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Continuous Variable Quantum Entanglement and Its applications

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Outline

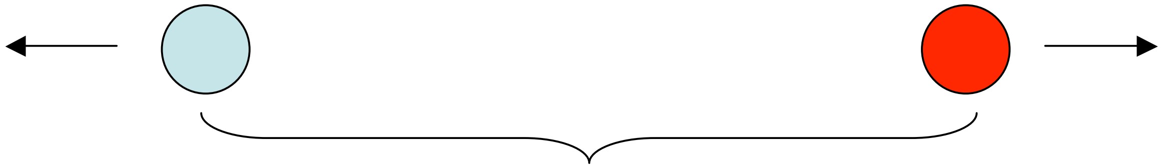
- Entanglement in General
- Continuous variable optical entanglement
- Entanglement measures
- Other types of entanglement
- Applications of entanglement
- Quantum teleportation

What is entanglement?

- Two objects are said to be entangled when their total wave-function is not factorizable into wave-functions of the individual objects.

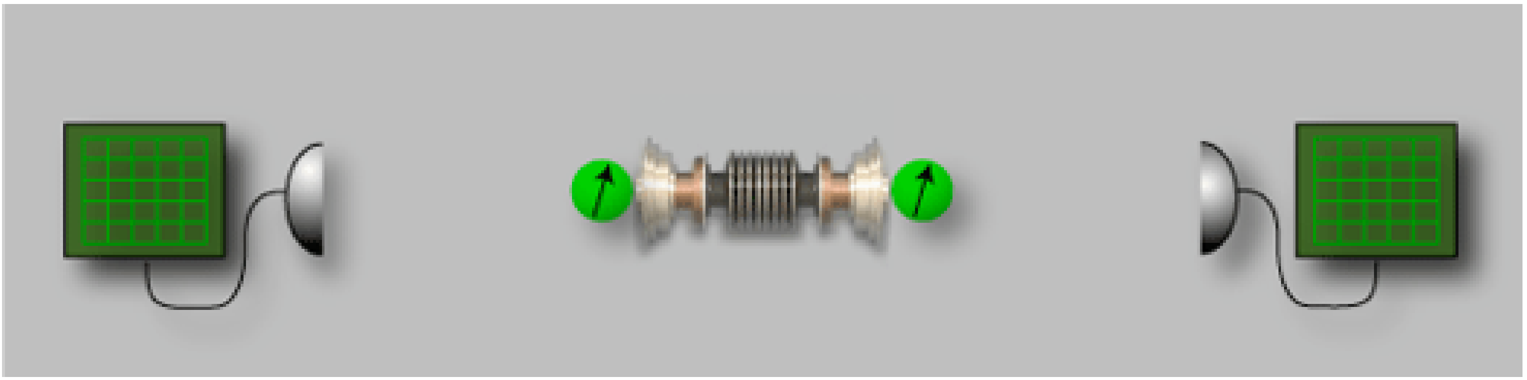
- Not entangled

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}\left(|H_1\rangle + |V_1\rangle\right) \qquad |\psi_2\rangle = \frac{1}{\sqrt{2}}\left(|H_2\rangle + |V_2\rangle\right)$$



- Entangled

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|H_1H_2\rangle + |V_1V_2\rangle\right)$$



- Note: Entanglement is different to superposition.

Why is it weird?

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|H_1H_2\rangle + |V_1V_2\rangle\right) = \frac{1}{\sqrt{2}}\left(|D_1D_2\rangle + |A_1A_2\rangle\right) = \frac{1}{\sqrt{2}}\left(|L_1L_2\rangle + |R_1R_2\rangle\right)$$

- P1 measures HV and get H
- P2 measuring HV MUST get H

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(\boxed{|H_1H_2\rangle} + |V_1V_2\rangle\right)$$

- P1 measures HV and get V
- P2 measuring HV MUST get V

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|H_1H_2\rangle + \boxed{|V_1V_2\rangle}\right)$$

- P1 measures HV and get V
- P2 measuring DA can get D or A

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(\boxed{|H_1H_2\rangle} + |V_1V_2\rangle\right)$$

- P1 measures DA and get D
- P2 measuring DA MUST get D

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(\boxed{|D_1D_2\rangle} + |A_1A_2\rangle\right)$$

- Wave-function of the system collapses in a way that is completely determined by the measurement outcome of P1.

How to create entanglement?

- Use conservation laws. Start with one system that can break up into sub-systems.
- Eg. Nuclear fission with conservation of energy and momentum
- Eg. Parametric down conversion. Split one photon into two photons.
- Look at two non-commuting observables and “prove” via inference that Heisenberg Uncertainty Principle (HUP) can appear to be violated.

$$[X,P] = i\hbar \qquad\qquad \Delta X^2 \Delta P^2 = \frac{\hbar}{2}$$

- We get $\Delta X_{inf} \bullet \Delta P_2$ < HUP Limit?

Measure position ΔX_1

Position inferred ΔX_{inf}

Measure momentum ΔP_2

- Resolution: Inference does not count!
- After particle 1 has been measured, the wave-function of particle 2 (or even the system) is changed. This new wave-function still obeys the HUP.

A brief history of entanglement

1935: Einstein-Podolsky-Rosen's proposal to prove quantum mechanics is incomplete
1935: Schrödinger coined the word "entanglement" - Verschränkung
1950: Gamma-ray pairs from positron & electrons produced by Wu & Shaknov.
1964: J. S. Bell proposed a theorem to exclude hidden variable theories.
1976: Entanglement between protons observed by Lamehi-Rachti & Mittig.
1980s: Low-energy photons from radiative atomic cascade by Aspect et al. Close a lot of loopholes in a series of experiments.
1988: Light entanglement from crystals by Shih & Alley.
1989: Greenberger-Horne-Zeilinger entanglement.
1992: Entanglement from continuous-wave squeezers by Ou & Kimble et al.
1999: Entanglement from optical fibre by Silberhorn & Lam et al.
2001: Entanglement of atomic ensembles by Julsgaard & Polzik et al.
2002: Entanglement by a New Zealander, Bowen et al.

Future: Inter-species entanglement?
- Entanglement of light beams of different wavelengths
- Atom-light entanglement.

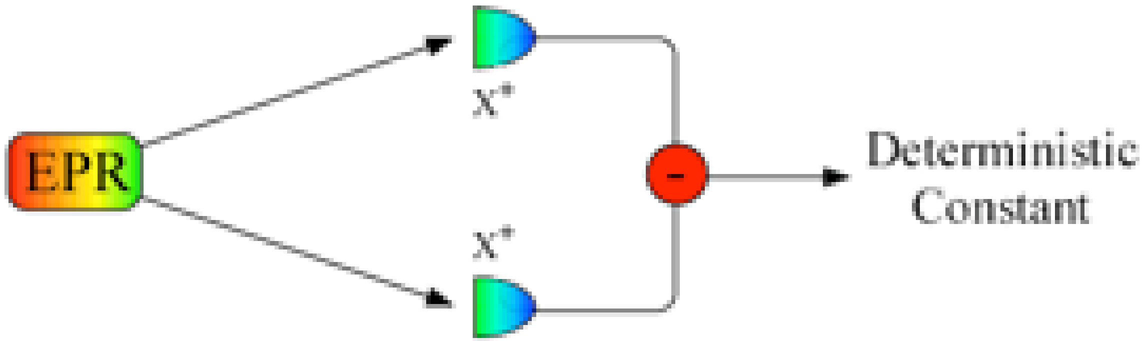
Future: Entanglement of Bose-Einstein Condensates?
Future: Macroscopic entanglement?
Future: Long lived entanglement?

Continuous variable optical entanglement

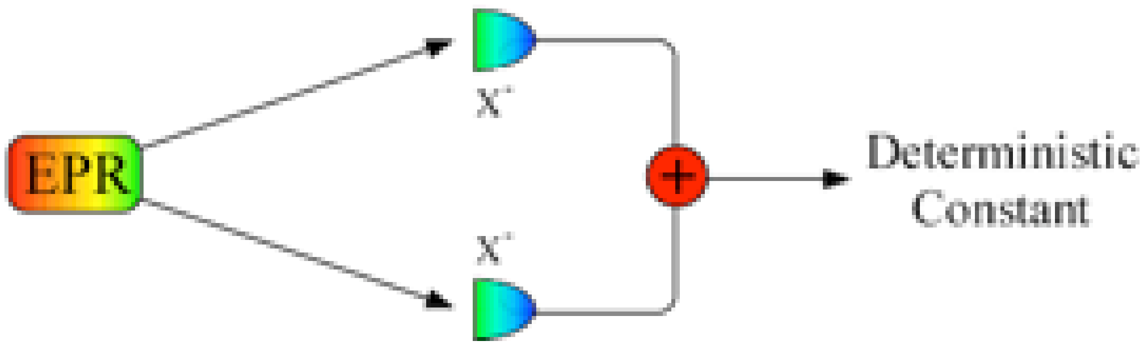
- We want to look at the amplitude and the phase quadrature only.

$$\left[X^{+}, X^{-}\right] = 2i \qquad V(X^{+})V(X^{+}) = 1$$

- Subtract the intensities (amplitudes) of the two beams gives a very quiet measurement: Intensity difference squeezing.

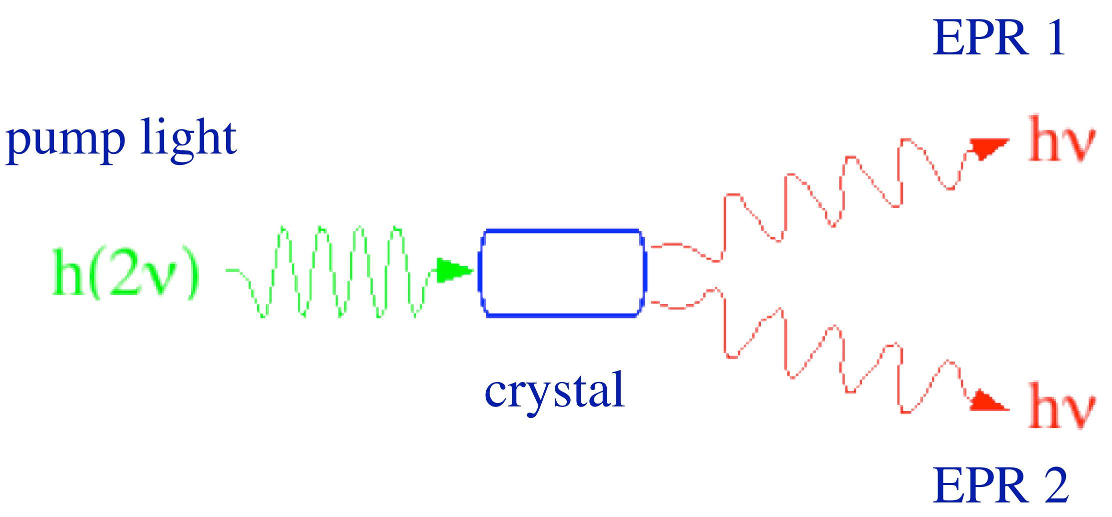


- Sum the phases of the two beams gives a very quiet measurement as well.



- What is the limit for saying that there is optical entanglement?

Parametric down conversion

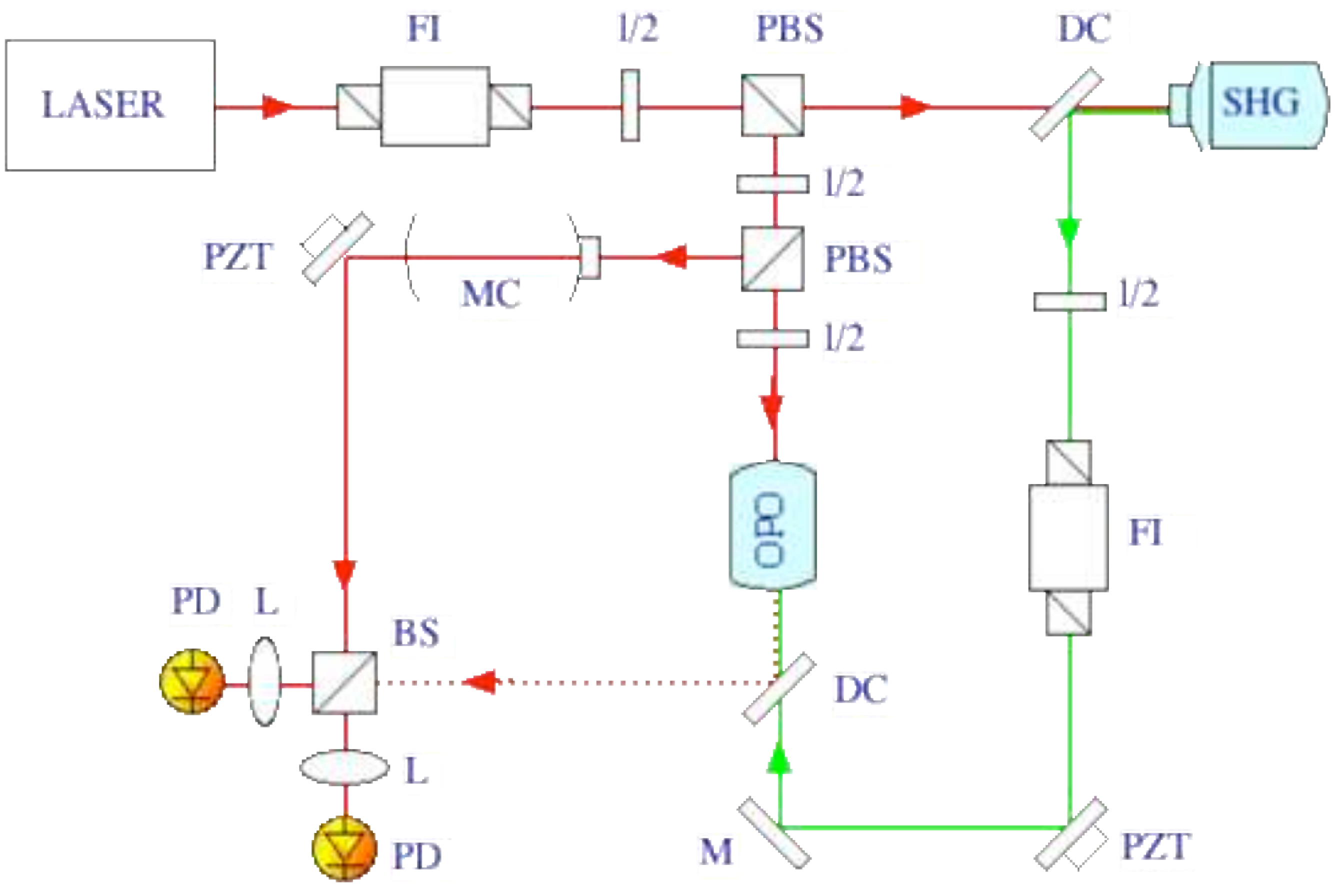


• Pair productions	=>	2 photons production for each pump photon
	=>	Amplitude correlation
• Conserv. of energy	=>	Anti-correlated k-vector
	=>	Phase anti-correlation

- One beam is vertically polarized and the other is horizontally polarized in Type II Optical parametric oscillator/amplifiers.
- These two beams are entangled.

Squeezing with OPO/A

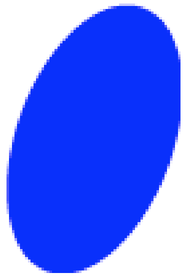
- For degenerate Type I OPO/A, the signal and idler beams have the same polarization.



- The single output of the OPO/A is squeezed.

