle = J(J+1)\hbar^2 \left| J, M_J L, S \right\rangle$$

$$J_z \left| J, M_J, L, S \right\rangle = M_J \hbar \left| J, M_J, L, S \right\rangle$$

$$L \bullet S = \frac{1}{2}\left(J^2 - L^2 - S^2\right)$$

$$H_{so} \propto J^2 - L^2 - S^2$$

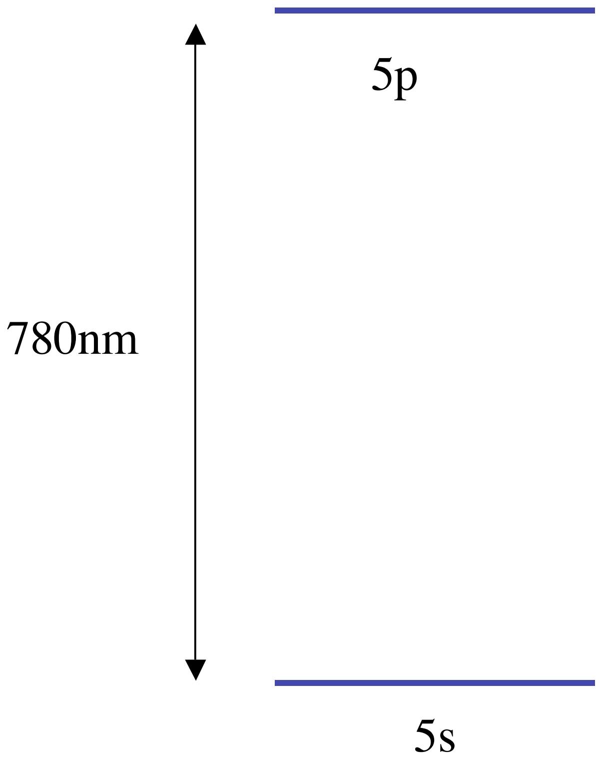
Relativistic correction

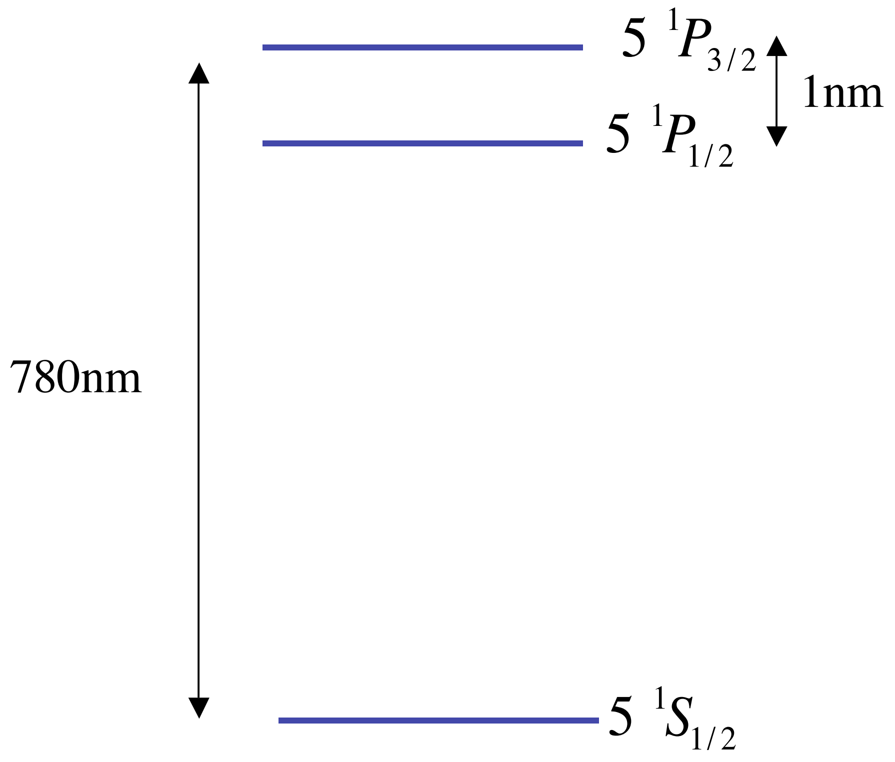
$$E_k = \sqrt{c^2 p^2 + m_e^2 c^4}$$

$$E_k = m_e c^2 + \frac{p^2}{2m} - \frac{p^4}{8 m_e^3 c^2} + \bullet\bullet\bullet$$

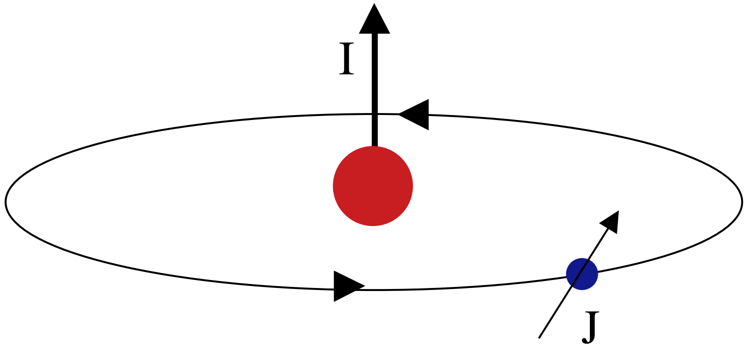
Fine Structure

$$H \approx \underbrace{\frac{\hat{p}^2}{2m_e} - \frac{Ze^2}{4\pi\varepsilon_0 r}}_{\text{hydrogen like}} + \underbrace{\frac{e^2}{4\pi\varepsilon_0}\sum_{i=1}^{N-1}\int d^3 r_i \frac{\left|\Psi(r_i)\right|^2}{\left|\vec{r}_i - \vec{r}\right|}}_{\text{core electrons}} + \underbrace{\frac{e^2}{8\pi\varepsilon_0 m_e^2 c^2}\frac{1}{r^3}\vec{S}\bullet\vec{L}}_{\text{spin orbit}} - \underbrace{\frac{p^4}{8m_e^3 c^2}}_{\text{relativistic}}$$





