Lecture 1

BCS-BEC CROSSOVER PHYSICS IN ATOMIC SUPERFLUID FERMI GASES

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Since 1995, we have become familiar with **BEC in trapped gases of Bose atoms**. These atomic Bose condensates are **macroscopic matter waves** which behave like other waves. How to manipulate these matter waves is one of the major themes of this summer school .

Fermi gases have become the hot topic in **ultracold atom physics** in the last year.The goal of these four lectures is to introduce you to the **new Bose condensates** that arise in such **trapped atomic Fermi gases.**

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THE ESSENTIAL IDEAS

Interactions between the Fermi atoms can produce **bound states** (pairs) which act as **Bosons**. These **molecules** can then Bose condense below a certain transition temperature, giving rise to what is called a **Fermi superfluid**.

These **molecular Bose condensates** are the analogue of **metallic superconductors**, which involves a Bose condensate of **Cooper pairs** (a bound state of two electrons). Indeed, one of the goals of these lectures is to argue that there is **no** real distinction between a **Fermi superfluid** with a **molecular condensate** and one with a **BCS Cooper pair condensate.** This is what is involved in the famous **BCS-BEC crossover.**

The end result is that we now have **molecular matter waves** which involve a condensate of Bosons composed of bound states of atoms (**molecules**). These are like **atomic matter waves.**

Plan of four lectures:

1. Compare the properties of non-interacting Bose and Fermi gases. General discussion of modern theory of a **Bose** superfluid.
2. Introduction to idea of a condensate in a interacting **Fermi gas**
   - **Two-body effects** : A **Feshbach resonance** between two atoms, and the formation of molecules.
   - **Many-body effects**: some key ideas of the BCS **theory of superconductivity** in Fermi gases.
3. Review the crossover from the usual **BCS state** of large overlapping Cooper pairs to a **BEC gas** with small Cooper pairs, the **BCS-BEC crossover**.
4. Discuss an interacting Fermion-Boson model which allows one to neatly **imbed molecules** (produced by a two-body Feshbach resonance) into an interacting **Fermi gas of atoms.**

How do we produce quantum fluids and matter waves?

- Quantum effects are **smeared out at high temperatures** due to thermal motion. This is why physicists have been on a long quest to go to lower and lower temperatures. Life is more **interesting** as $T \to 0$, where more **delicate phases** of matter can become **stabilized**.
Superfluidity only lives at low T.

- BEC and ultracold atomic gases are just the latest spectacular discovery in this **quest for absolute zero** over the last 100 years.
The first quantum fluid systems studied were **superfluid ^4He** and **superconductors**.

What do we mean by low temperatures?

		milli	micro	nano
273 K	**1K**	**10^{-3} K**	**10^{-6} K**	**10^{-9} K**
(< 1875)	(1910 -1960)	~ 1970	~ 1980	> 1995

Quantum statistics

- Two classes of particles in nature: ***Bosons*** and ***Fermions***
- ***Fermions***: - integral spin particles – electrons, protons, neutrons, **atoms** with an odd number of neutrons.
- ***Bosons***: integral spin particles – photons, mesons, **atoms** with an even number of neutrons.

What is the origin of the physical difference between Fermions and Bosons?

At the microscopic level, it is associated with the behaviour of the many-body wave function under the exchange of ***identical*** *particles:*

$|\Psi(\mathbf{r}_1, \mathbf{r}_2, ..., \mathbf{r}_N)|^2$ - probability of finding 1 at $\mathbf{r}_1$, 2 at $\mathbf{r}_2$, …

$|\Psi(\mathbf{r}_2, \mathbf{r}_1, ..., \mathbf{r}_N)|^2$ - probability of finding 1 at $\mathbf{r}_2$, 2 at $\mathbf{r}_1$, …

Particles are ***indistinguishable***, these two probabilities

must be the ***same***, and hence we must have:

$$\Psi(\mathbf{r}_2, \mathbf{r}_1, ..., \mathbf{r}_N) = \pm\Psi(\mathbf{r}_1, \mathbf{r}_2, ..., \mathbf{r}_N)$$