*

Squeezing the vacuum

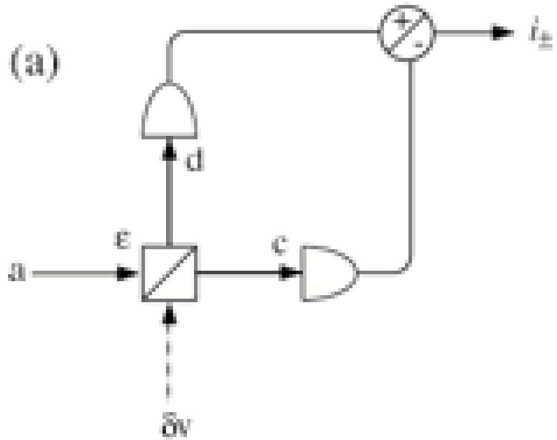


Squeezing and entanglement

- Can we use squeezed light to generate entanglement?

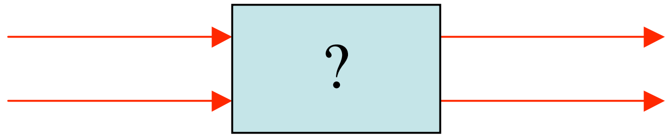
- Squeezing:
- One beam only

- Sub-quantum noise stability (quantum correlations) exists in one quadrature at the expense of making the orthogonal quadrature very noisy

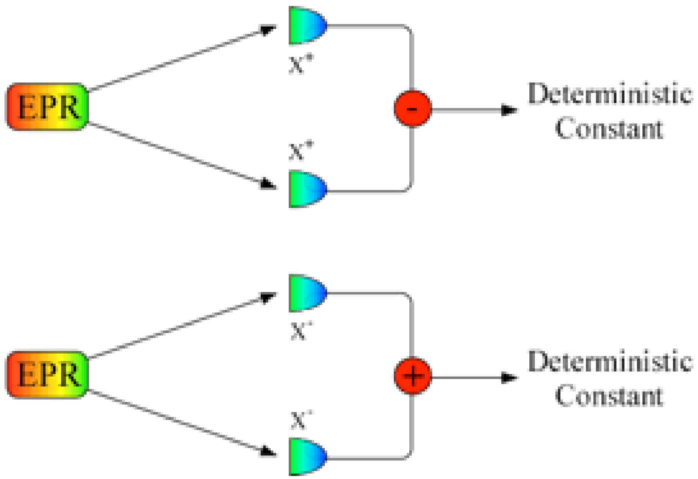


- Completely un-interested in the other quadrature => Do not really care whether state is minimum uncertainty limited. Do not care about state purity.

- Entanglement
- Must be between 2 beams



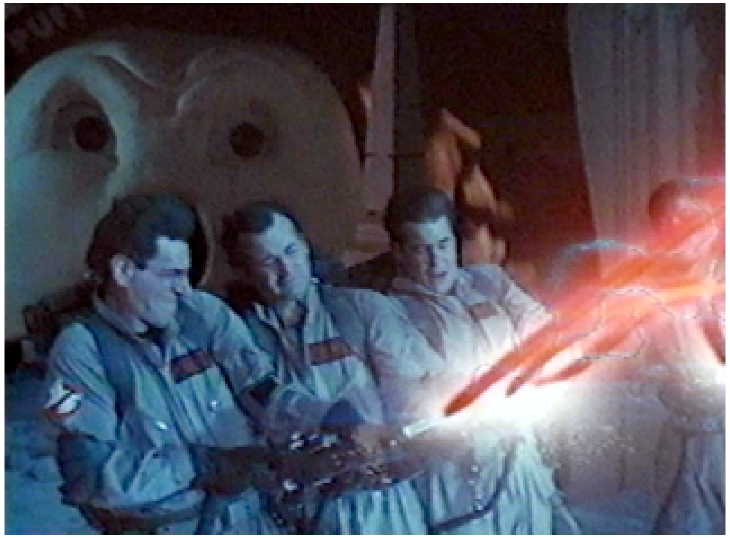
- Must have quantum correlations established on both non-commuting quadratures



- Does worry about all quadratures! Purity matters.

Ghostbusters Entanglement

- Don’t cross the streams!



Ghostbusters generating Einstein-Podolsky-Rosen entanglement.

Dr. Egon Spengler : There's something very important I forgot to tell you.

Dr. Peter Venkman : What?

Dr. Egon Spengler : Don't cross the streams.

Dr. Peter Venkman : Why?

Dr. Egon Spengler : It would be bad.

Dr. Peter Venkman : I'm fuzzy on the whole good/bad thing. What do you mean "bad"?

Dr. Egon Spengler : Try to imagine all life as you know it stopping instantaneously and every molecule in your body exploding at the speed of light.

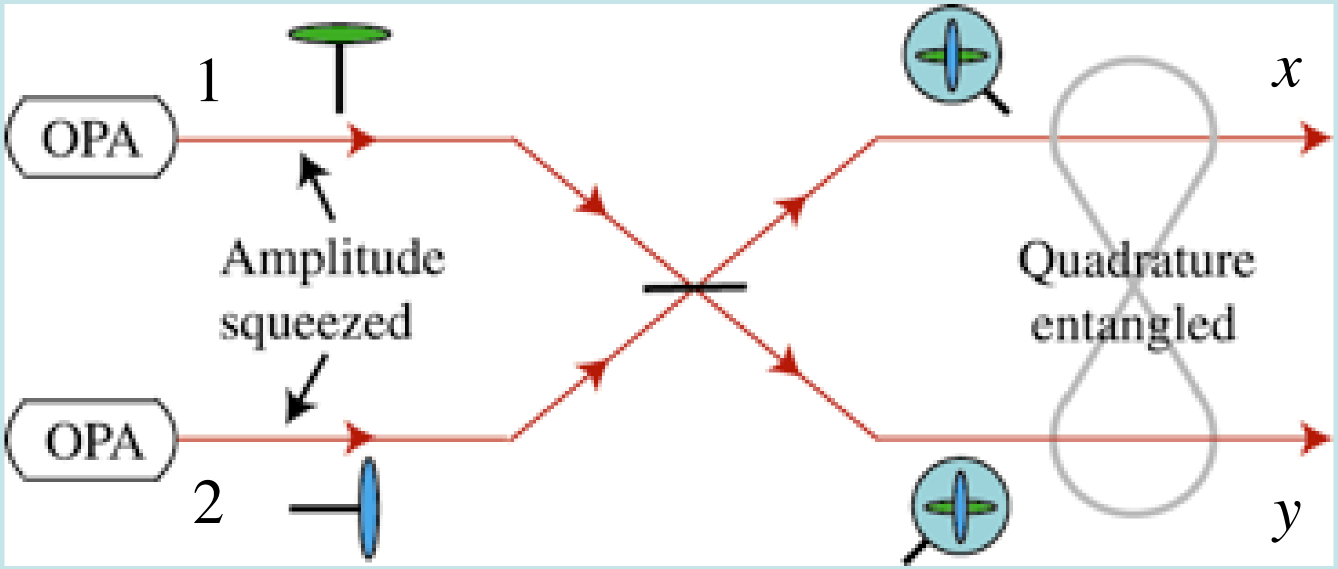
Dr. Raymond Stantz : Total protonic reversal.

Dr. Peter Venkman : That's bad. Okay. Alright, important safety tip, thanks Egon.

Generating quadrature entanglement

- Need to mix two squeezed beams with a 90 degree phase difference on a 50/50 beam splitter

$$X_{1,2}^{+} < 1 < X_{1,2}^{-}$$

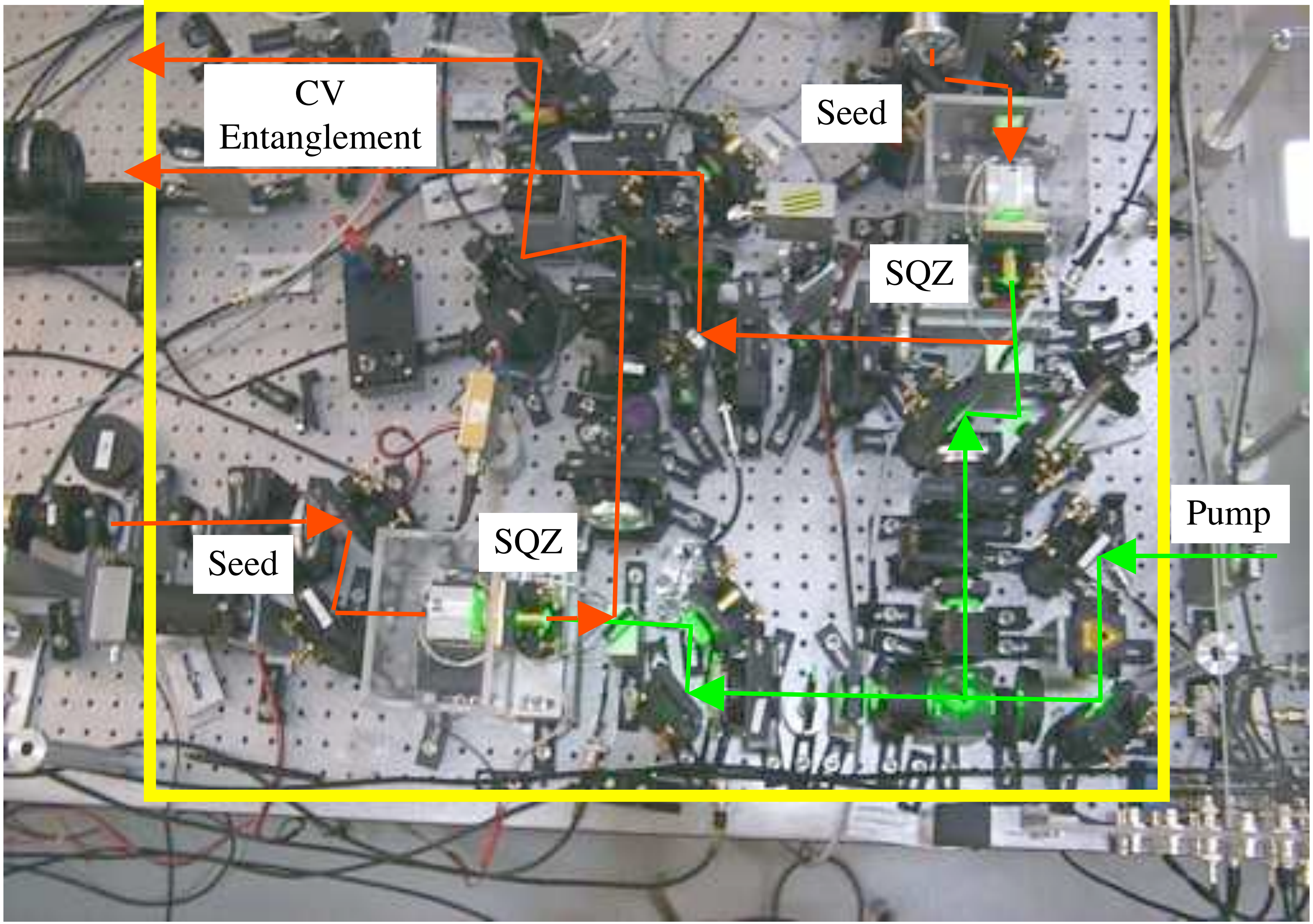


$$X_x^{+} = \frac{1}{\sqrt{2}}\left(X_1^{+} + X_2^{-}\right) \qquad X_y^{+} = \frac{1}{\sqrt{2}}\left(X_1^{+} - X_2^{-}\right)$$

$$X_x^{-} = \frac{1}{\sqrt{2}}\left(X_1^{-} + X_2^{+}\right) \qquad X_y^{-} = \frac{1}{\sqrt{2}}\left(X_1^{-} - X_2^{+}\right)$$

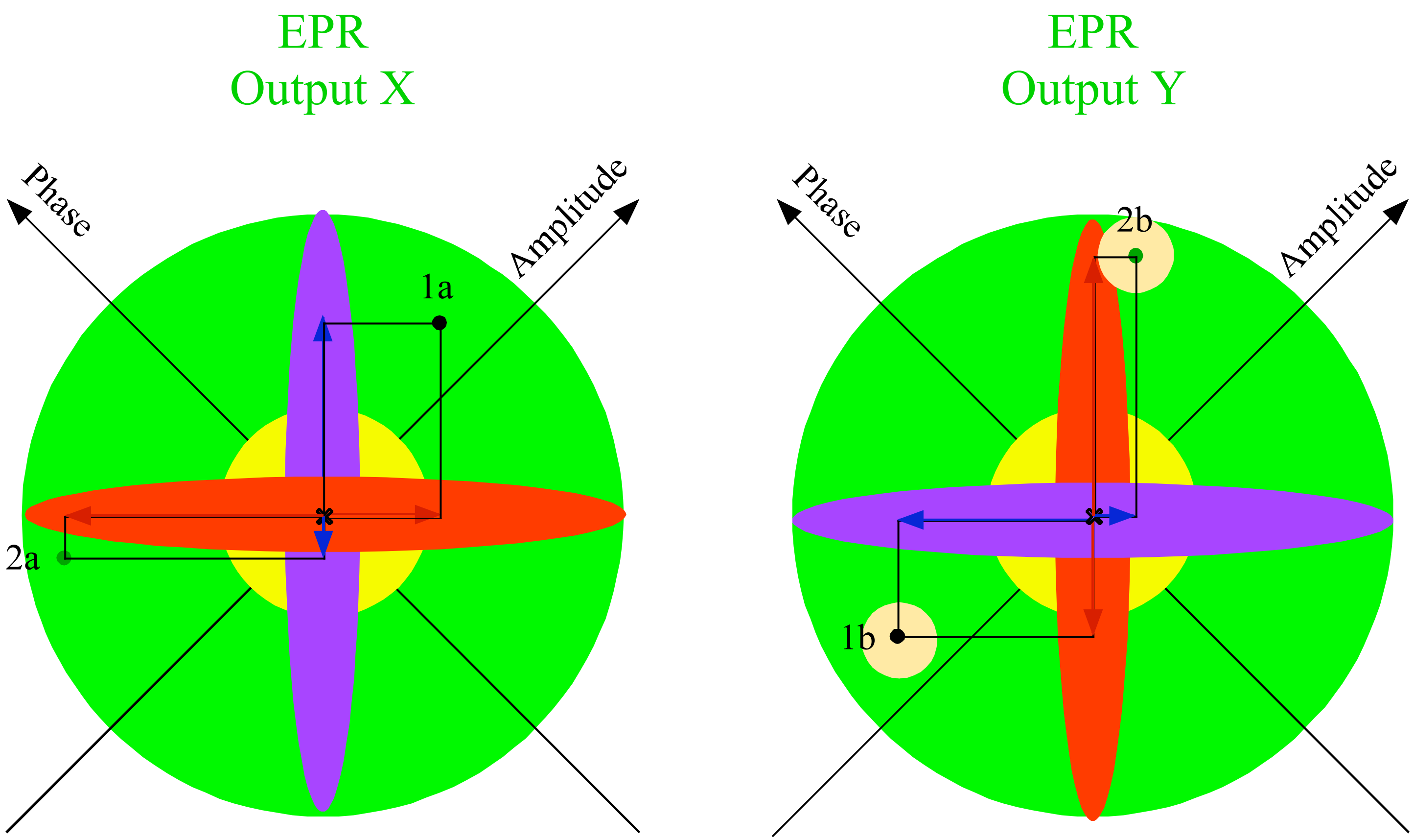
Ou *et al.*, Phys. Rev. Lett. **68**, 3663 (1992)