Hyperfine Interaction

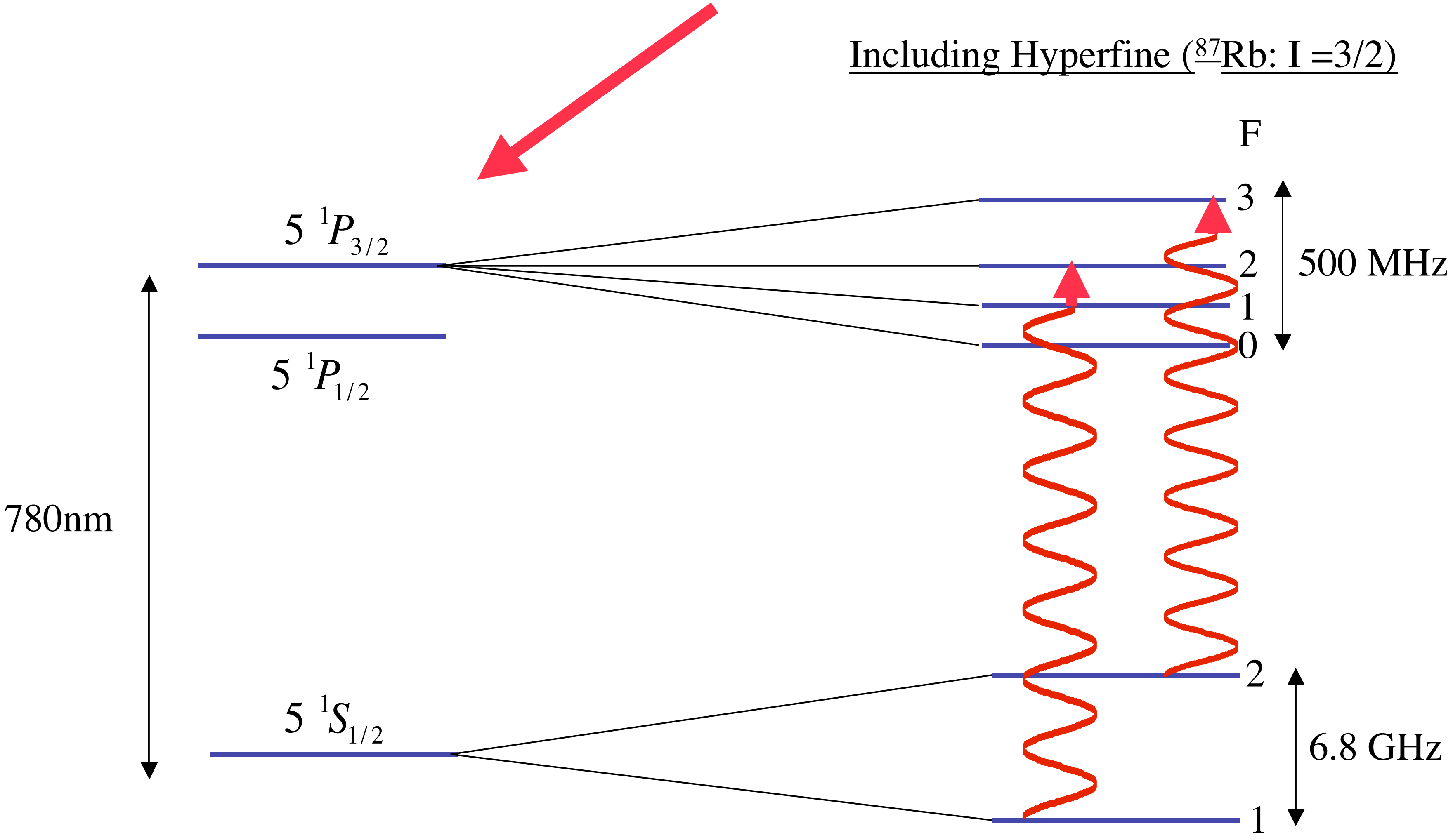


$$H_{HF} = A(J) J \bullet I$$

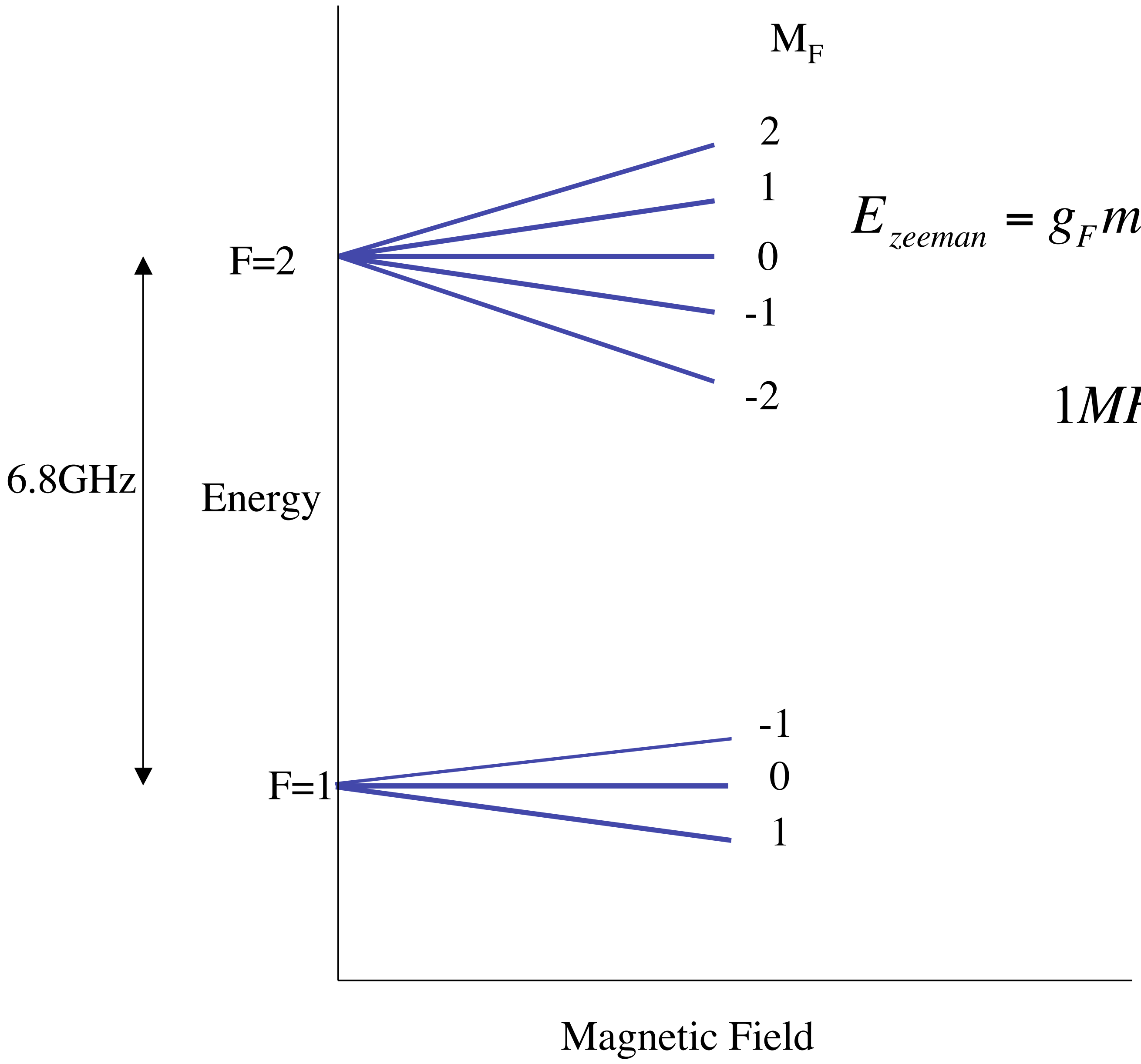
$$F = J + I$$

$$H_{HF} = A(J)\left(F^2 - J^2 - I^2\right)$$

$$H = \underbrace{\underbrace{\frac{\hat{p}^2}{2m_e} - \frac{Ze^2}{4\pi\varepsilon_0 r}}_{hydrogen\ like} + \underbrace{\frac{e^2}{4\pi\varepsilon_0}\sum_{i=1}^{N-1}\int d^3r_i \frac{\left|\Psi(r_i)\right|^2}{\left|\vec{r}_i - \vec{r}\right|}}_{core\ electrons} + \underbrace{\frac{e^2}{8\pi\varepsilon_0 m_e^2 c^2}\frac{1}{r^3}\vec{S}\bullet\vec{L}}_{spin\ orbit} - \underbrace{\frac{p^4}{8m_e^3c^2}}_{relativistic}} + \underbrace{\xi J \bullet I}_{hyperfine}$$