+ sign: Bosons

– sign: Fermions

Some consequences

- For non-interacting atoms, each atom occupies some quantum state φ_α. For example, atoms in a box of volume V are in a planewave eigenstate, specified by the momentum ***k***

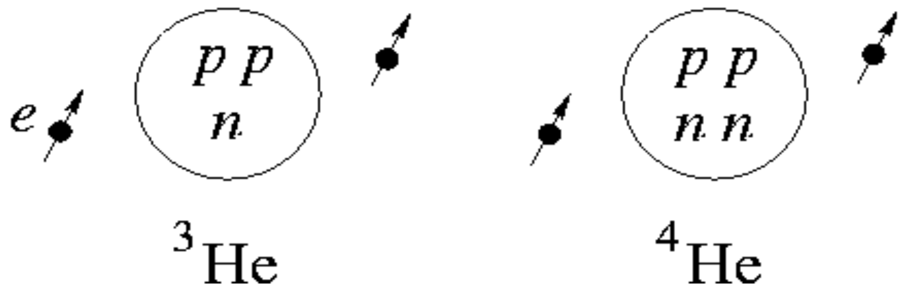
$$\phi_{\mathbf{k}}(\mathbf{r}) = \frac{1}{\sqrt{V}} e^{i\mathbf{k}\cdot\mathbf{r}}$$

- Two such non-interacting particles have a wave function

$$\Psi(\mathbf{r}_1, \mathbf{r}_2) = \frac{1}{\sqrt{2}} [\phi_\alpha(\mathbf{r}_1)\phi_\beta(\mathbf{r}_2) \pm \phi_\alpha(\mathbf{r}_2)\phi_\beta(\mathbf{r}_1)]$$

- If $\alpha = \beta$, this two-particle wave function **vanishes** for **Fermions**, but not for **Bosons.** Thus any **number** of **Bosons** can occupy a given single particle state but only ***one*** **Fermion** can occupy this state.We see the origin of the **Pauli exclusion principle** for **Fermions**, which determines everything!

This distinction leads to two different kinds of quantum fluids: Bose and Fermi



^{4}He (2 protons, 2 neutrons and 2 electrons) → **Boson**

^{3}He (2 protons, 1 neutron, 2 electrons) → **Fermion**

Both isotopes are chemically equivalent and have the ***same*** atomic spectra

- However, in liquid form at low temperatures, they behave ***completely differently!***
- ^{4}He becomes a ***superfluid*** below T ~ 2 K while ^{3}He is a ***viscous liquid***, before finally becoming a (different kind of) superfluid below T ~ 2 mK

Atoms, atoms and more atoms!

Most recent ultracold atom studies have used **alkali atoms,** since they have a very **simple electronic structure**, ie, one *s*-electron outside a closed shell. **Net spin** of an alkali atom is F = I ±1/2 .

Bose atoms of choice: ^{87}**Rb** , ^{85}**Rb** ^{23}**Na** , ^{7}**Li** , ^{133}**Cs**

Fermi atoms of choice: ^{6}**Li** , ^{40}**K**

Rule: Even number of neutrons gives a **Boson**, since the number of protons equals the number of electrons. Check this! Because an even number of neutrons gives a more stable nucleus, about **80%** of the periodic table are Bose atoms.