Entanglement generation experiment



Looking within the uncertainty circle

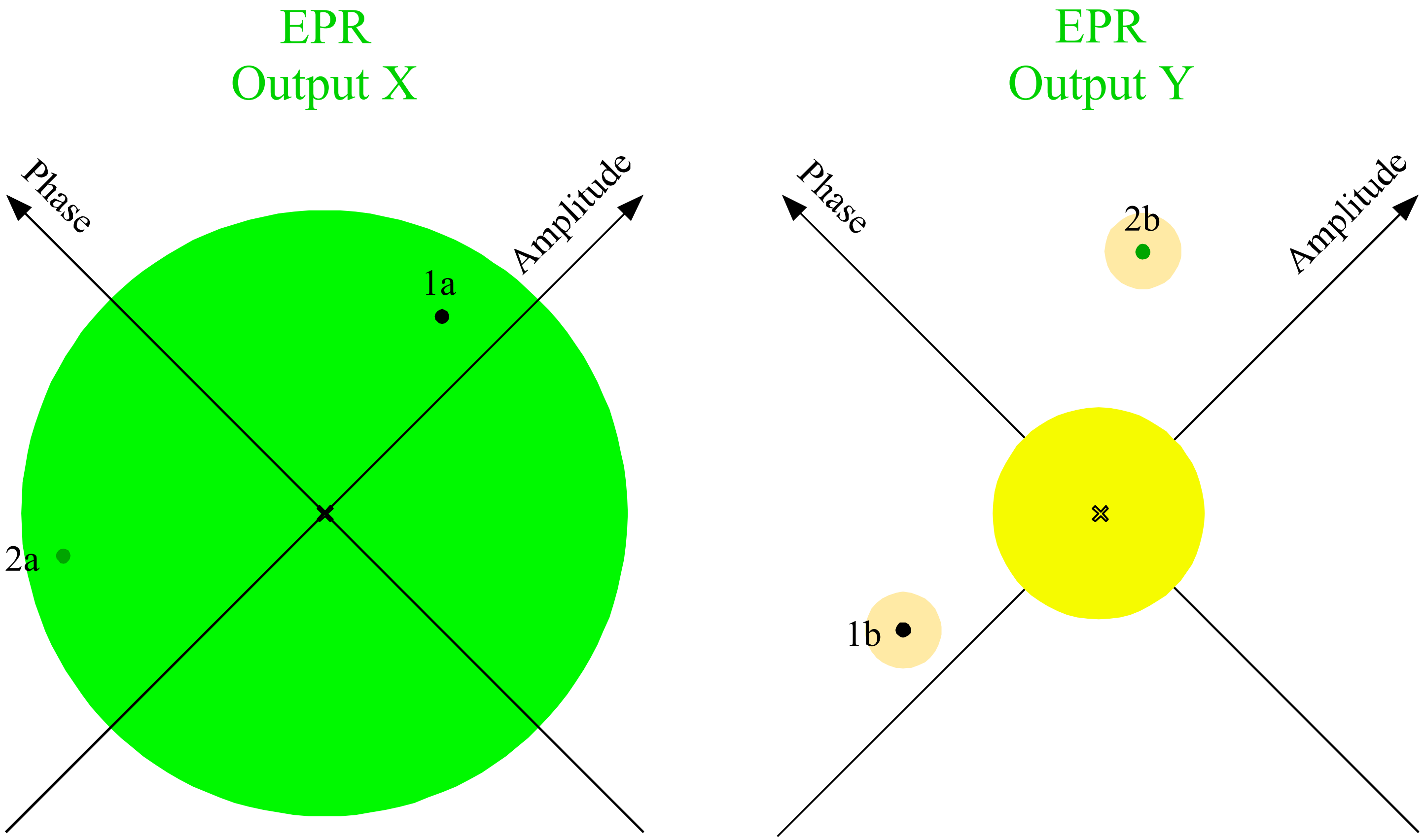
Individually, each beam is very noise in every quadrature



Combined, they are correlated in phase, and anti-correlated in amplitude beyond the quantum limit.

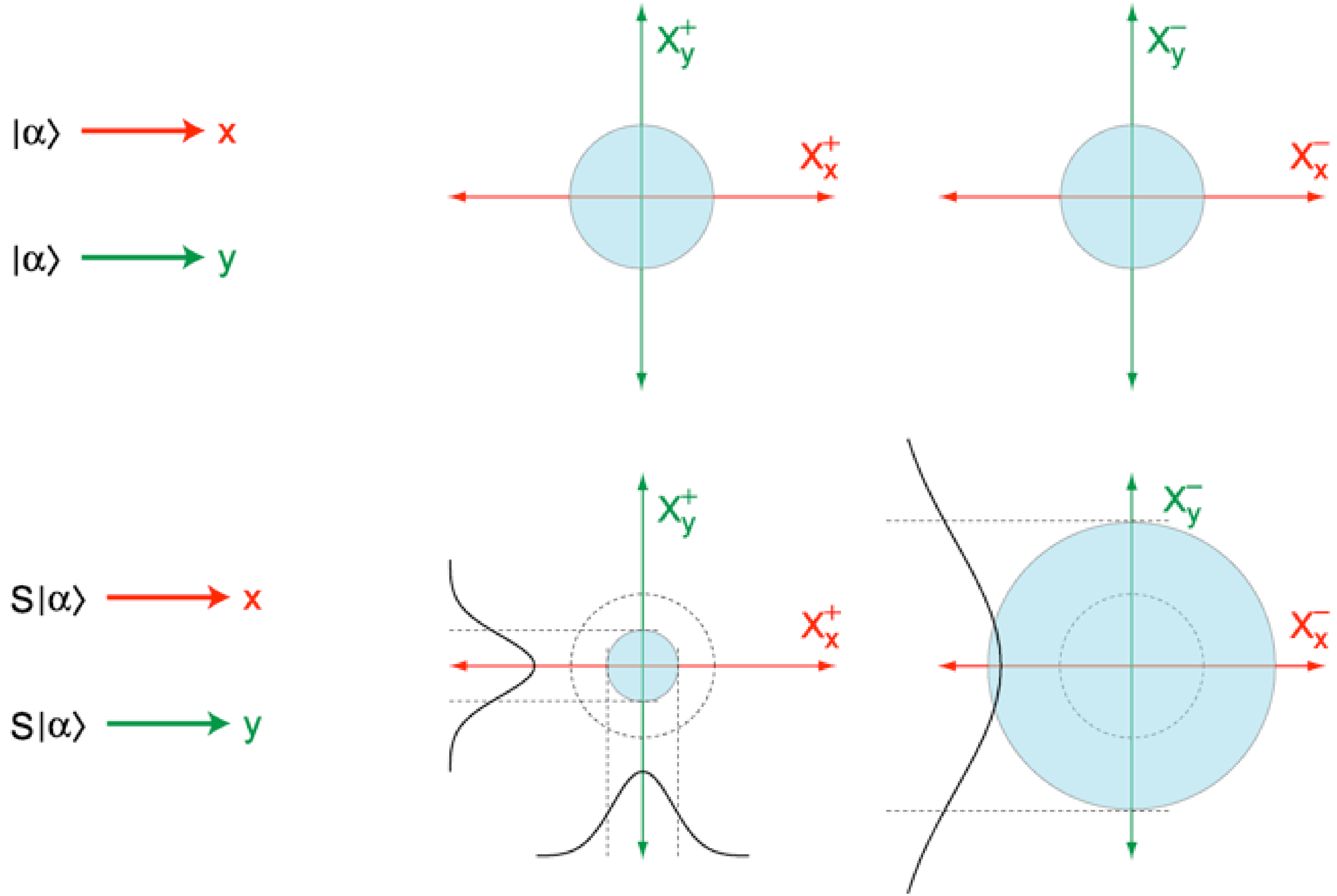
Looking within the uncertainty circle

Seems to demonstrate that EPR's idea is right?
Is Heisenberg Uncertainty Principle being violated?

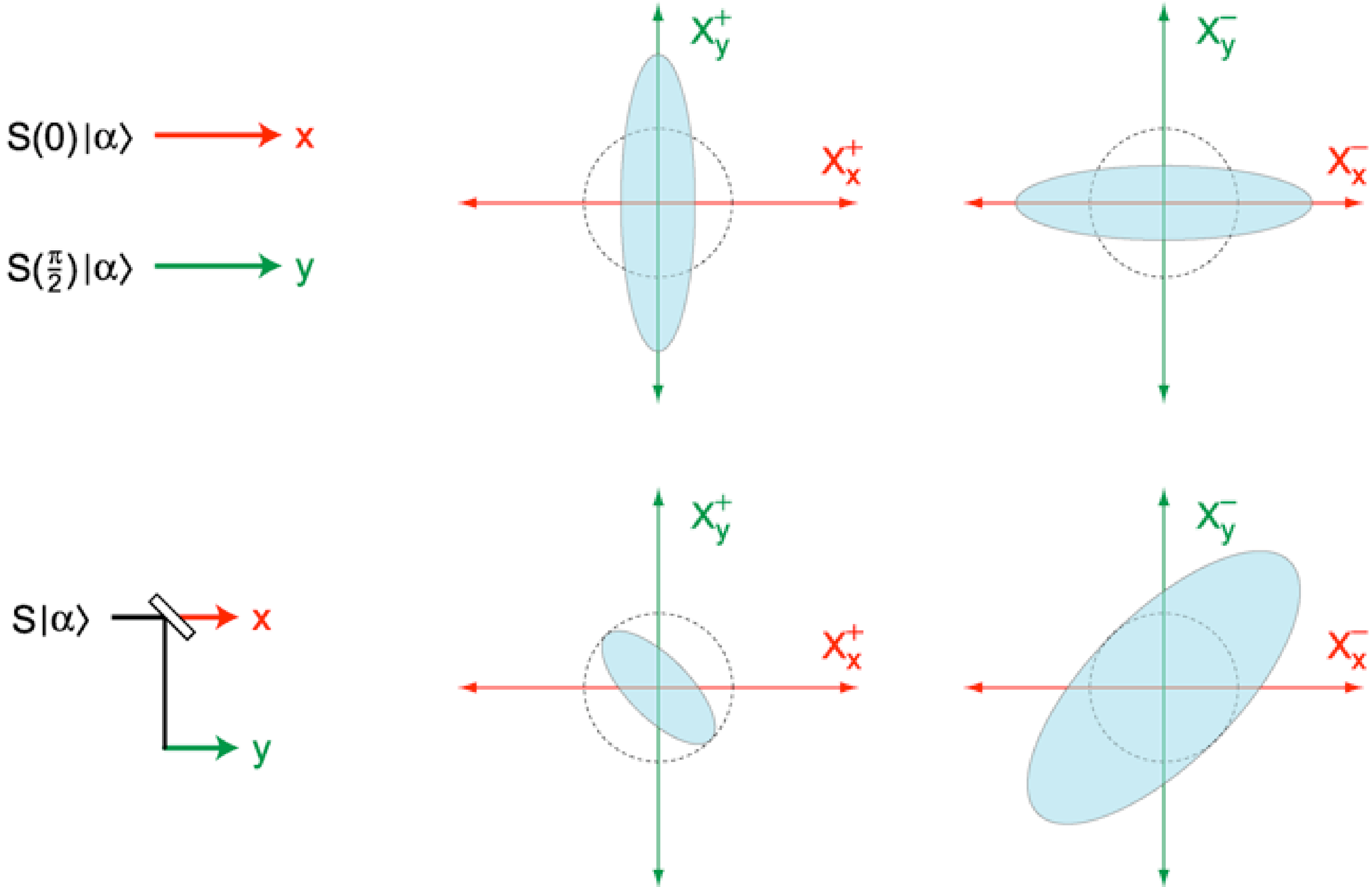


Resolving the paradox:

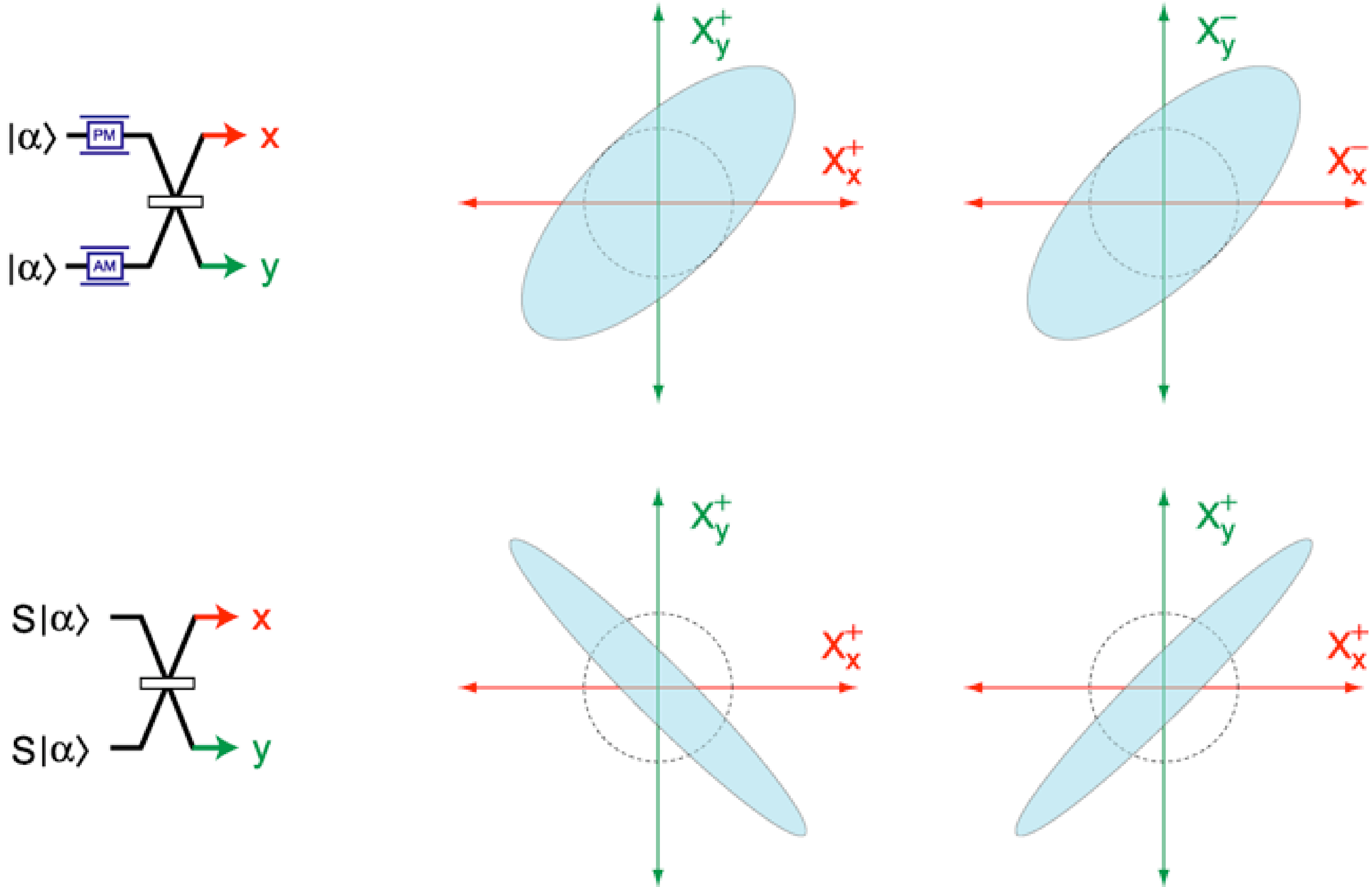
Cross correlations between beams



Cross correlations between beams



Cross correlations between beams



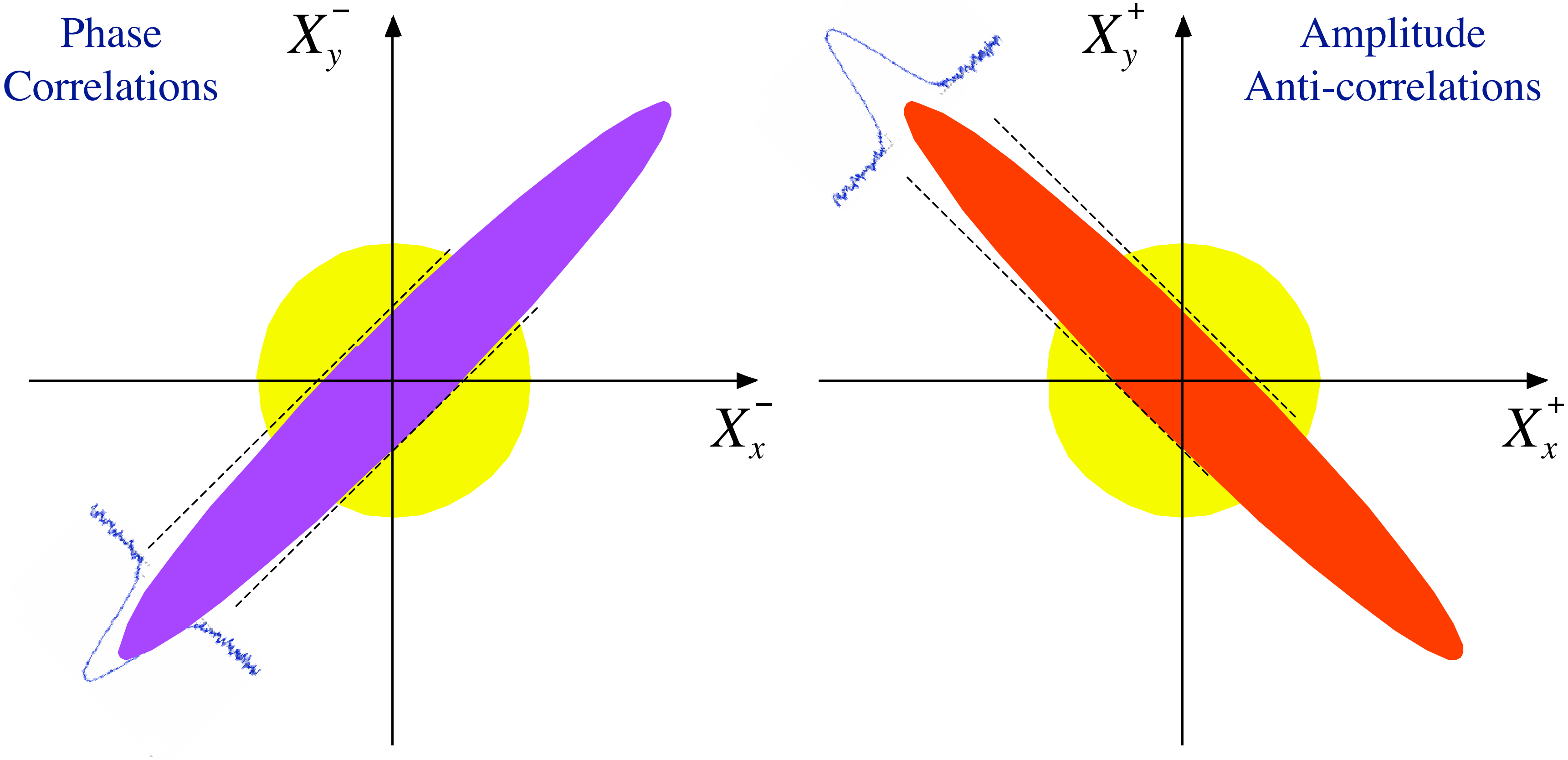
The sum and difference variances

$$X_{1,2}^{+} < 1 < X_{1,2}^{-}$$

$$X_x^{+} = \frac{1}{\sqrt{2}}\left(X_1^{+} + X_2^{-}\right) \qquad X_y^{+} = \frac{1}{\sqrt{2}}\left(X_1^{+} - X_2^{-}\right)$$

$$X_x^{-} = \frac{1}{\sqrt{2}}\left(X_1^{-} + X_2^{+}\right) \qquad X_y^{-} = \frac{1}{\sqrt{2}}\left(X_1^{-} - X_2^{+}\right)$$

$$V\left(X_x^{+} + X_y^{+}\right)/2 = V\left(X_x^{+}\right) \qquad V\left(X_x^{-} - X_y^{-}\right)/2 = V\left(X_x^{+}\right)$$



Inseparability Criterion

- In the spirit of the Schrödinger Picture
- Measures the degree of inseparability of two entangled wavefunctions
- Looks at the quadrature amplitudes' quantum correlations

$$V\left(X_x^+ + X_y^+\right)/2 = V\left(X_1^+\right) \qquad V\left(X_x^- - X_y^-\right)/2 = V\left(X_2^+\right)$$

- The sum/difference correlations of the amplitude/phase between the two sub-systems must both be less than the HUP

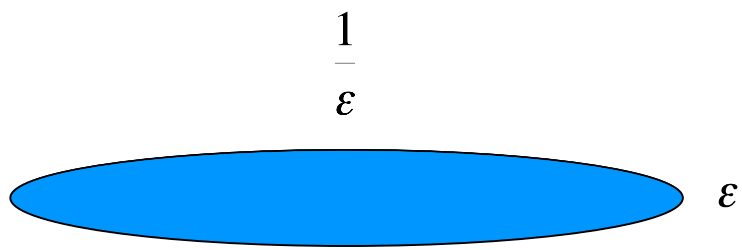
$$\sqrt{V\left(X_x^+ + X_y^+\right)V\left(X_x^- - X_y^-\right)}/2 < 1$$

- Insensitive to the purity of states.

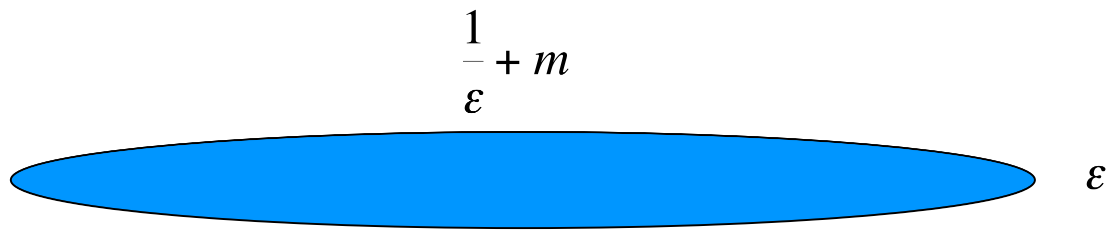
Duan *et al*., Phys. Rev. Lett. **84**, 4002 (2001)

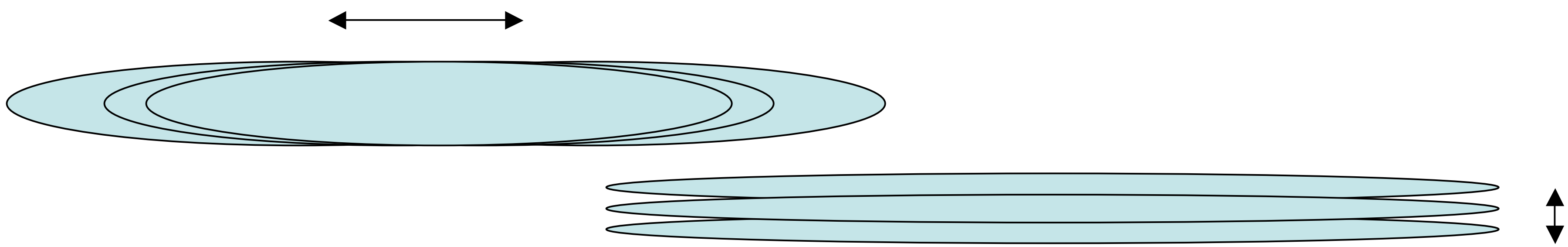
State purity

- Minimum uncertainty states are pure



- Mixed states of squeezed light

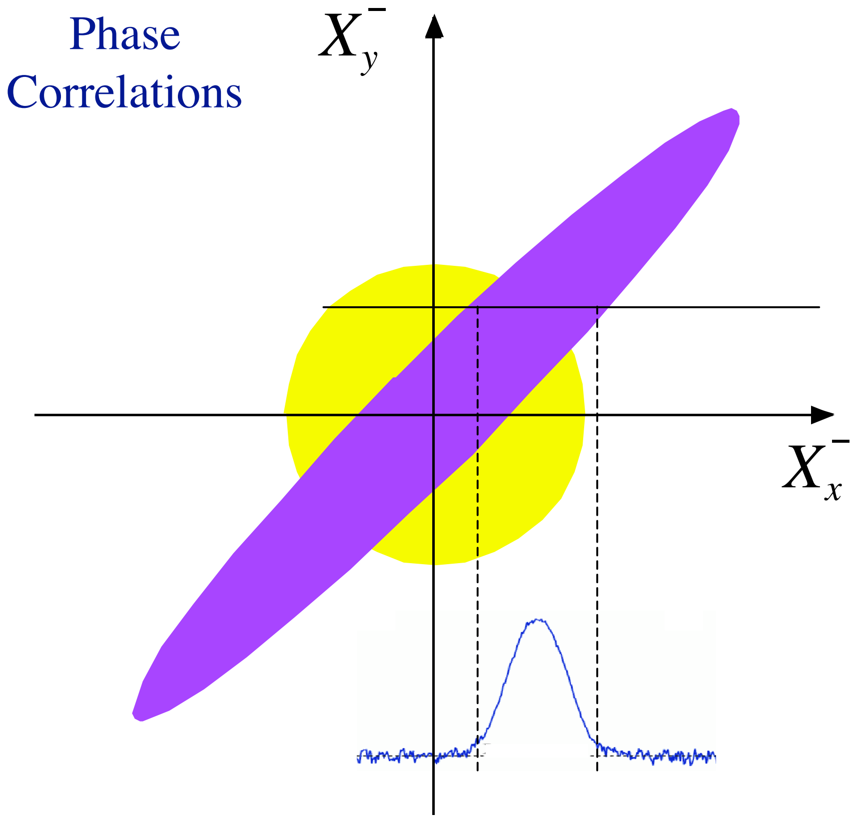


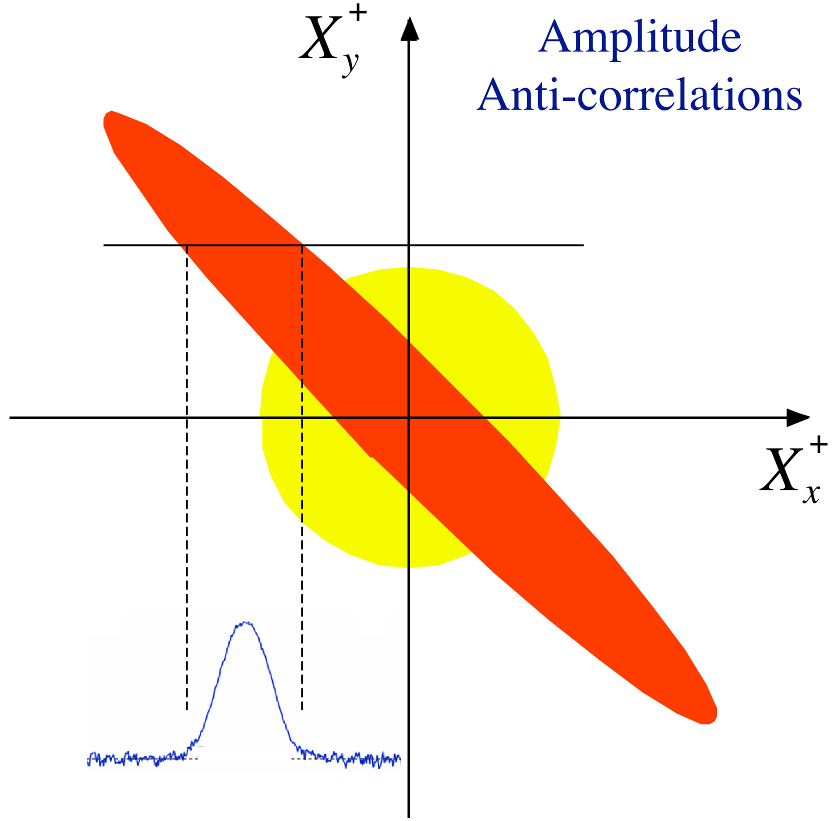


The conditional variances

$$\varepsilon = X^{+}_{1,2} < 1 < X^{-}_{1,2} = \frac{1}{\varepsilon}$$

$$V^{+}_{x|y} = V\left(X^{+}_{x}\right) - \frac{\left|\left\langle \delta X^{+}_{x} \delta X^{+}_{y} \right\rangle\right|^{2}}{V\left(X^{+}_{y}\right)} \qquad V^{-}_{x|y} = V\left(X^{-}_{x}\right) - \frac{\left|\left\langle \delta X^{-}_{x} \delta X^{-}_{y} \right\rangle\right|^{2}}{V\left(X^{-}_{y}\right)}$$





EPR criterion

- More in the spirit of the Heisenberg Picture
- Measures how well we can demonstrate the EPR paradox
- Looks at conditional variances of the quadrature amplitudes

$$V_{x|y}^{+} = V\left(X_x^{+}\right) - \frac{\left|\left\langle \delta X_x^{+} \delta X_y^{+} \right\rangle\right|^2}{V\left(X_y^{+}\right)} \qquad V_{x|y}^{-} = V\left(X_x^{-}\right) - \frac{\left|\left\langle \delta X_x^{-} \delta X_y^{-} \right\rangle\right|^2}{V\left(X_y^{-}\right)}$$

- The product of the amplitude and phase quadratures conditional variances must be less than the Heisenberg Uncertainty Limit

$$V_{x|y}^{+} V_{x|y}^{-} < 1$$

- Takes into account the purity of the entanglement

Reid and Drummond, Phys. Rev. Lett. **60**, 2731 (1988)

Other forms of quadrature entanglement

- Can we have entanglement that has cross quadrature correlations between beams?

$$V^{+}_{x+|y-} = V\left(X^{+}_{x}\right) - \frac{\left|\left\langle \delta X^{+}_{x} \delta X^{-}_{y} \right\rangle\right|^{2}}{V\left(X^{-}_{y}\right)} < 1 \qquad V^{-}_{x-|y+} = V\left(X^{-}_{x}\right) - \frac{\left|\left\langle \delta X^{-}_{x} \delta X^{+}_{y} \right\rangle\right|^{2}}{V\left(X^{+}_{y}\right)} < 1$$

$$V\left(X^{+}_{x} + X^{-}_{y}\right)/2 < 1 \qquad V\left(X^{-}_{x} - X^{+}_{y}\right)/2 < 1$$

- Can we have entanglement that has same sign correlations for both quadratures?

$$V^{+}_{x|y} = V\left(X^{+}_{x}\right) - \frac{\left|\left\langle \delta X^{+}_{x} \delta X^{+}_{y} \right\rangle\right|^{2}}{V\left(X^{+}_{y}\right)} < 1 \qquad V^{-}_{x|y} = V\left(X^{-}_{x}\right) - \frac{\left|\left\langle \delta X^{-}_{x} \delta X^{-}_{y} \right\rangle\right|^{2}}{V\left(X^{-}_{y}\right)} < 1$$

$$V\left(X^{+}_{x} - X^{+}_{y}\right)/2 < 1 \qquad V\left(X^{-}_{x} - X^{-}_{y}\right)/2 < 1$$

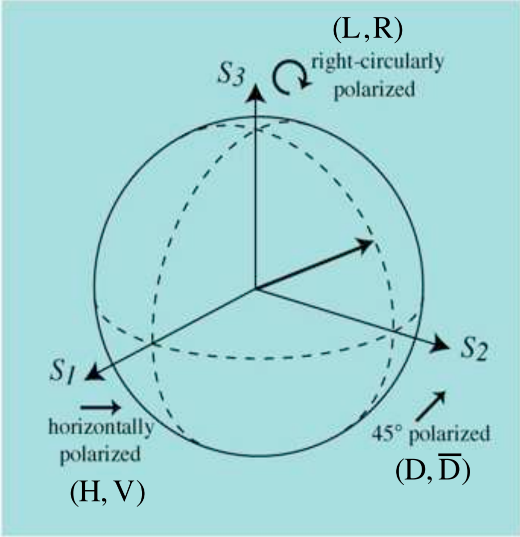
No cloning theorem

Let U be the cloning operator such that
U $|\psi\rangle = |\psi\rangle * |\psi\rangle$ and
U $|\phi\rangle = |\phi\rangle * |\phi\rangle$

For a state in superposition $|\xi\rangle = 1/\sqrt{2}\,(|\phi\rangle + |\psi\rangle)$, we have
U $|\xi\rangle = 1/\sqrt{2}\,(U|\phi\rangle + U|\psi\rangle) = U\, 1/\sqrt{2}\,(|\phi\rangle + |\psi\rangle)$
Assuming QM is linear

Should the answer be:
U $|\xi\rangle = 1/\sqrt{2}\,(|\phi\rangle * |\phi\rangle + |\psi\rangle * |\psi\rangle)$
or
U $|\xi\rangle = 1/\sqrt{2}\,[(|\phi\rangle + |\psi\rangle) * (|\phi\rangle + |\psi\rangle)]$ (q.e.d.)