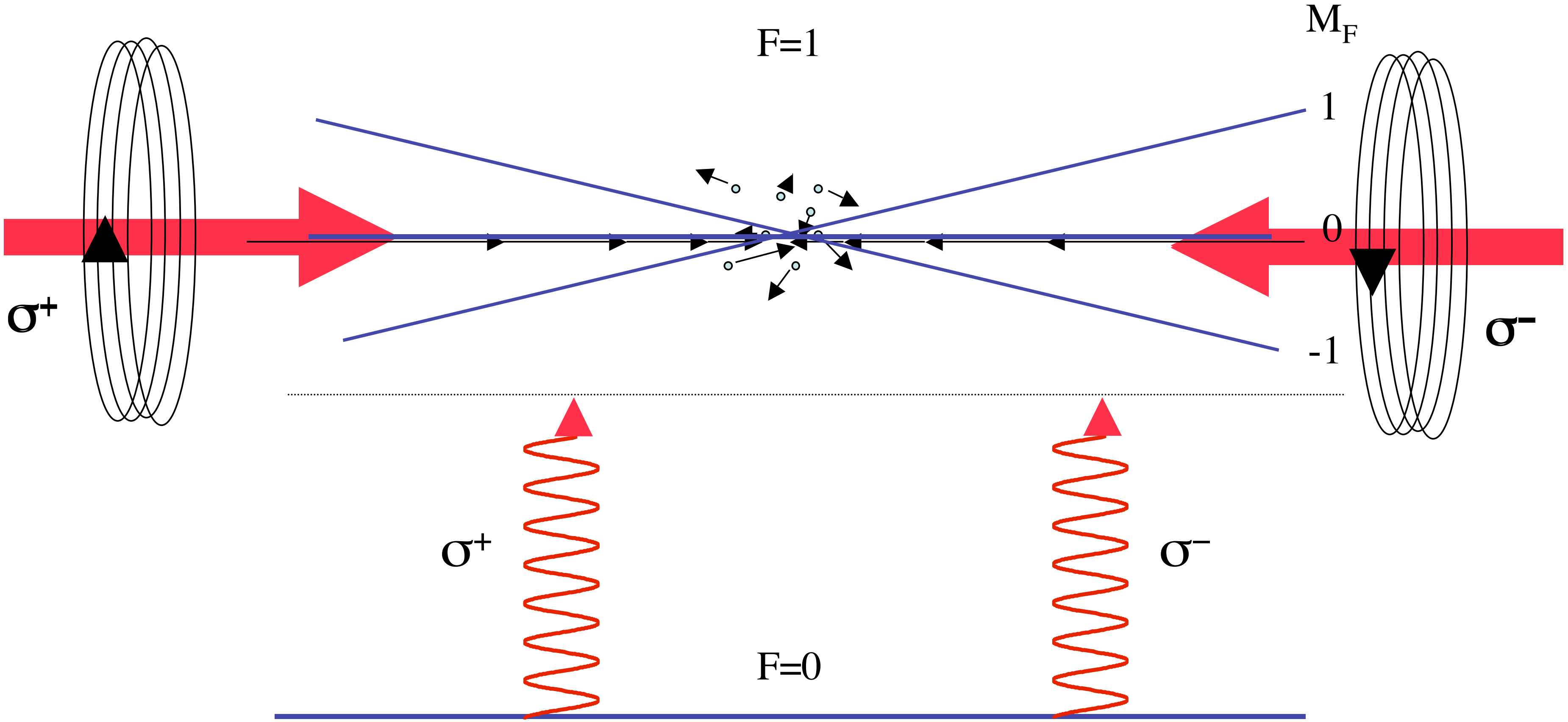
The Zeeman Effect and Magnetic Trapping



$$E_{zeeman} = g_F m_F \mu_b B \approx 1MHz/Gauss$$

$$1MHz \approx 100\mu K$$

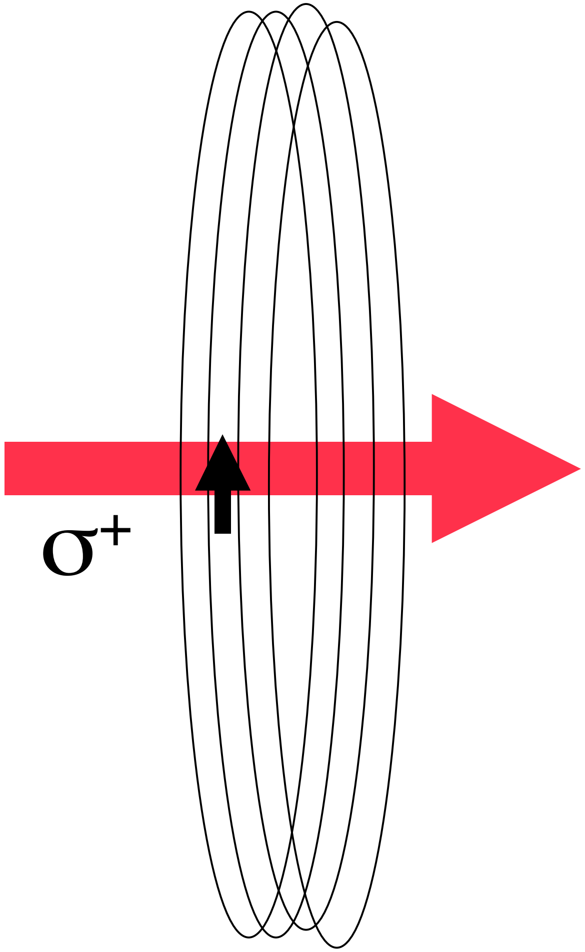
The Magneto Optic Trap

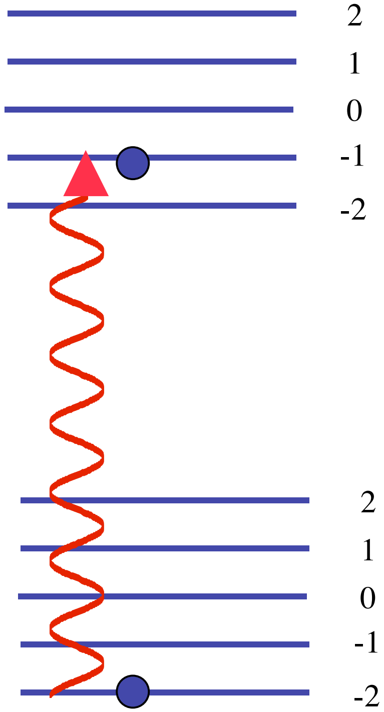


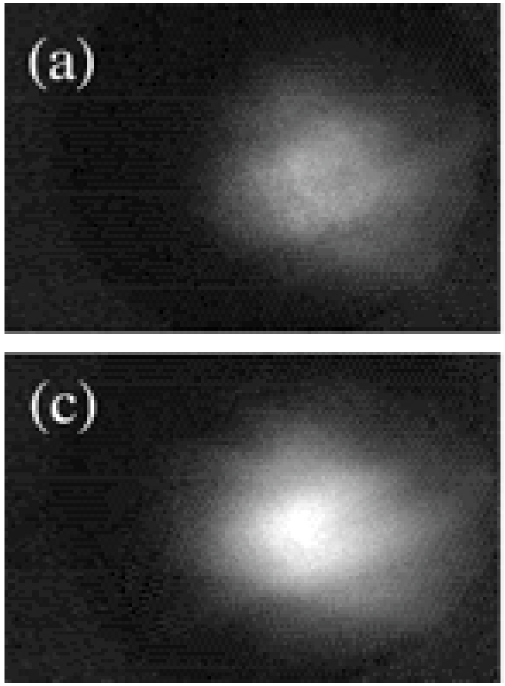
Typical Numbers

$n=10^{10}$ cm^{-3}
T=200 μK
$\rho=10^{-6}$

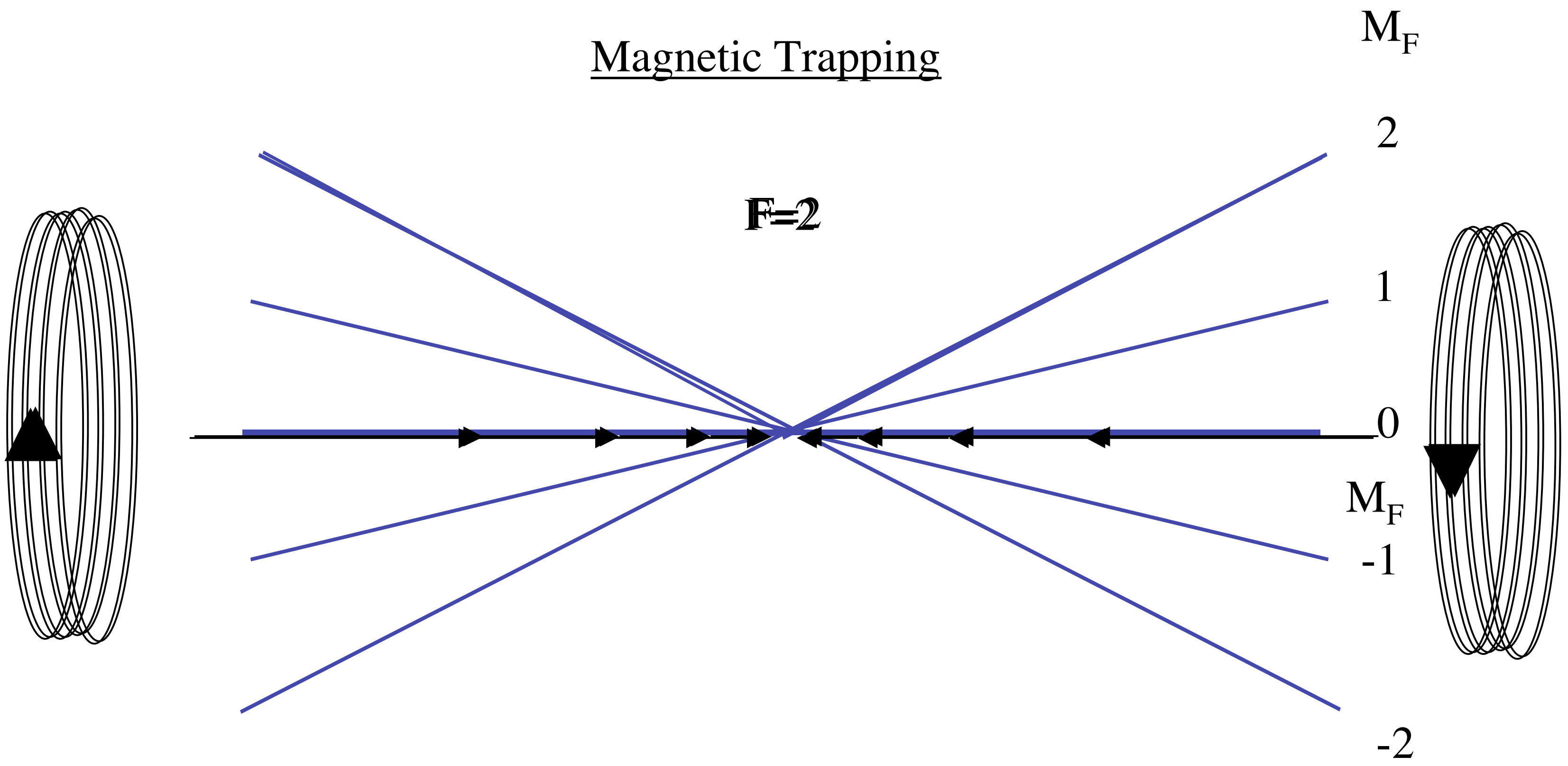
Optical Pumping





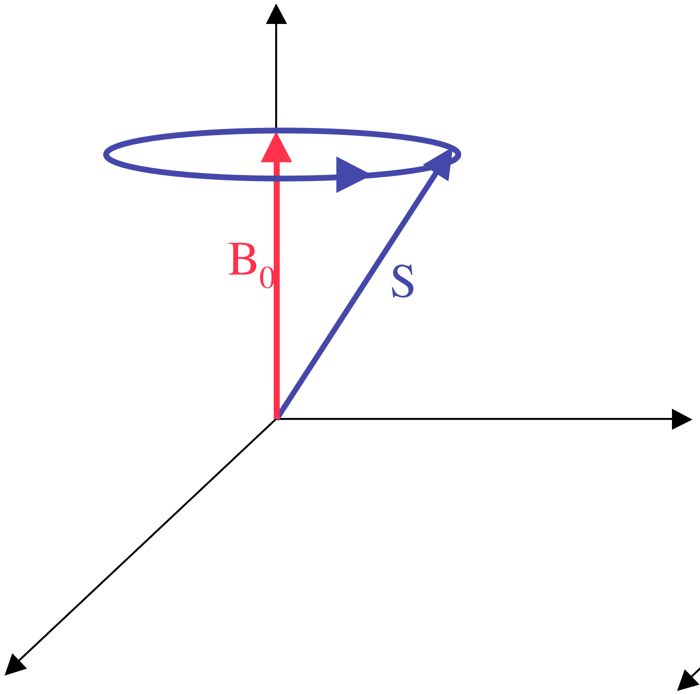