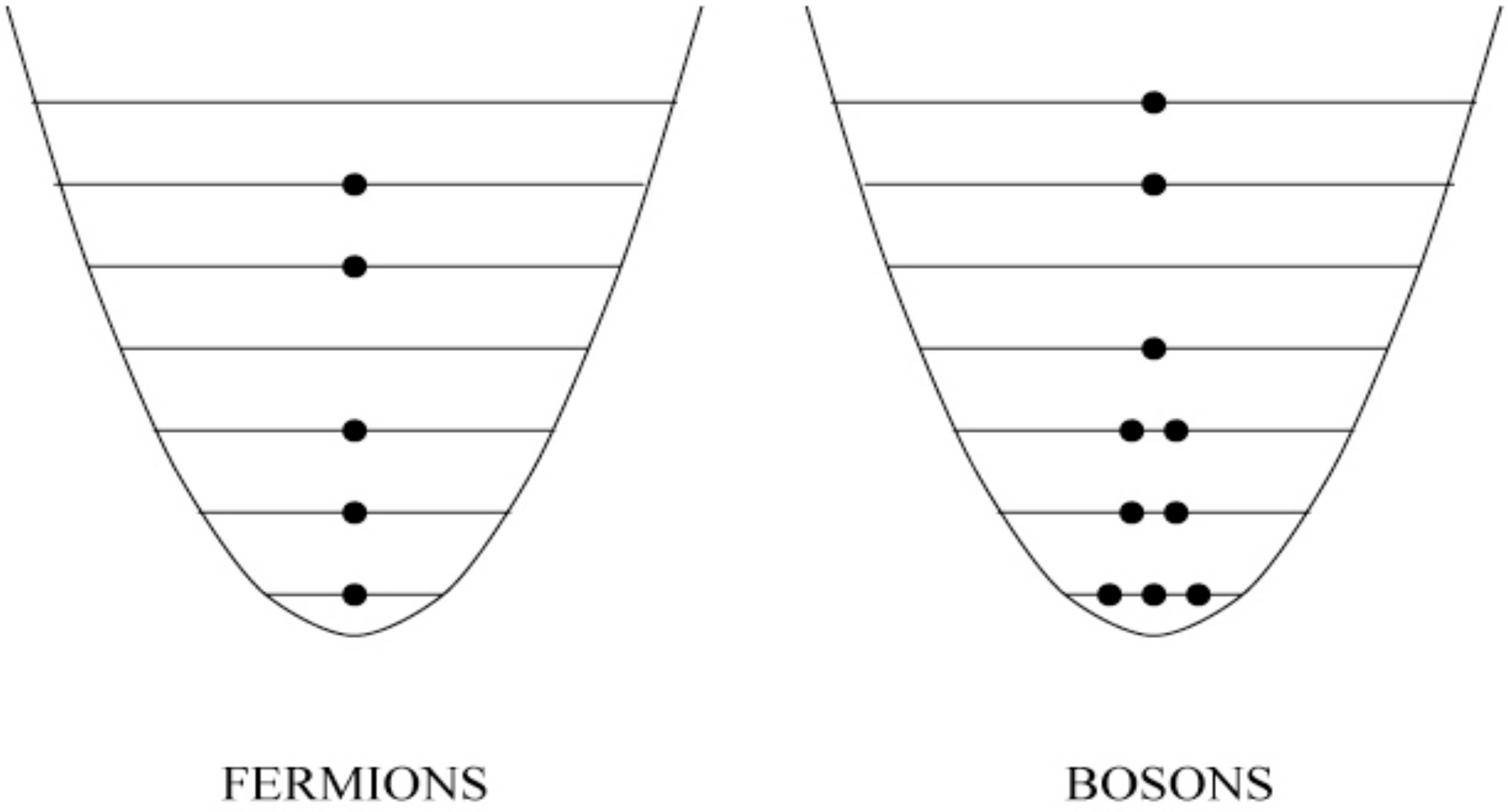
Trapped Atomic Fermi Gases

- Many groups can now cool Fermi gases down to **$0.05T_F$**, where **T_F** is the Fermi temperature (10^{-6} K).

- Two different atomic hyperfine states $| F, m_F \rangle$ are used, where m_F = - F, .., -1, 0, 1,.., F denotes the different Zeeman levels. F denotes the total spin of the atom(nuclear and electronic). For **Fermions**, F must be an odd multiple of 1/2.