Polarization entanglement



$$\hat{S}_0 = \hat{a}_H^\dagger \hat{a}_H + \hat{a}_V^\dagger \hat{a}_V,$$

$$\hat{S}_1 = \hat{a}_H^\dagger \hat{a}_H - \hat{a}_V^\dagger \hat{a}_V,$$

$$\hat{S}_2 = \hat{a}_H^\dagger \hat{a}_V e^{i\theta} + \hat{a}_V^\dagger \hat{a}_H e^{-i\theta}$$

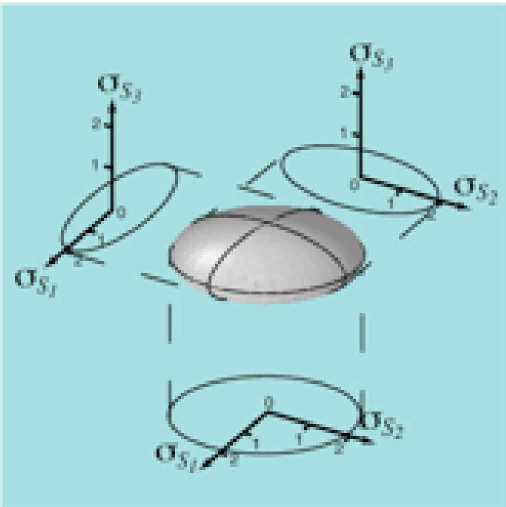
$$\hat{S}_3 = i\hat{a}_V^\dagger \hat{a}_H e^{-i\theta} - i\hat{a}_H^\dagger \hat{a}_V e^{i\theta}.$$

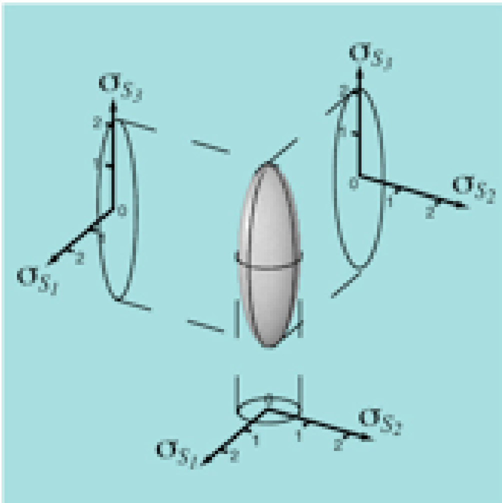
Commutation relations of Stokes operators

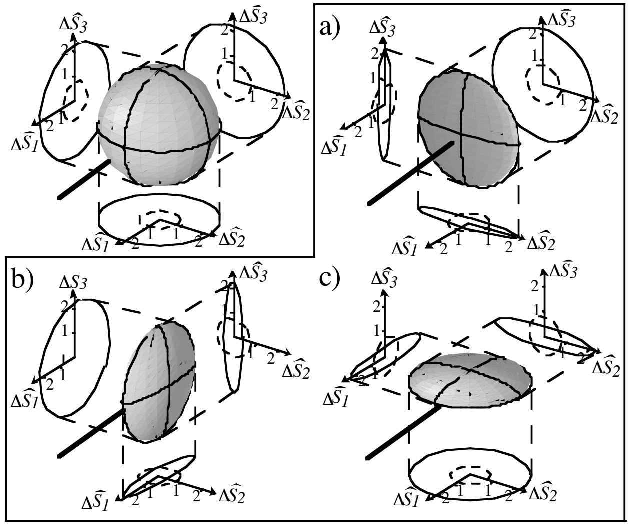
$$\left[\hat{S}_1, \hat{S}_2\right] = 2i\hat{S}_3$$

$$\left[\hat{S}_2, \hat{S}_3\right] = 2i\hat{S}_1$$

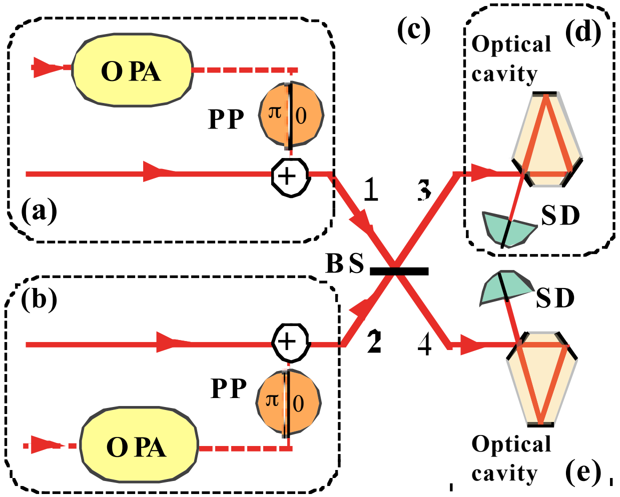
$$\left[\hat{S}_3, \hat{S}_1\right] = 2i\hat{S}_2$$

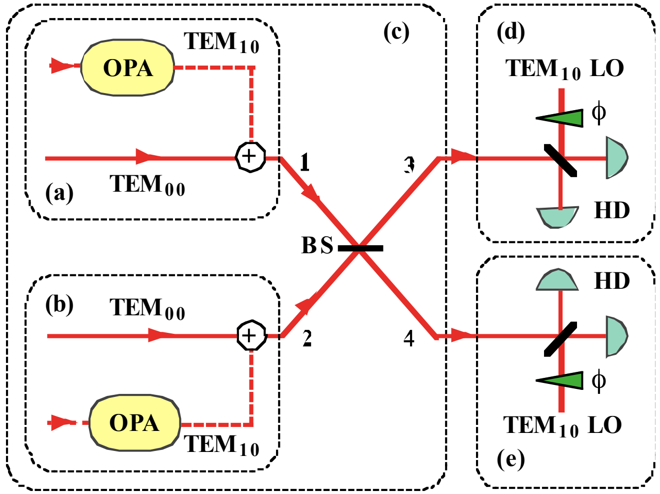






Spatial entanglement





- Near field-Far field entanglement
  - Squeeze 2 TEM_{10} modes and interfere on a beam splitter

$$\left[x, p_x\right] = 1$$

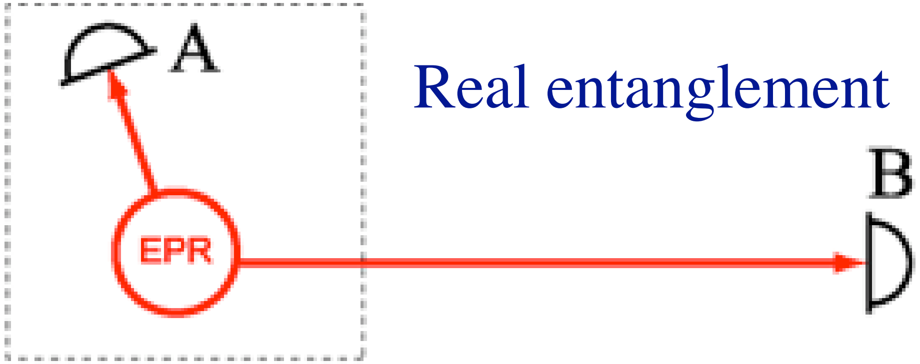
- Position-momentum entanglement
  - equivalent to near field-far field entanglement

$$\left[y, p_y\right] = 1$$

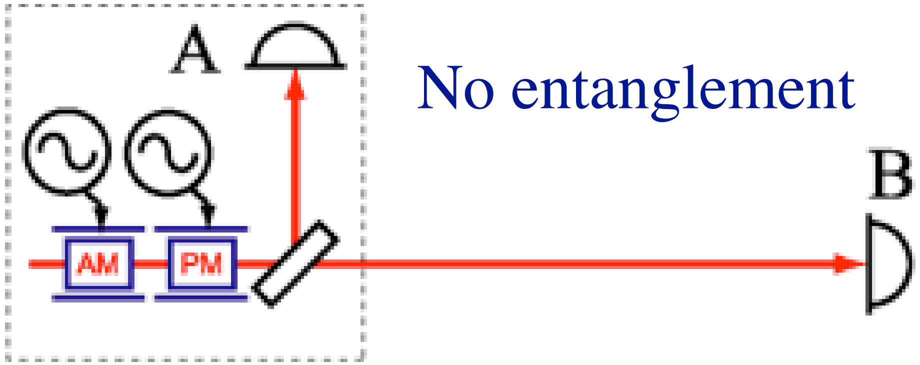
- Split detector entanglement
  - Squeeze 2 flipped modes and interfere on a beam splitter

Virtual entanglement

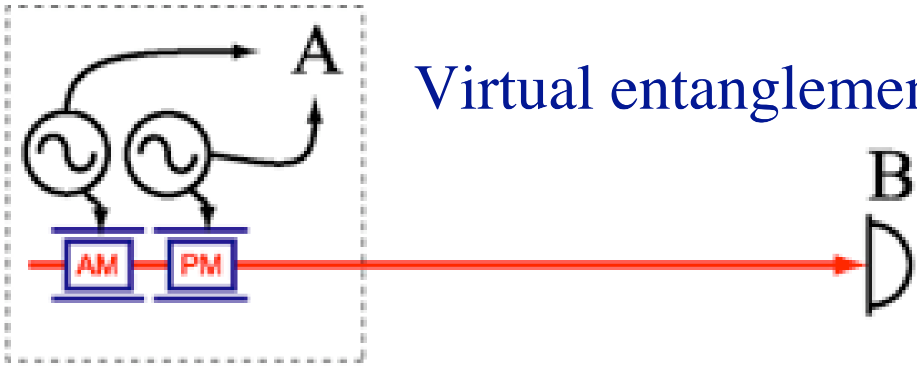
Real entanglement



No entanglement



Virtual entanglement



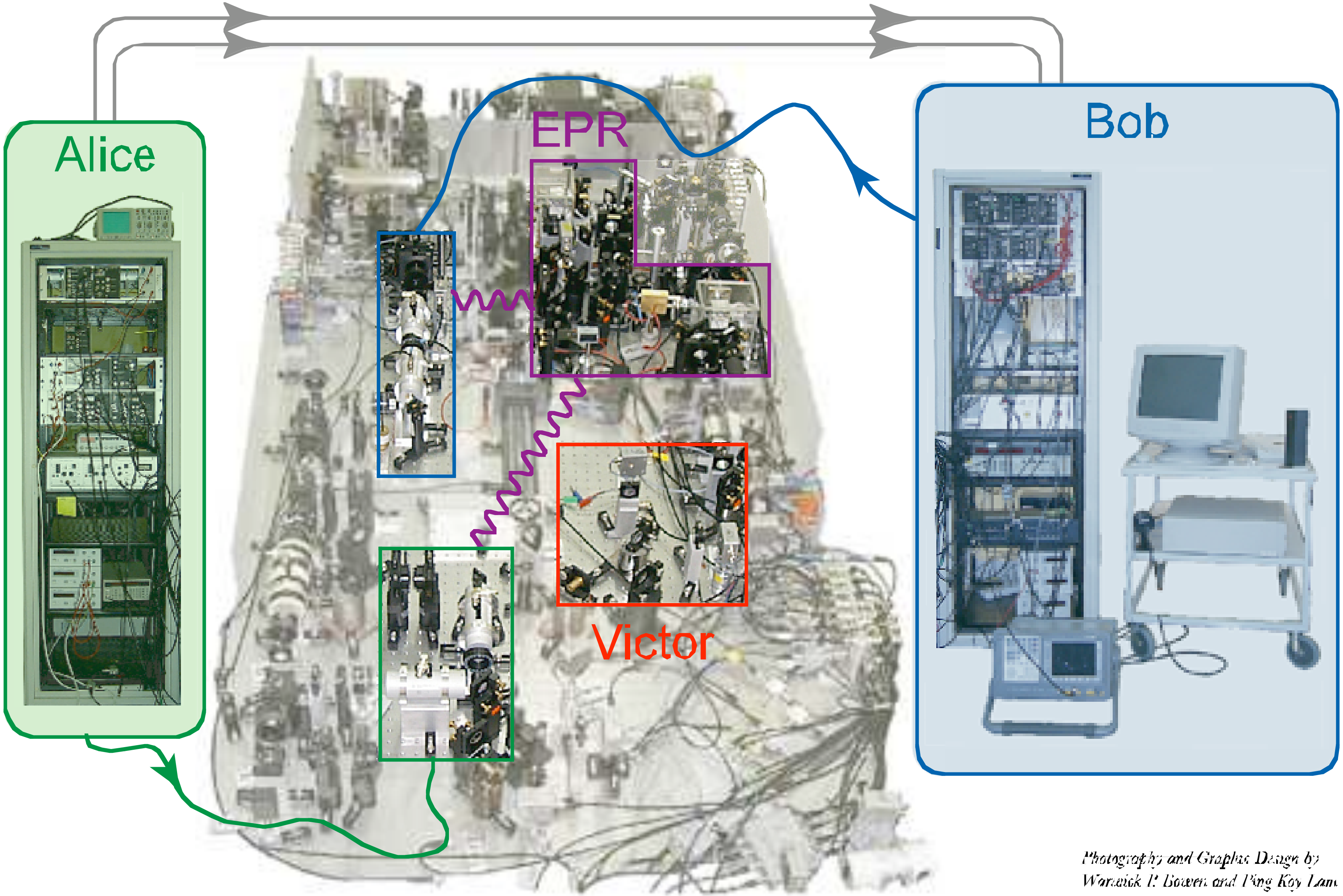
Applications of entanglement

- Quantum information processing
  - C-not gates in quantum computation
  - Grover’s algorithm
  - Shor’s algorithm
  - Quantum games

- Quantum communication and cryptography
  - Quantum key distribution
  - Super-dense coding
  - Secret sharing network

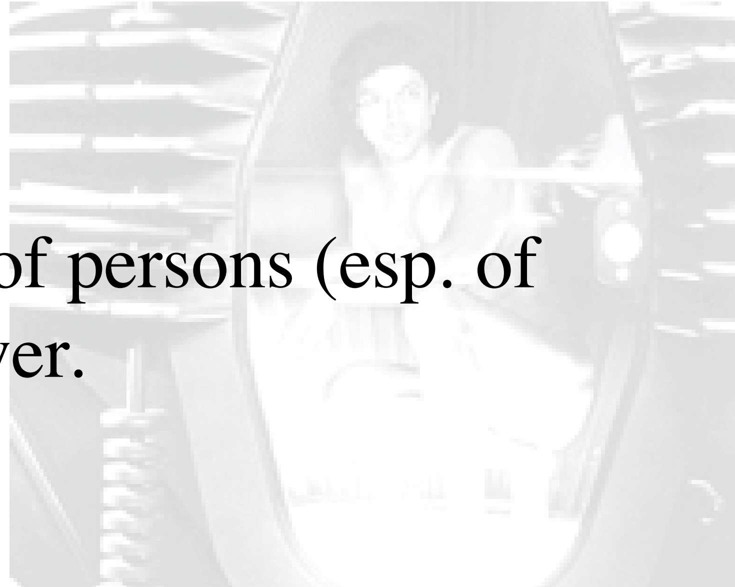
- Quantum metrology
  - Ultra-sensitive interferometric measurements
  - Sub-diffraction limited imaging resolution
  - Time keeping, lithography, etc.