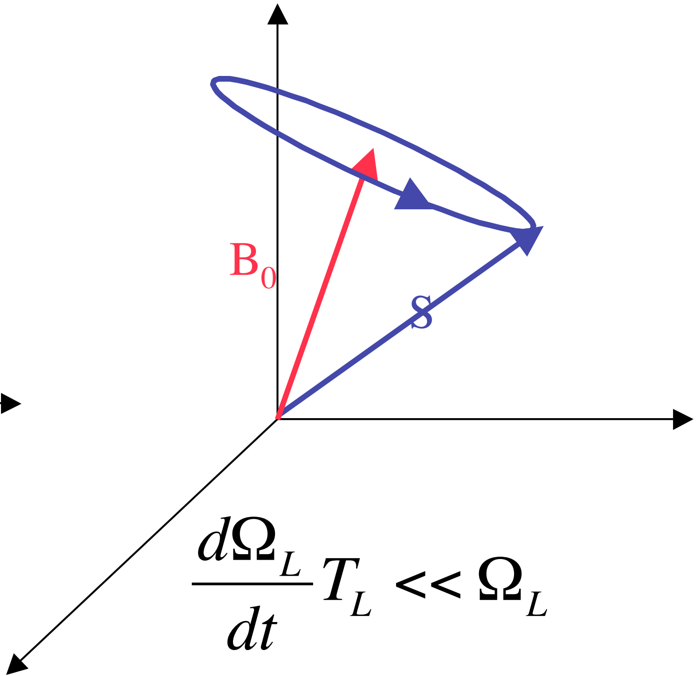
Magnetic Trapping



$$U(\vec{r}) = g_F m_F \mu_b \left|B(\vec{r})\right| \approx 1 MHz / Gauss$$

Classical Spin in a field





$$\frac{d\vec{S}}{dt} = \vec{\mu} \times \vec{B} = \gamma \vec{S} \times \vec{B}$$

The spin precesses around $\mathbf{B_0}$ at the Larmour precession frequency

$$\Omega_L = \gamma B_0 = g_F m_F \mu_b B \approx 1 MHz / Gauss$$

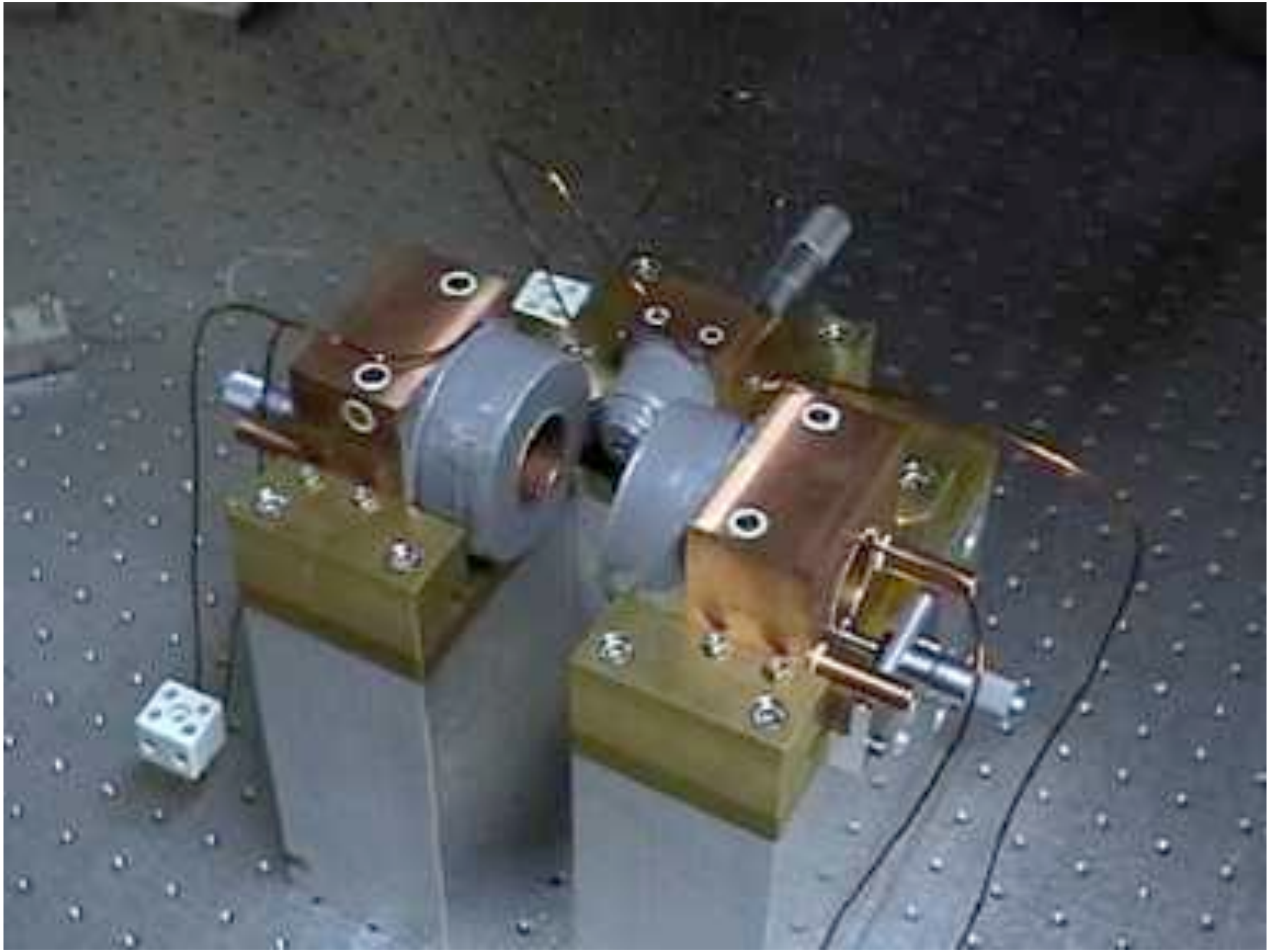
$$\frac{d\Omega_L}{dt} T_L << \Omega_L$$