- ^{40}K (F = 9/2) - Jin (JILA, Boulder).
  ^{6}Li (F = 1/2) - Grimm (Innsbruck), Ketterle (MIT) ,
  Hulet (Rice) , Salomon (ENS, Paris).

Occupation of single particle quantum states in traps



Finite Temperatures

The average number of particles in a given single particle state is given by either the Bose of Fermi distribution function:

$$f(\epsilon_\alpha) = \frac{1}{e^{\beta(\epsilon_\alpha - \mu)} \pm 1}$$

(+) Fermions

(−) Bosons

$$\beta = \frac{1}{k_B T}$$

The chemical potential **μ(n)** is determined by

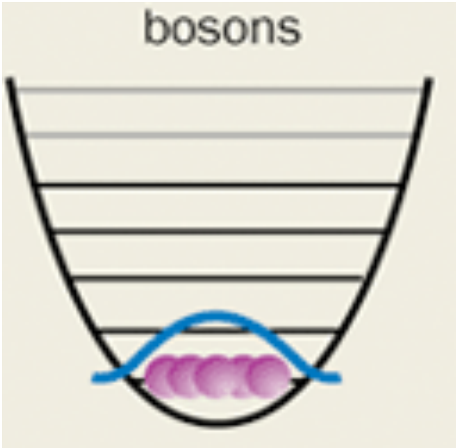
$$\langle N \rangle = \sum_\alpha f(\epsilon_\alpha)$$

Since the way the states are occupied is different in the two cases, the thermodynamic properties of systems of Bosons and **Fermions** will be quite different.

At temperatures just above T_{BEC} , all the the lowest lying energy states are occupied according to the **Bose distribution function**

$$N(E_n) = \frac{1}{e^{\beta(E_n-\mu)}-1}, \quad E_n = n\hbar\omega_0, \quad n = 0,1,2,..$$

Below T_{BEC}, the number of atoms in the **lowest** energy level of the harmonic trap **abruptly** starts to increase(**Einstein, 1925**) All the atoms occupy this state when $T \ll T_{BEC}$.These **atoms** are the **Bose condensate** of a trapped non-interacting gas , with $N \sim 10^6$ atoms in the same single-particle state.This is the **atomic matter wave**, given by the Gaussian ground state wavefunction of a SHO.



$\leftarrow \quad \hbar\omega_0$

A Bose-condensed gas is NOT a gas!!

When does a gas become a quantum gas?

- The **average** distance between the atoms in the gas = $d(n)$
- The **average** thermal de Broglie wavelength = $\lambda(T)$

$$\lambda(T) = \sqrt{\frac{2\pi\hbar^2}{mk_B T}}$$

The average density $n \sim$ (the volume per atom) $^{-1} \sim 1/d^3$

At high T $\rightarrow$ $\lambda(T) << d(n)$
At low T $\rightarrow$ $\lambda(T) \approx d(n)$ $\leftarrow$ ***quantum features***
At ultralow T $\rightarrow$ $\lambda(T) >> d(n)$

- Clearly the wavelike nature of the atoms in the gas becomes crucial.

The wavefunctions of different atoms now **overlap**, and
the atoms move in a **highly correlated** collective manner.

$$\lambda(T) \approx d(n) \leftarrow \textit{quantum features}$$

This simple formula contains all the physics!

$$k_B T_c = 2\pi\hbar^2 / m\lambda^2 \approx 2\pi\hbar^2 / md^2$$

$$Bosons \Rightarrow \frac{2\pi\hbar^2 n^{2/3}}{m} \approx T_{BEC} = Einstein, 1925$$

$$Fermions \Rightarrow \frac{2\pi\hbar^2 k_F^2}{m} \approx T_F \equiv \frac{(\hbar k_F)^2}{2m}$$

In the case of a **Fermi gas**, we have used the well known result that at T = 0, Fermions fill all the momentum states up to the **Fermi momentum** p_F, with an atom density

$$n = \frac{N}{V} = \int \frac{dk^3 f(\varepsilon_k)}{(2\pi)^3} = \frac{k_F^3}{6\pi^2}$$

It is useful to study some single-particle properties of a non-interacting Fermi gas in a harmonic trap.