The ANU Quantum Optics Teleporter

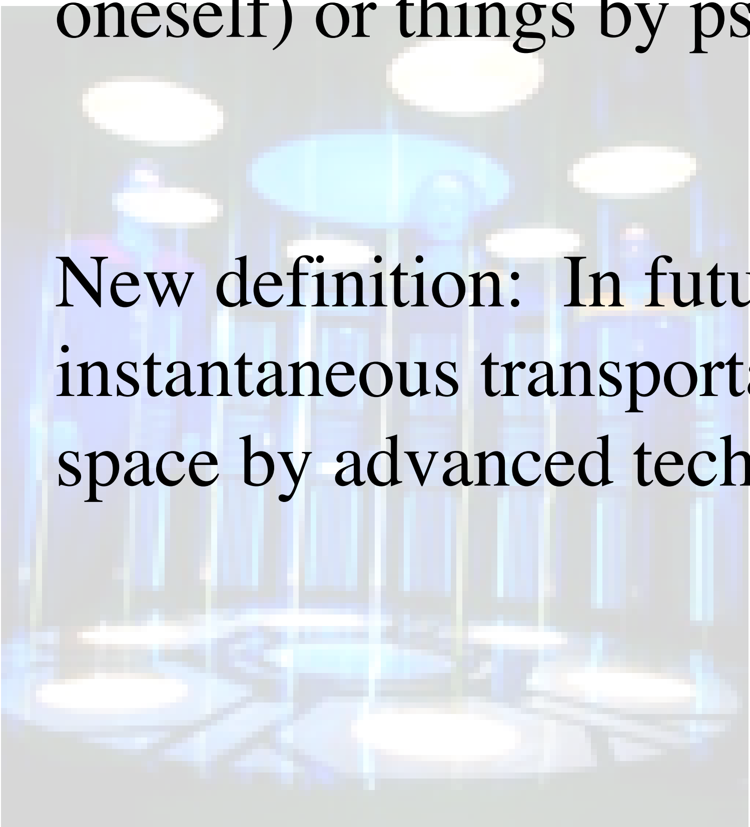


Photography and Graphic Design by Warwick P. Bowen and Ping Koy Lam

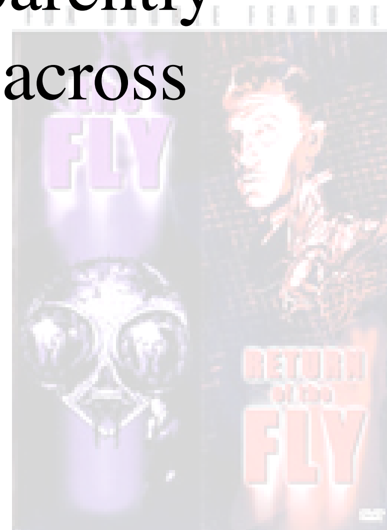
Oxford English Dictionary:



Old definition: The conveyance of persons (esp. of oneself) or things by psychic power.



New definition: In futuristic description, apparently instantaneous transportation of persons, etc., across space by advanced technological means.



The Fly

Teleportation definition

- Teleportation is the disembodied transportation of an object that involves
  - Thorough measurements of an input
  - Transmission of the measured results
  - Perfect reconstruction of the input at a different location

‘Alice’

(1) Measurement

(2) Transmission

(3) Reconstruction

‘Bob’

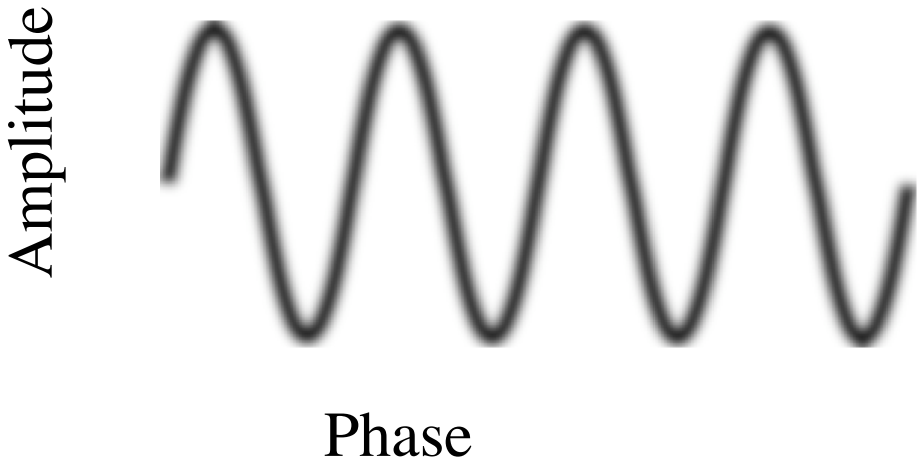
Alice and Bob are the names given to:

“A” the sender and

“B” the receiver.

Teleportation objective

- To prove that we can reconstruct the quantum state of light at a distance without paying any “quantum duty” of measurements.



- To teleport a laser beam that carries information.
- We encode small signals on the sideband frequencies of the light beam both on the phase as well as the amplitude quadratures.
- Equivalent to AM and FM simulcast.
- Need to show that information on both quadrature can in principle be perfectly reconstructed at a distance.

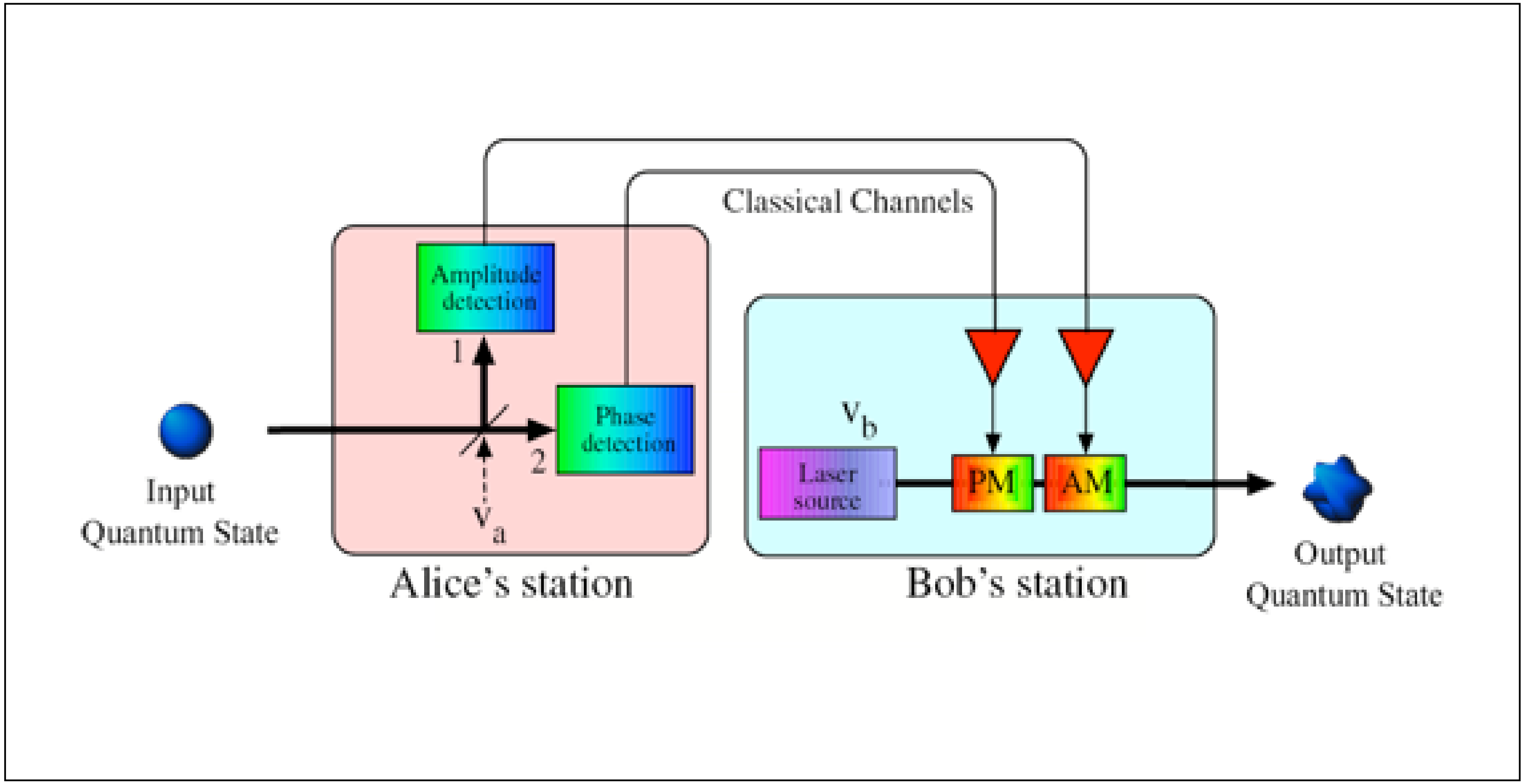
How big is the quantum noise

- If the intensity “stick” is the width of Australia, then how big is the quantum noise?

Assuming our experimental parameters (5kHz linewidth, 10mW @ 1064nm) then the quantum noise is a 1m gym ball.

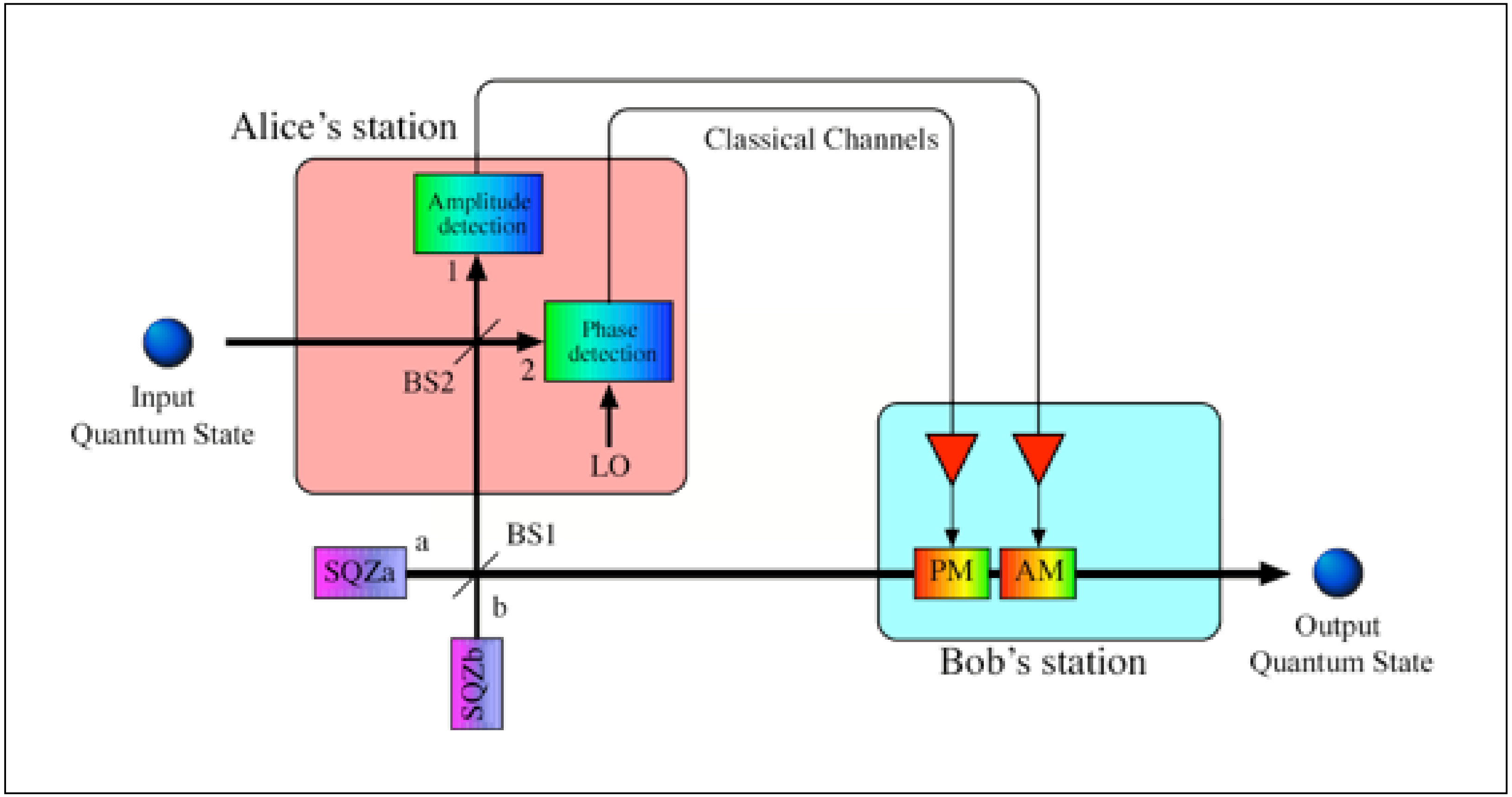
The classical teleporter

- Simultaneous measurements of the conjugate observables will introduce vacuum noise.



The quantum teleporter

- Plug the vacuum noise input with entanglement!