$$\frac{d\Omega_L}{dt} \frac{1}{\Omega_L} << \Omega_L$$

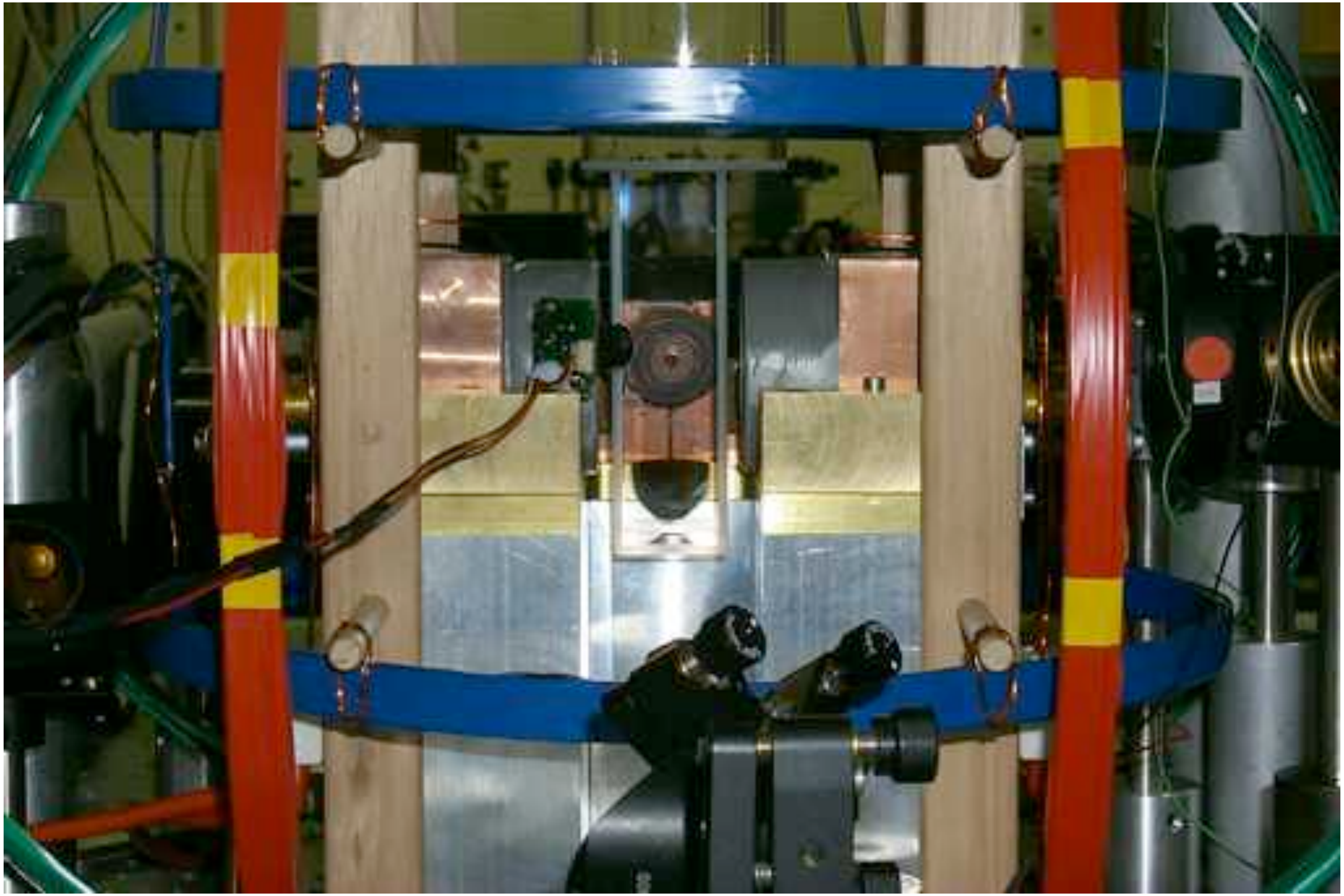
$$\frac{d\Omega_L}{dt} << \Omega_L^2$$

$$\left(V \bullet \nabla\right) B << \gamma B^2$$

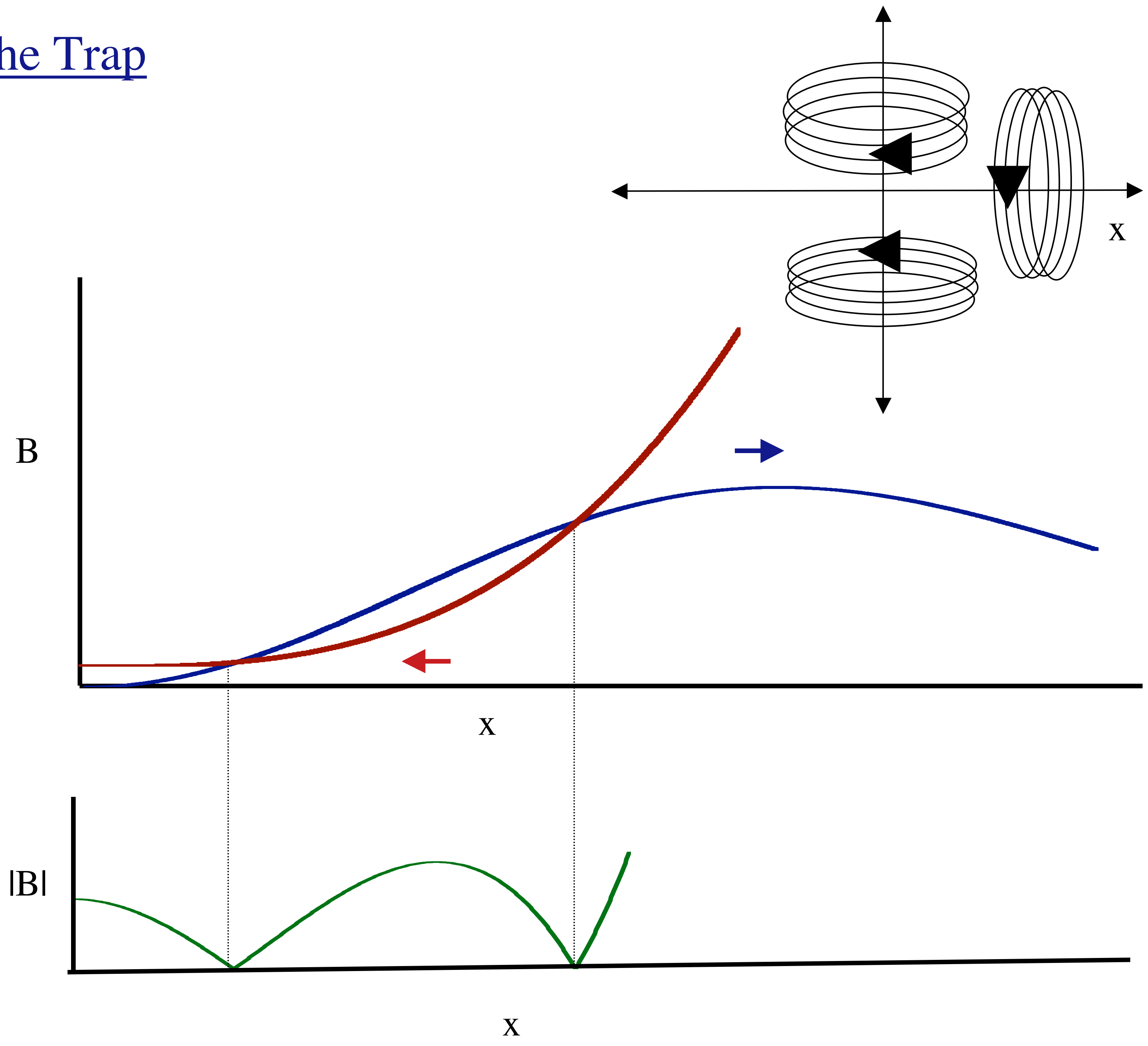
The Magnetic Trap



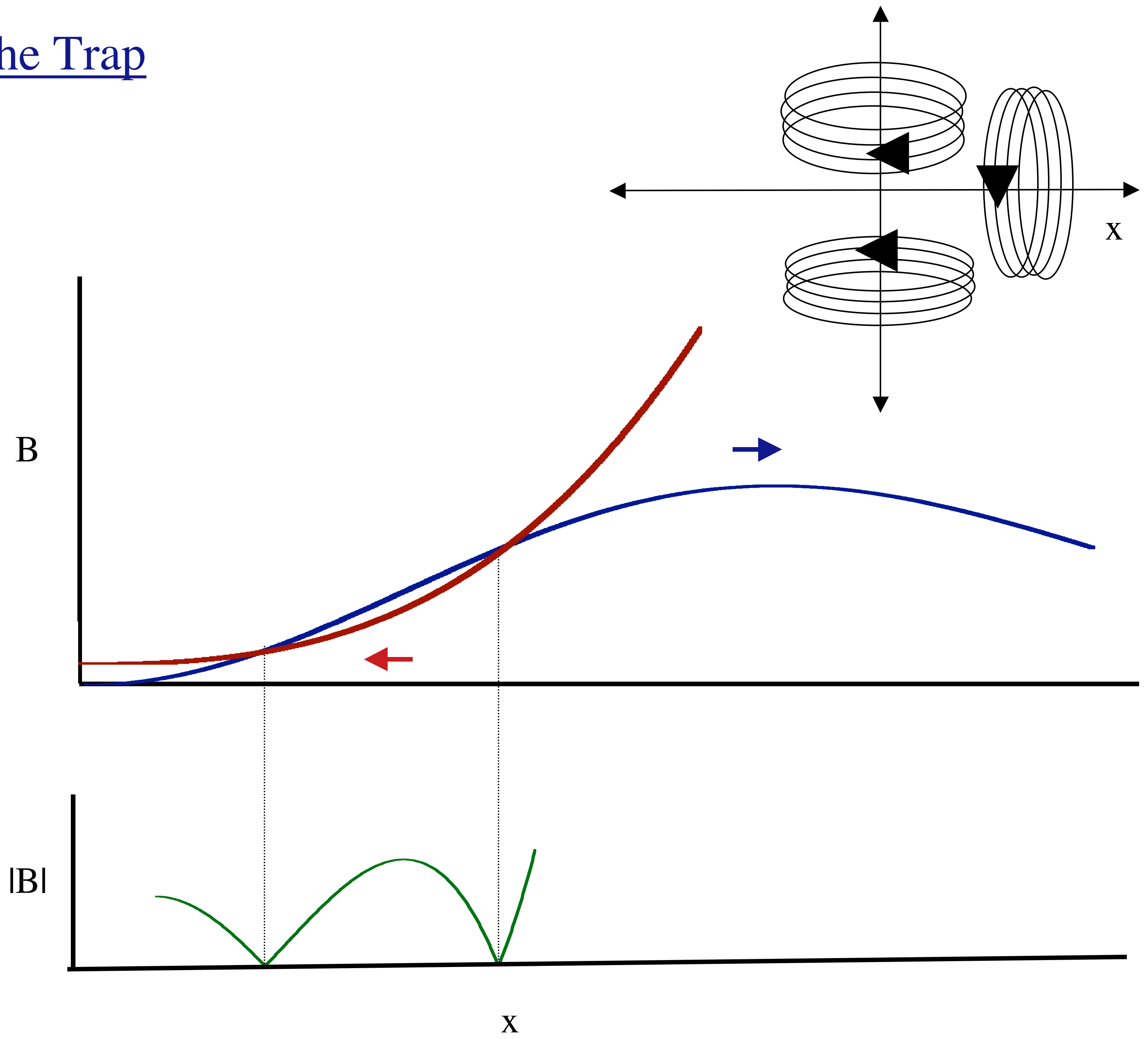
The trap



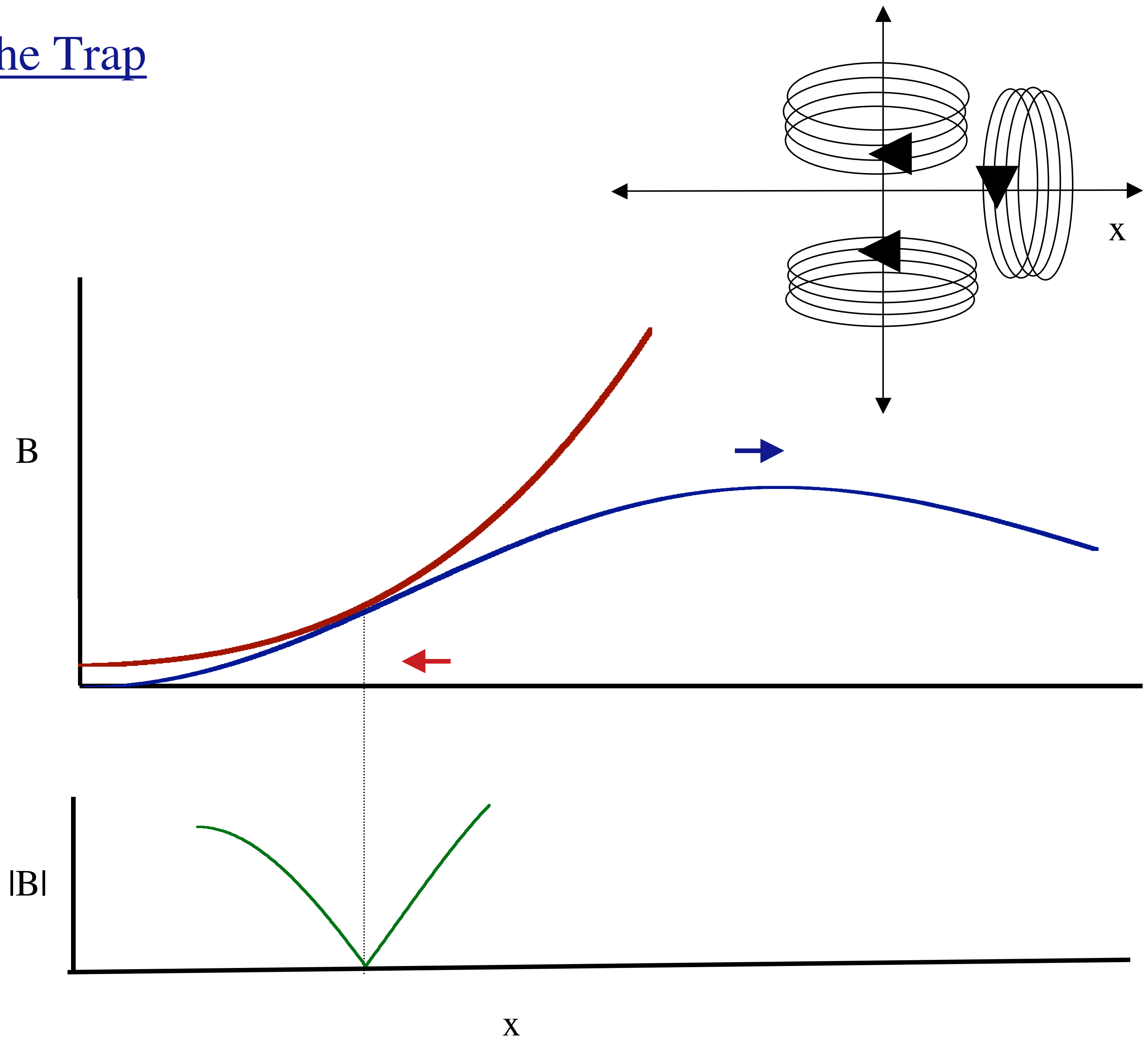
Forming the Trap



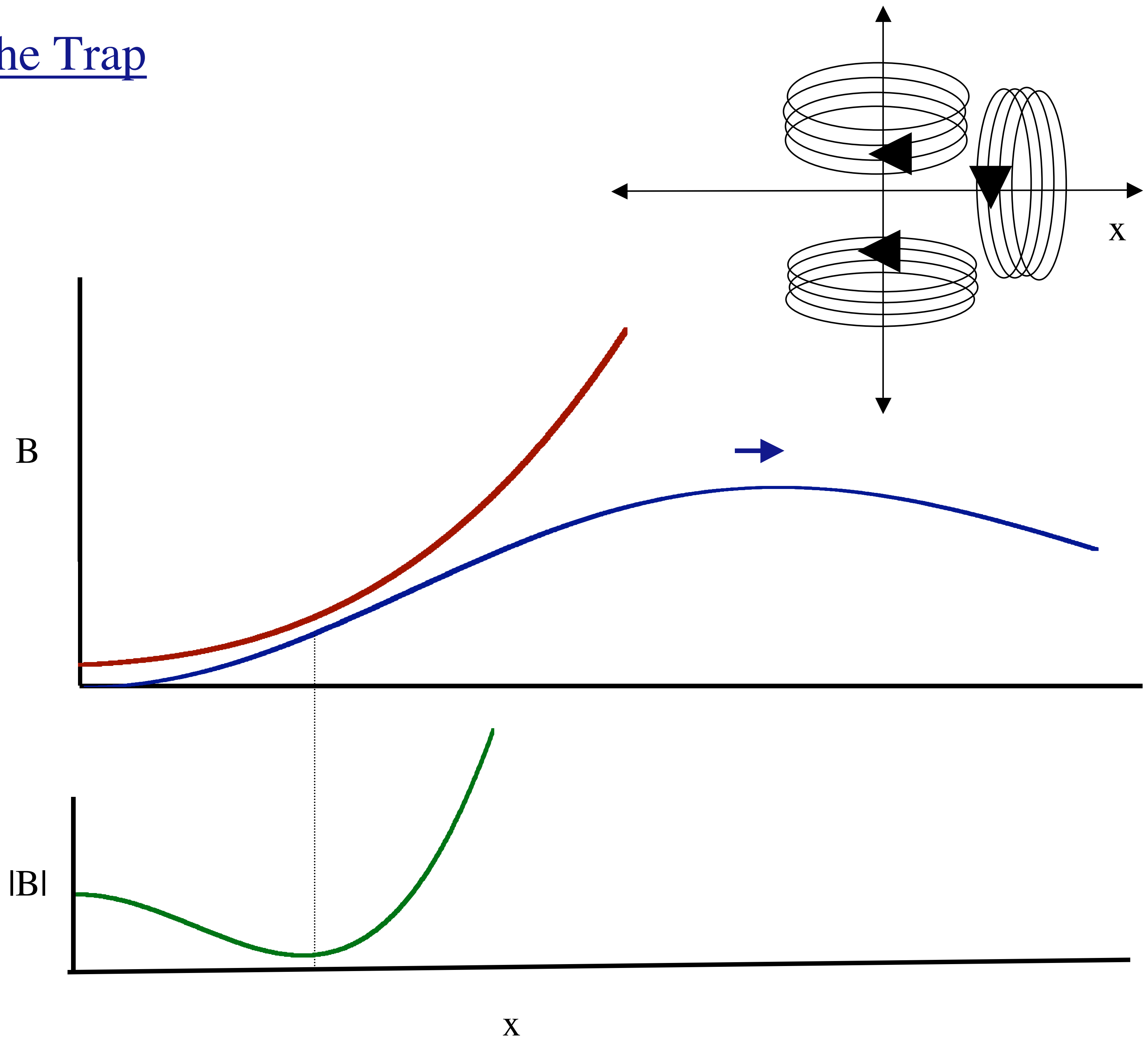
Forming the Trap



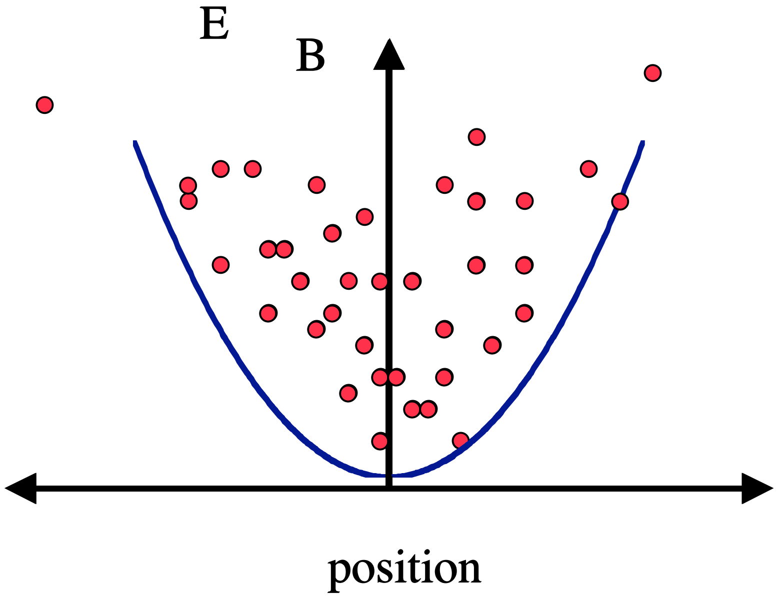