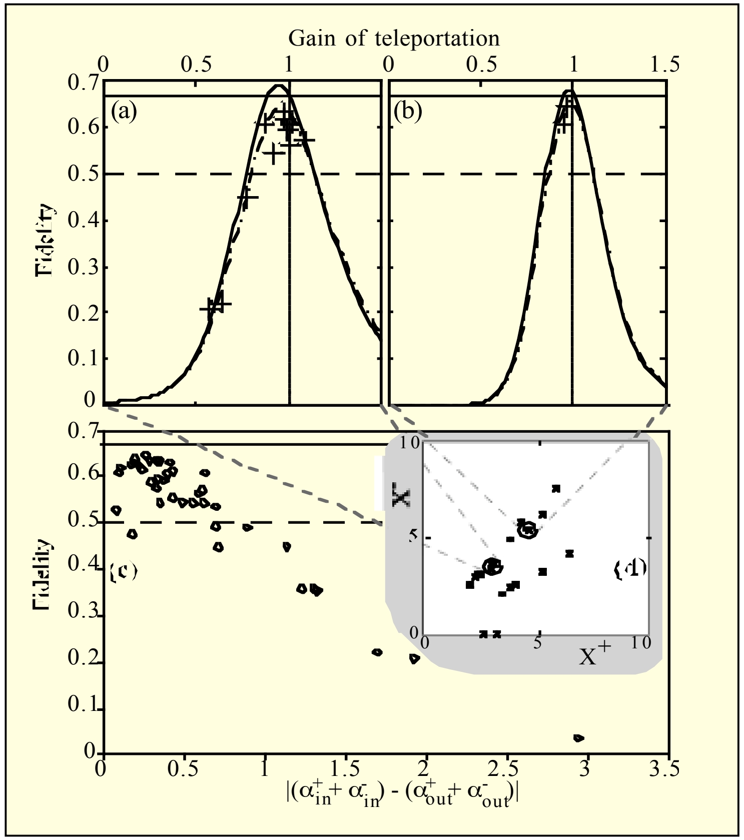


Teleportation fidelity

- Results can be analysed by comparing an ensemble of the input and the output states.
- We can use fidelity = $<\psi_{in}|\rho_{out}|\psi_{in}>$

- F = 1 is perfect teleportation
- F = 0.67 is the no-cloning limit
- F = 0.5 is the classical limit

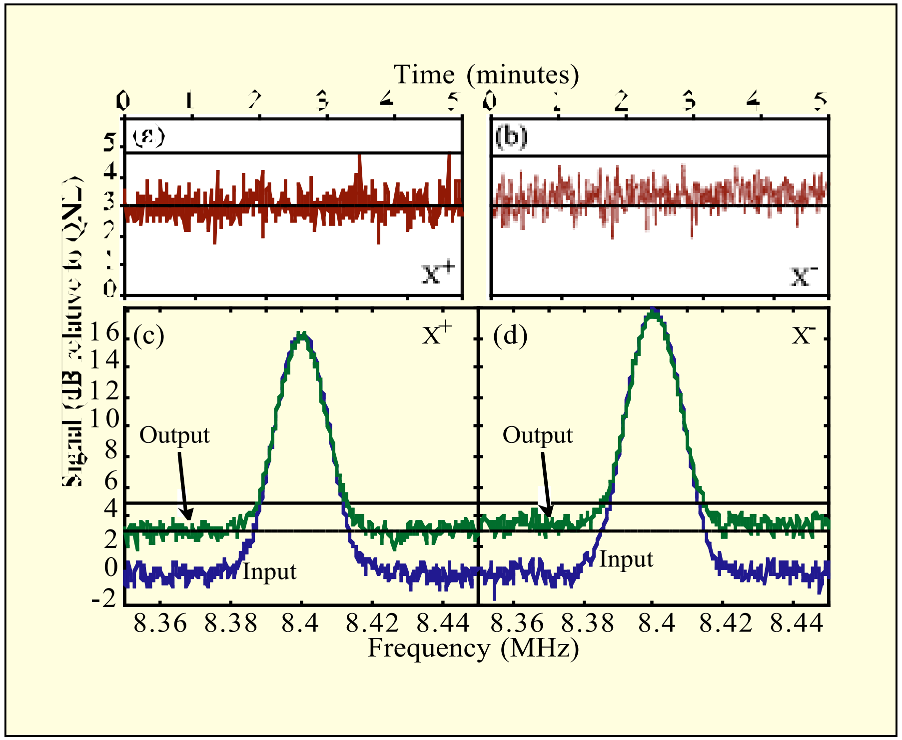
- Problem: Cannot tell whether there are quantum correlations between two objects via fidelity.



Teleportation information

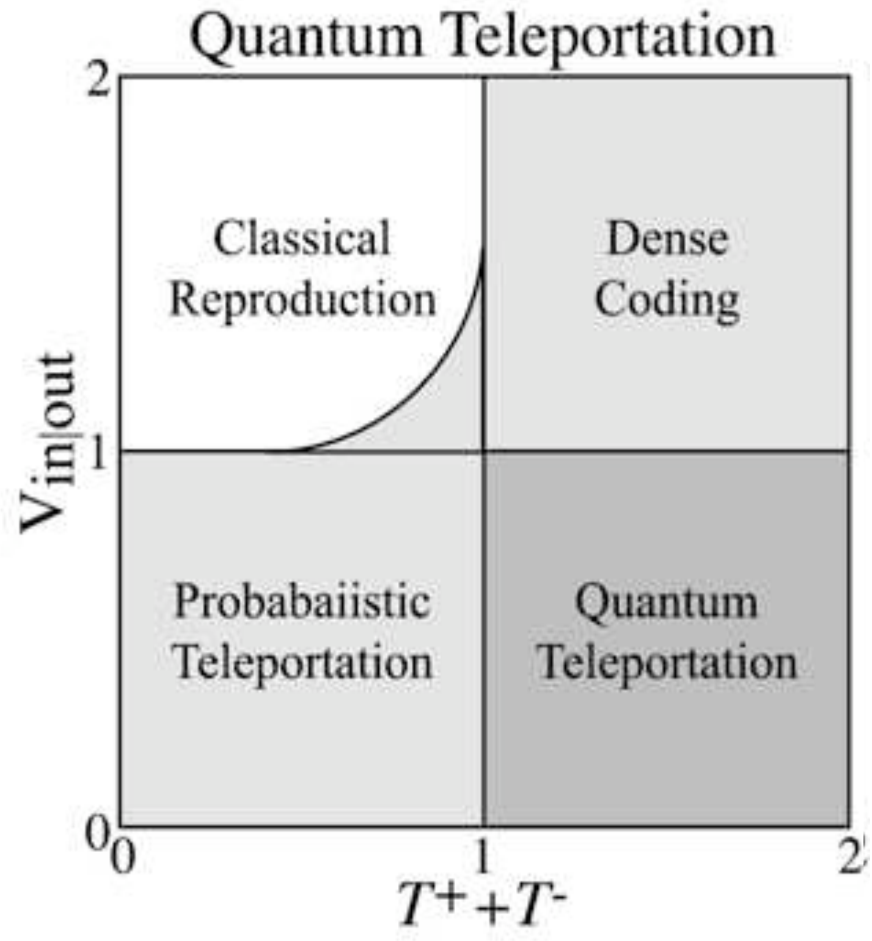
- Results can also be analysed by measuring the signal-to-noise of both amplitude and phase quadratures.
- Need to encode signal on both quadratures and measure the ratio of signal and noise power

- SNR_{out} = 100% (0 dB) of SNR_{in} is perfect teleportation
- SNR_{out} = 50% (3 dB) of SNR_{in} is the no-cloning limit
- SNR_{out} = 33% (4.8 dB) of SNR_{in} is the classical limit



T-V diagram (Ralph-Lam criteria)

- Analyse teleportation in terms of signal transfer coefficients and quantum correlations.
- Horizontal Axis: $T = SNR_{ampli} + SNR_{phase}$ information axis
- Vertical Axis: $V = V_{x+|y+} + V_{x-|y-}$ correlation axis

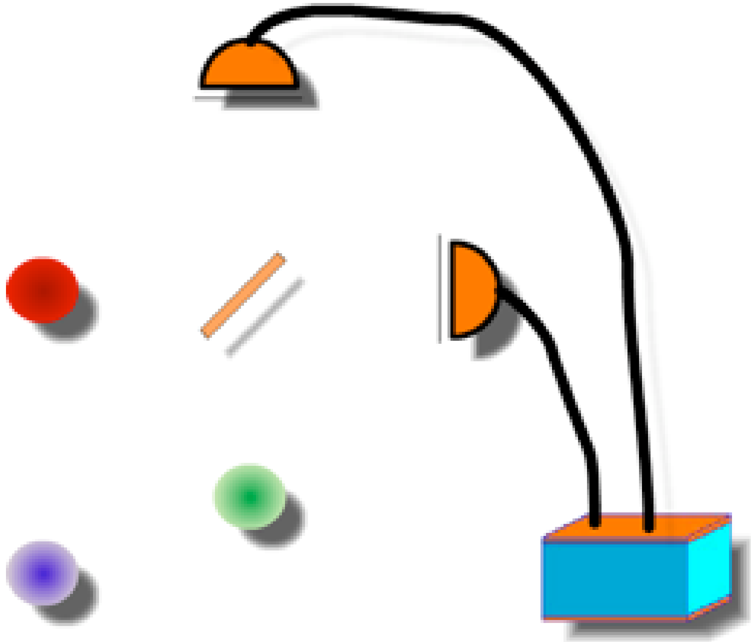


Grangier et. al, Nature 396, 537 (1998).

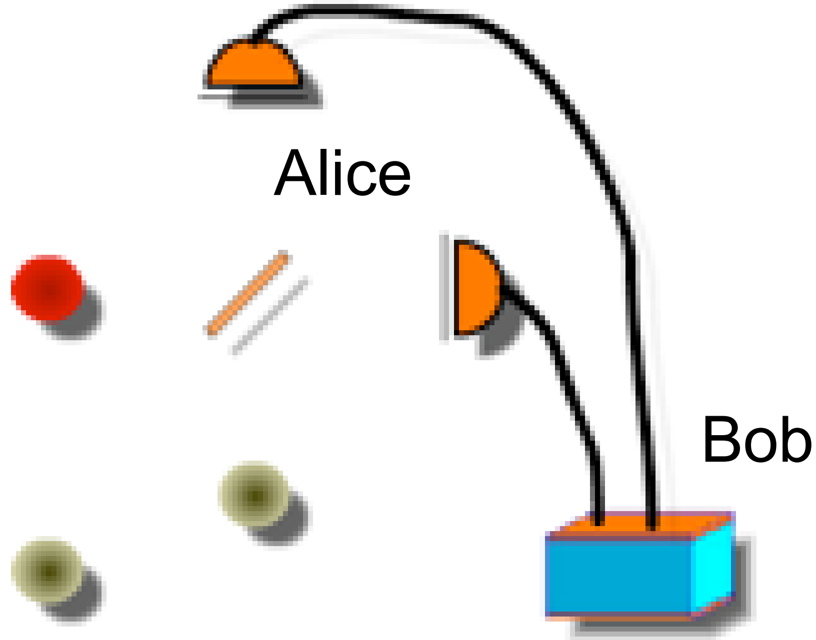
Ralph and Lam, Phys. Rev. Lett. 81, 5668 (1998

The Copenhagen explanation

No Entanglement



Entanglement

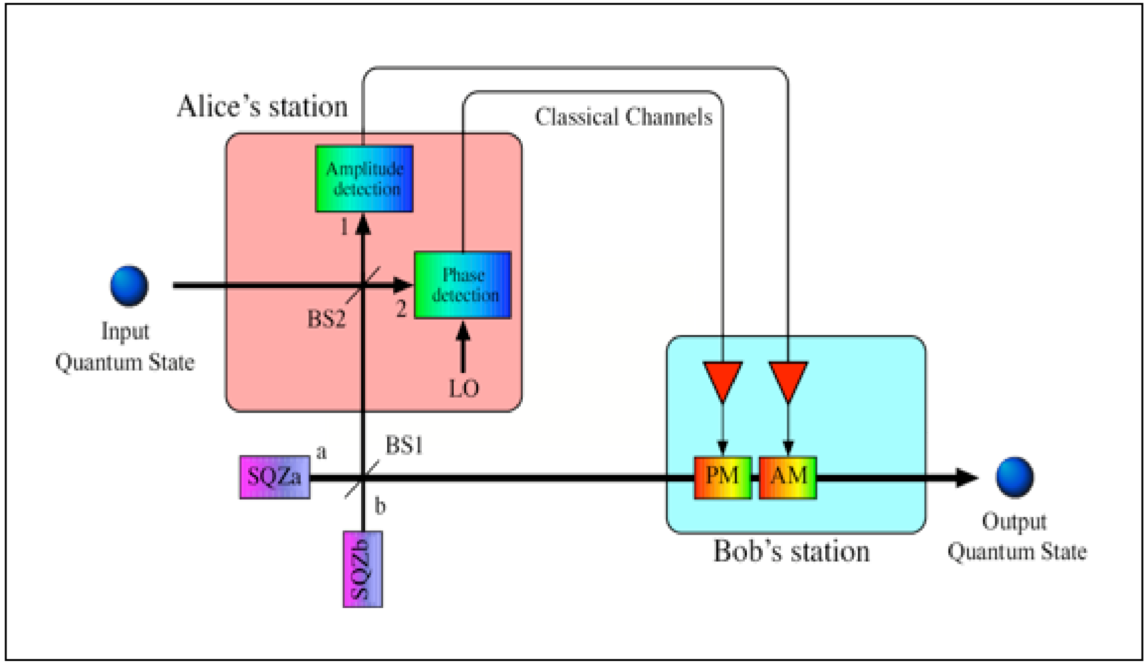


Wavefunctions collapse instantaneously.

Cramer's transactional interpretation

Quantum events can be describe by the interferences of advanced and retarded waves.

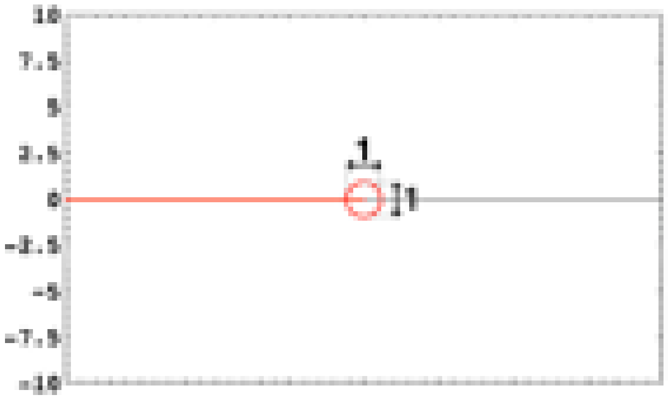
How can two bits produce one qubits?



In a quantum teleporter, information has to travel backward in time from Alice to the source of the EPR and then forward in time from the EPR to Bob.