Forming the Trap

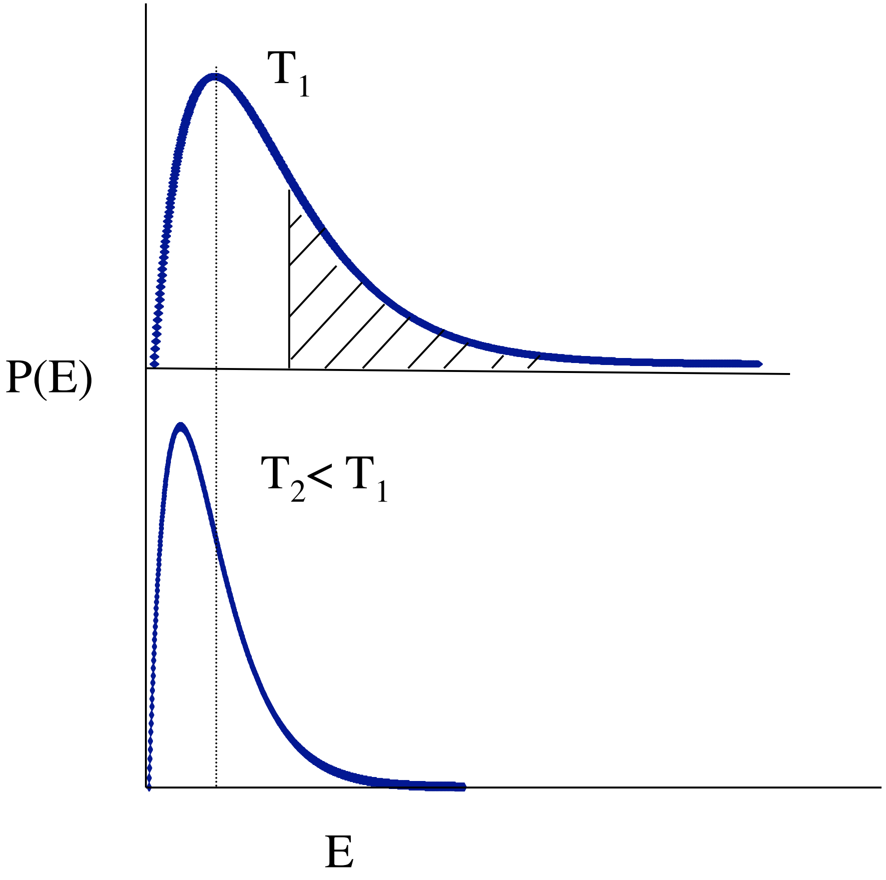


Forming the Trap

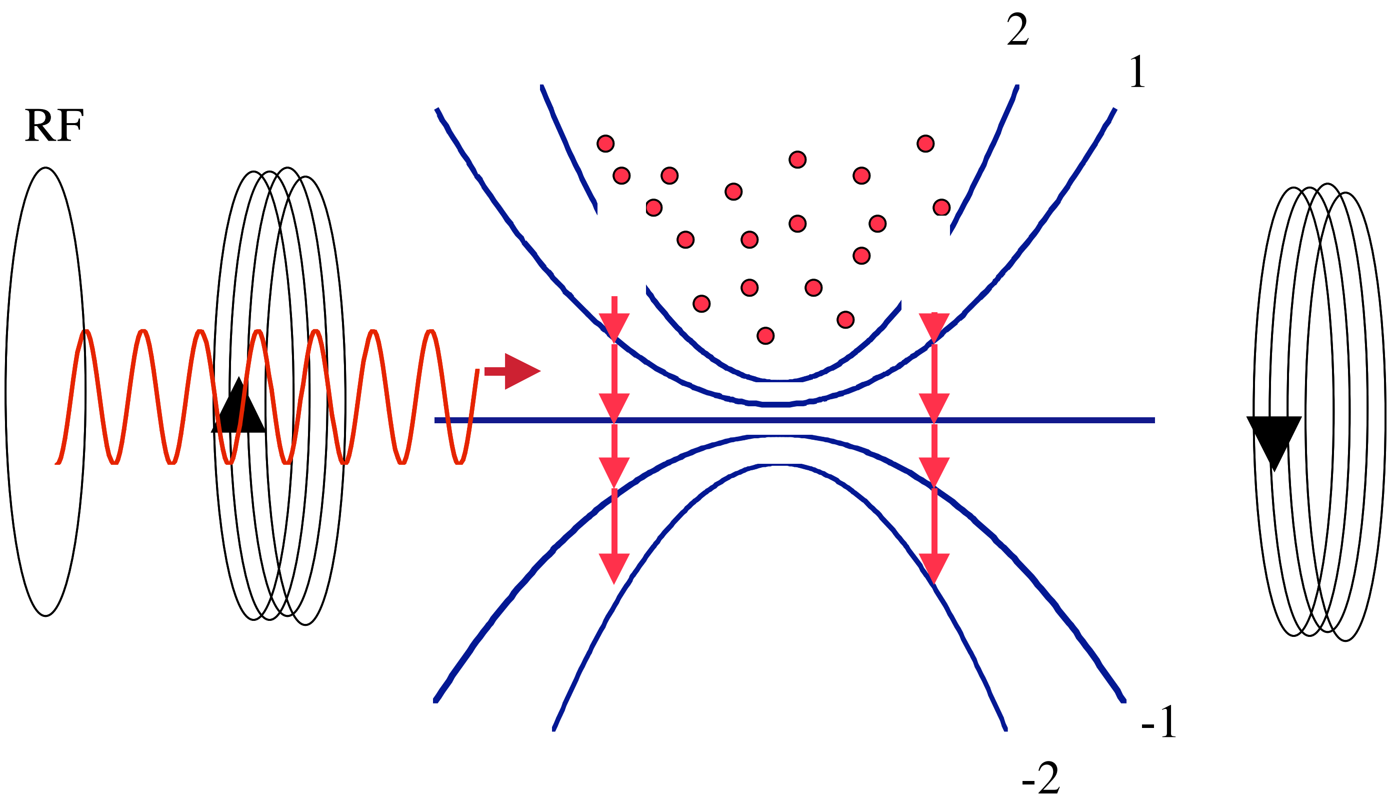


Evaporative Cooling: Truncating the Boltzmann Distribution





RF forced Evaporation



Elastic Collision Rate and Run Away Evaporation

$$\gamma_{el} = n\sigma_s V$$

$$n(\vec{r}) \propto N\,C\exp(-E/kT) \propto N\,C\exp\left[-U_{trap}(r)/kT\right]$$

$$C\iiint d^3r\exp\left[-\beta r^2/kT\right] = 1$$

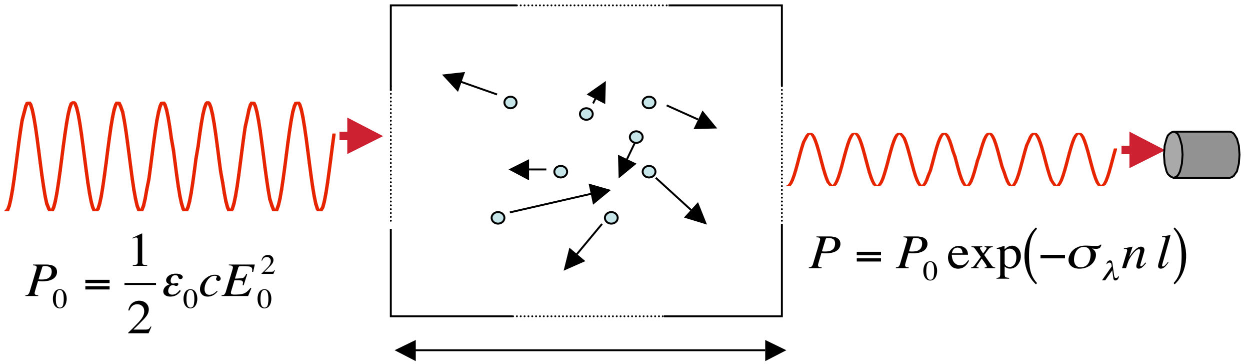
$$C = \left(\frac{\beta}{kT}\right)^{\frac{3}{2}}$$

$$n(0) \propto NT^{-\frac{3}{2}}$$

$$V \propto T^{\frac{1}{2}}$$

$$\gamma_{el} = n\sigma_s V \propto \frac{N}{T} \qquad\qquad \rho_{phase\ space} \propto NT^{-3}$$

Optical Depth and Runaway Evaporation



$$P_0 = \frac{1}{2}\varepsilon_0 c E_0^2$$

$$P = P_0 \exp\left(-\sigma_\lambda n\, l\right)$$

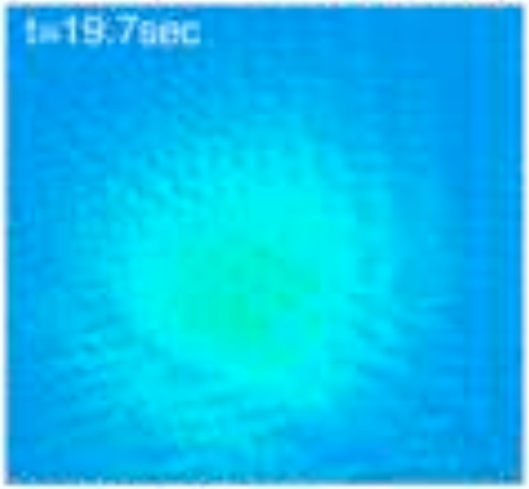
$$\sigma_\lambda n\, l = OD = \ln\frac{P_0}{P}$$