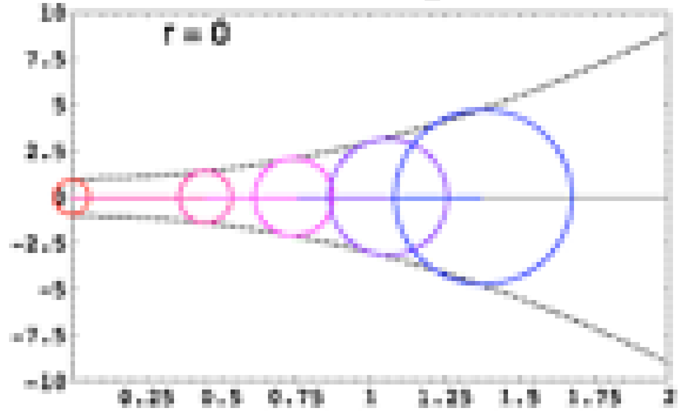
Qubits $\subseteq$ 2 bits + E-bit

Looking at the Wigner functions

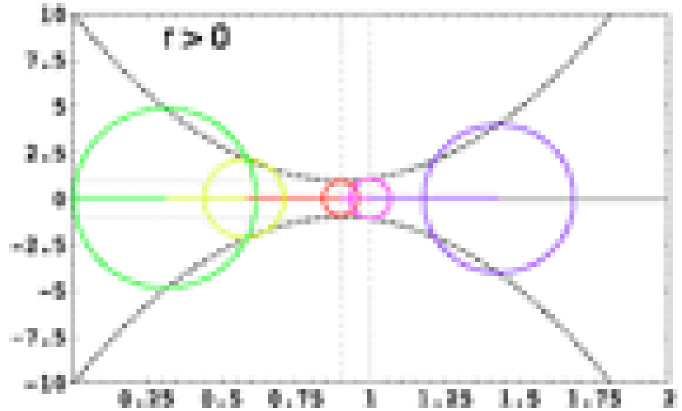


ALICE

Classical Teleportation



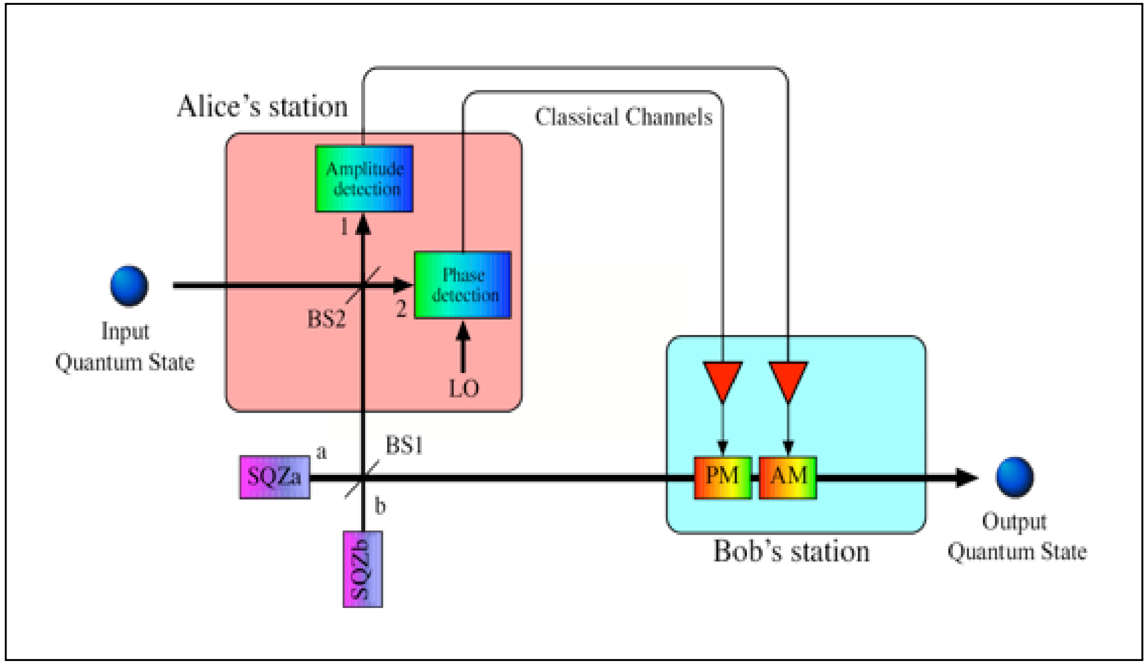
BOB



Quantum Teleportation

The Heisenberg Picture

Quantum information is contained in the classical channels. They are buried in the EPR noise.



We can think of the EPR source as being twin plasterers. One 'packed' the quantum information and the other 'unpacked' the quantum information.

Alice measured ($_{\text{signal}}$ + NOISE)
Bob reconstruct with: ($_{\text{signal}}$ + NOISE) - NOISE = $_{\text{signal}}$

Photonic description of entanglement

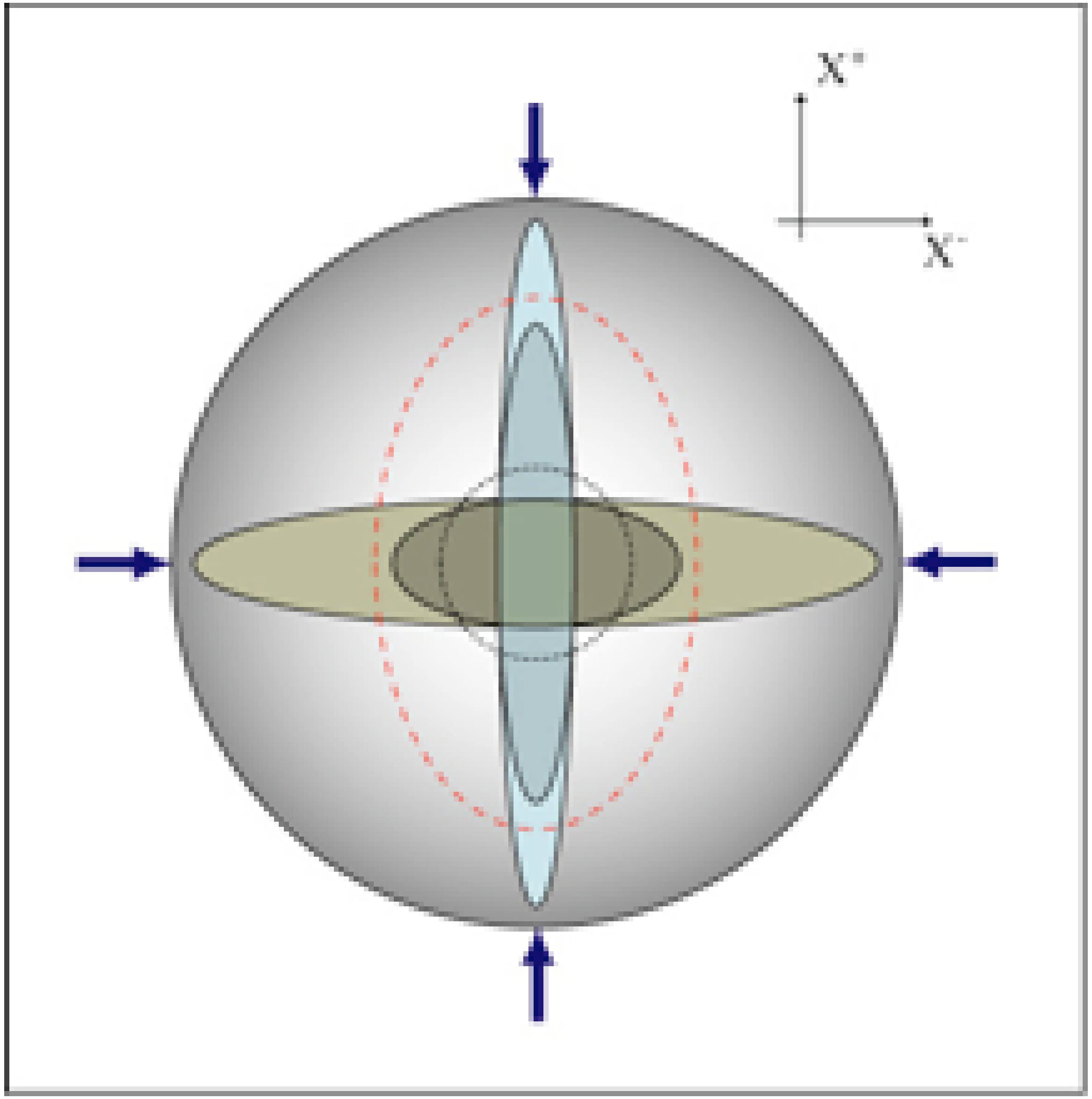
$$\boxed{\bar{n}_{\text{total}} = \bar{n}_x + \bar{n}_y = \frac{1}{4}\left(\Delta^2 X_x^+ + \Delta^2 X_x^- + \Delta^2 X_y^+ + \Delta^2 X_y^-\right) - 1}$$

$$\bar{n}_{\text{min}} = \sinh^2 r_1 + \sinh^2 r_2$$

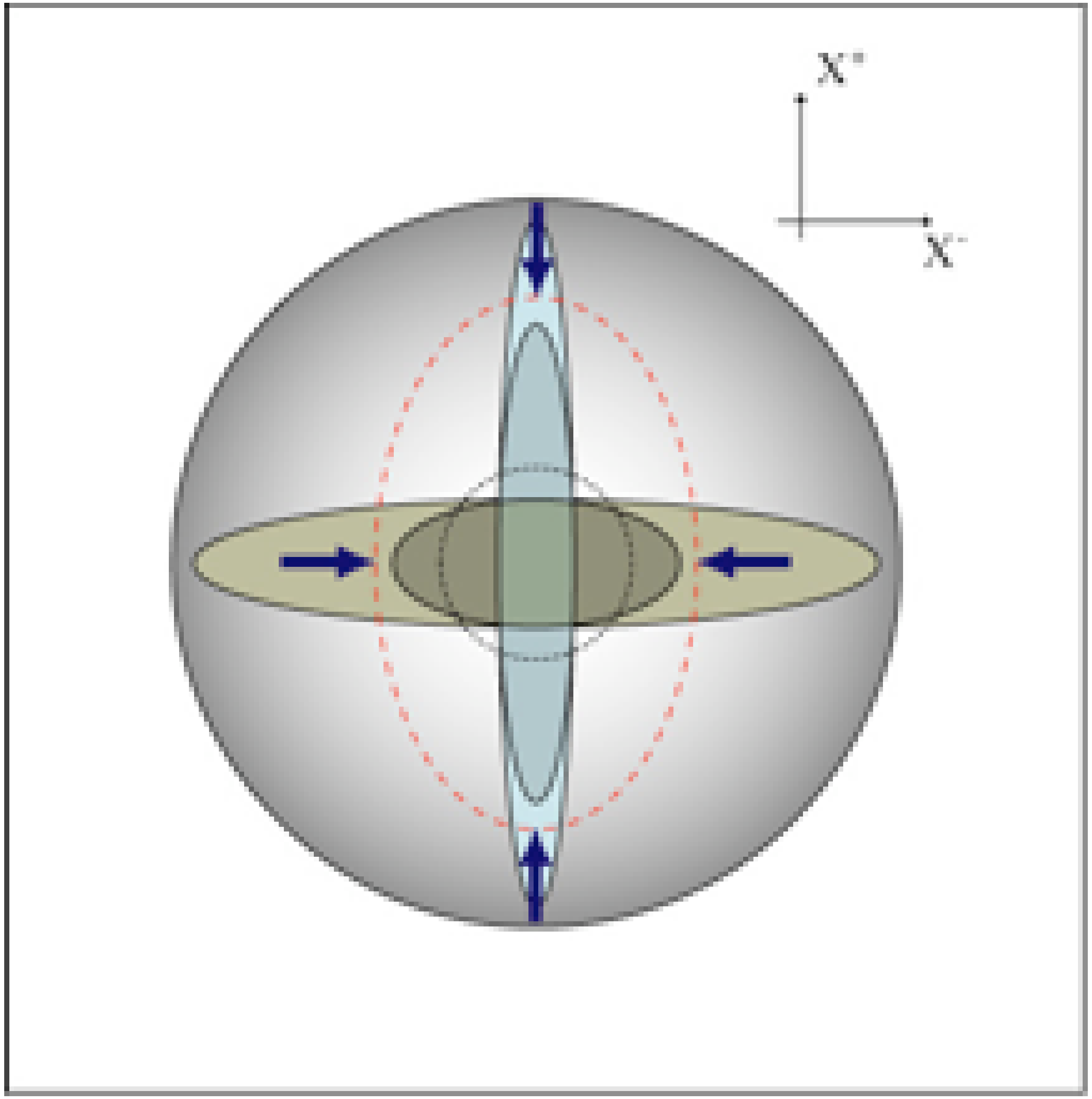
$$\bar{n}_{\text{bias}} = \frac{1}{2}\left(\frac{\Delta^2 X_{x\pm y}^+}{4} + \frac{1}{\Delta^2 X_{x\pm y}^+} + \frac{\Delta^2 X_{x\pm y}^-}{4} + \frac{1}{\Delta^2 X_{x\pm y}^-}\right) - \bar{n}_{\text{min}} - 1$$

$$\bar{n}_{\text{excess}} = \bar{n}_{\text{total}} - \bar{n}_{\text{min}} - \bar{n}_{\text{bias}}$$

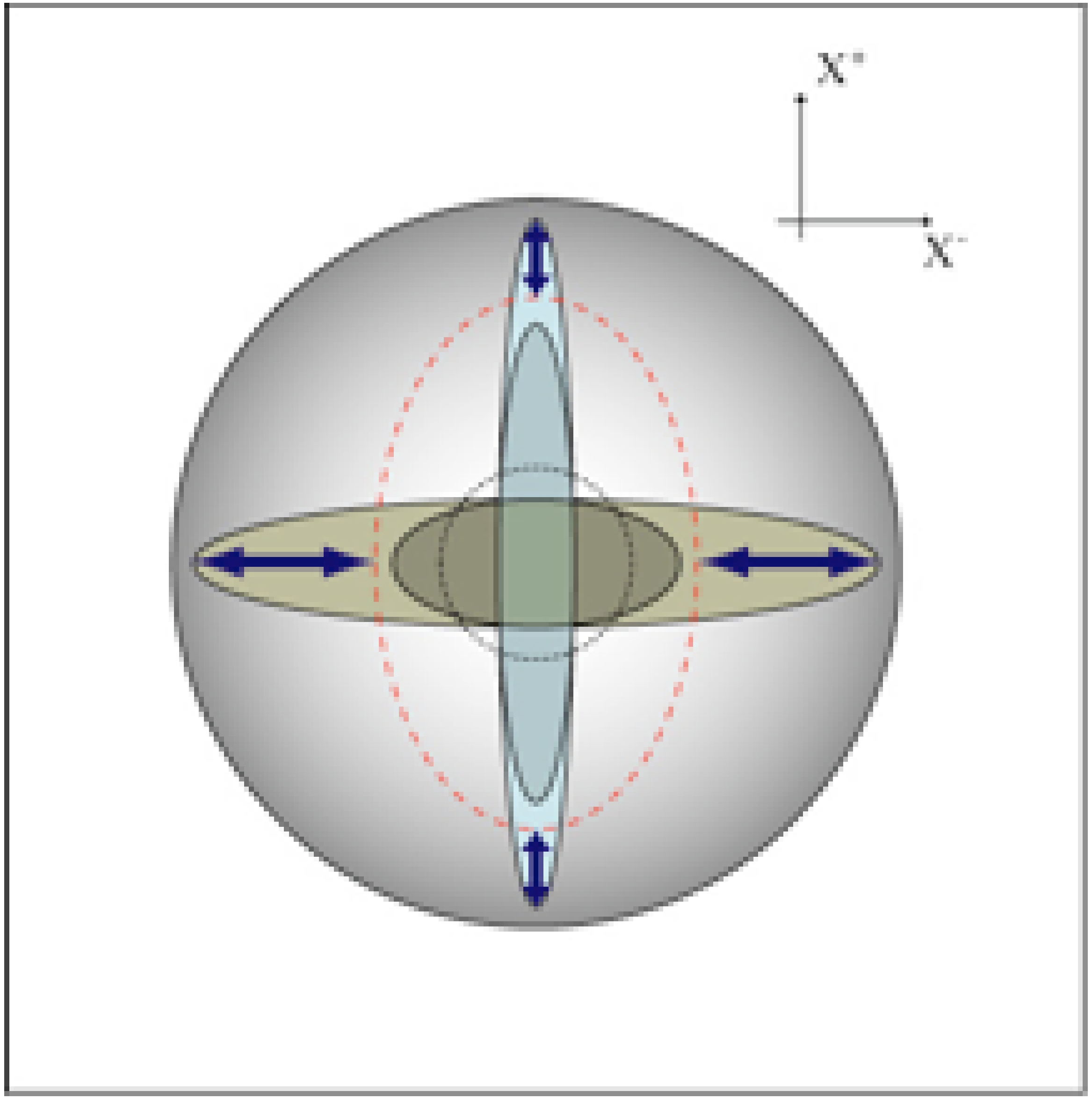
Total photons



Quantum photons



Biased photons



Excess photons