Following a similar procedure to that on the previous page we find

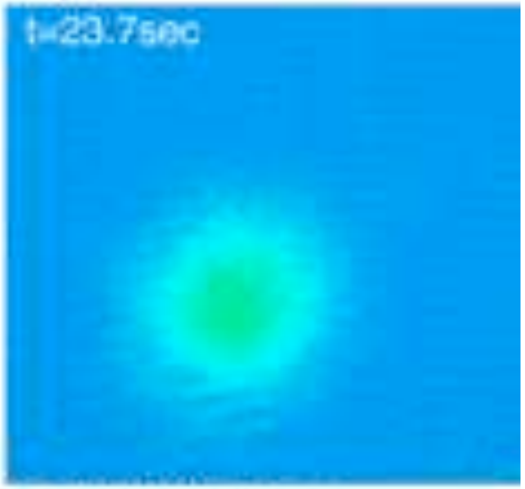
$$OD \propto \frac{N}{T} \propto \gamma_{el}$$

Linear Evaporative Cooling Ramp

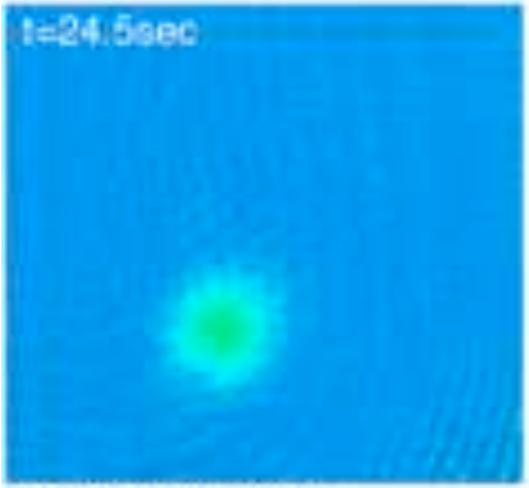
N=4.9x10^8 atoms
T=190μK D=1.3x10^-5



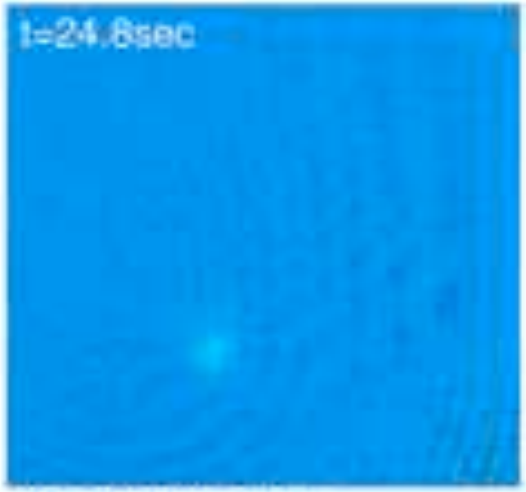
N=2.8x10^8 atoms
T=85μK D=8.7x10^-5



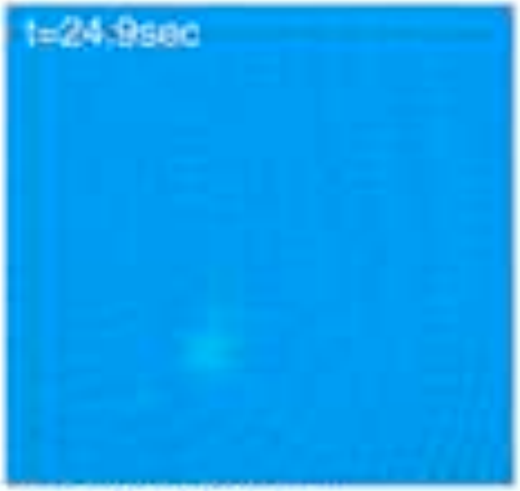
N=9.8x10^7 atoms
T=25μK D=1.2x10^-3



N=4.3x10^7 atoms
T=8.3μK D=0.014

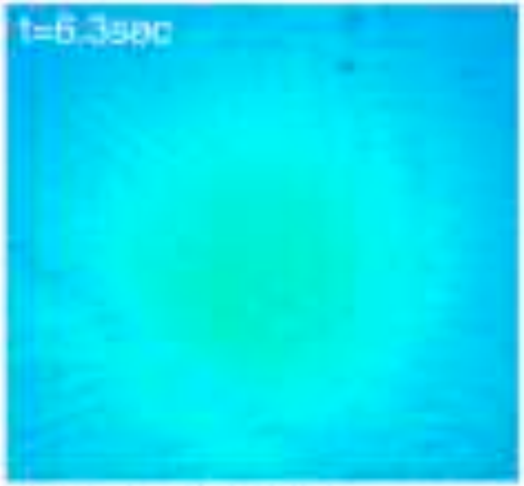


N=3.7x10^6 atoms
T=2.3μK D=0.058

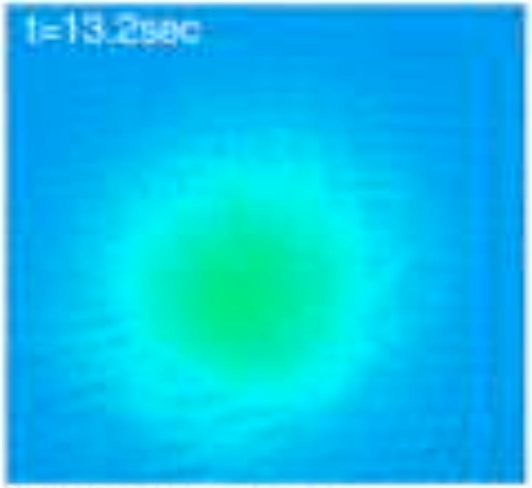


N=1.0x10^6 atoms
T=2.4μK D=0.014

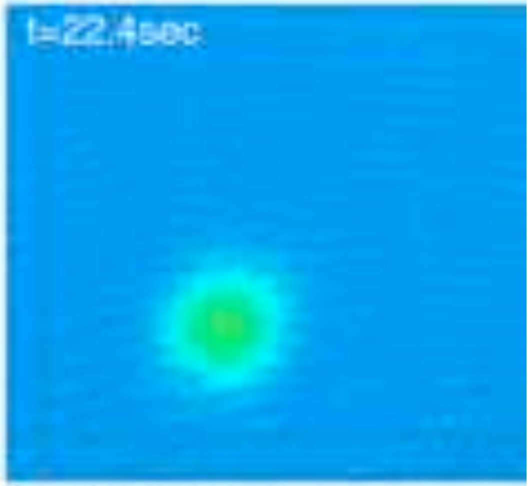
Logarithmic Evaporative Cooling Ramp



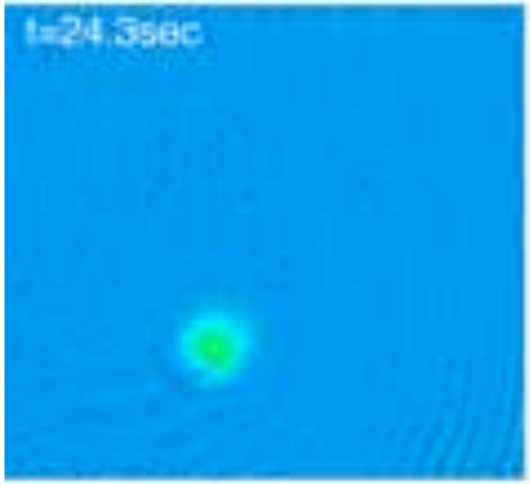
N=4.1x10^8 atoms
T=100μK D=7.8x10^-5



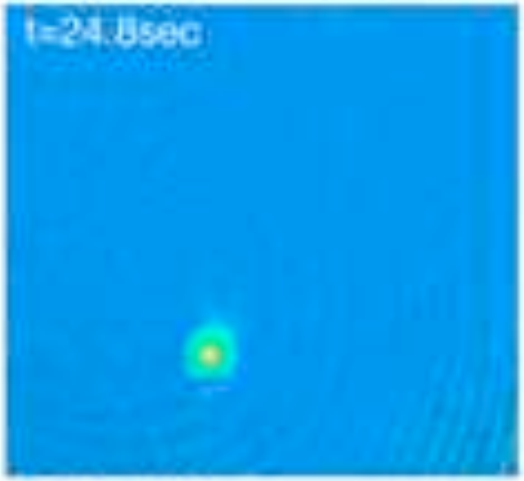
N=2.4x10^8 atoms
T=64μK D=1.7x10^-4



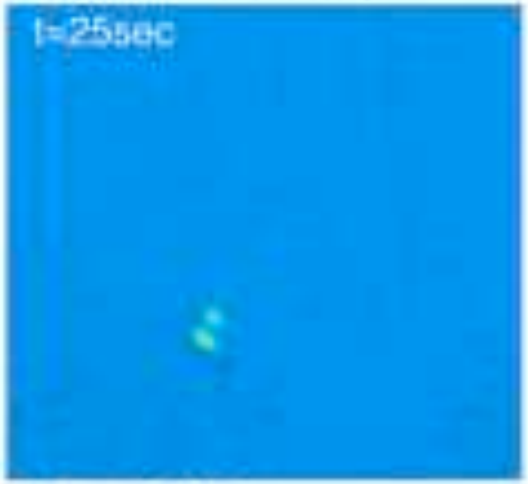
N=3.8x10^7 atoms
T=8.2μK D=0.013



N=1.3x10^7 atoms
T=2.2μK D=0.23



N=9.6x10^6 atoms
T=1.0μK D=1.83



N=1.4x10^6 atoms
T<400nK D>>2.6

END OF LECTURE 2