A good first approximation is to use the **semi-classical equilibrium distribution of atoms,**

$$f(r,p) = \frac{1}{e^{\beta(\varepsilon_p + \frac{1}{2}m\omega_0^2 r^2 - \mu)} + 1}$$

At T = 0, the **filled** momentum states are given by

$$\varepsilon_p \le \mu - \frac{1}{2}m\omega_0^2 r^2, \mu(T=0) = \varepsilon_F$$

The integral over ***p*** is easy to do, giving the **density profile**

$$n(r) = \frac{k_F^3}{6\pi^2}\left(1 - \frac{r^2}{R_F^2}\right)^{3/2} \quad \text{where} \quad \varepsilon_F = \frac{\hbar^2 k_F^2}{2m} \equiv \frac{1}{2}m\omega_0^2 R_F^2$$

The size of the trapped Fermi gas is determined by R_F.
It is **larger** than than the size of a trapped Bose gas, since Fermions of the same species tend to repel each other as a result of the **Pauli exclusion principle**.

You can check yourself that by integrating $f(\boldsymbol{r}, \boldsymbol{p})$ over both $\boldsymbol{r}$ and $\boldsymbol{p}$ that

$$N = \frac{1}{6}\left(\frac{\varepsilon_F}{\hbar\omega_0}\right)^3 \quad \text{and thus} \quad \varepsilon_F = (6N)^{1/3}\hbar\omega_0$$

The T = 0 density profile of a **degenerate Fermi gas** is not much modified at finite $T \ll T_F$ or by interatomic interactions. This is typical of what is called a **normal Fermi liquid** in condensed matter physics.

Quantum field theory(or second quantization)

The most efficient way of **handling quantum statistics** is to describe the interacting gas of atoms in terms of quantum field operators which creat and destroy atoms,

$\psi^+(r)$ **creats** an atom at position r

$\psi(r)$ **destroys** an atom at r

These **field operators** satisfy the key **commutation relations**:

$$\psi(r)\psi^+(r') \pm \psi^+(r')\,\psi(r) = \delta\,(r - r')$$
$$\psi(r)\psi(r') \pm \psi(r')\psi(r) = 0$$
$$\psi^+(r)\psi^+(r') \pm \psi^+(r')\psi^+(r) = 0$$

– sign for **Bosons**, operators **commute** at different positions.
+ sign for **Fermions**, operators **anticommute** at different positions.

These commutation relations **guarantee** the correct symmetry and antisymmetry of the quantum states in the two cases!

Each kind of atom α has its own field operators $\psi_\alpha(r)$. Moreover, if we are dealing with **molecules** which are **Bosons,** we can introduce **molecular** quantum field operators $\psi_M(r)$.

All observables of the system can be written in terms of these field operators

1. Local density operator is $n_\alpha(r) = \psi_\alpha^+(r)\psi_\alpha(r)$

2. The "grand canonical" Hamiltonian of a **non-interacting** gas in a trap $V_{tr}(r)$ is given by

$$H - \mu N = \sum_\alpha \int d\vec{r}\,\psi_\alpha^+(r)\left[-\frac{\hbar^2\nabla^2}{2m} + V_{tr}(r) - \mu\right]\psi_\alpha(r)$$

3. The **interaction energy** between atoms of type α is

$$V = \sum_\alpha \int dr \int dr'\,\psi_\alpha^+(r)\psi_\alpha^+(r')v_{\alpha\alpha}(r - r')\psi_\alpha(r')\psi_\alpha(r)$$

A new kind of condensed matter

matter waves

Atomic Bose condensates can involve millions of atoms all occupying the **identical** single-particle quantum state.This is because the atoms are **Bosons**, and the **Pauli exclusion** principle does not hold. As a result these millions of atoms are now described

by a macroscopic single particle wavefunction $\Phi(r)$.This is a **macroscopic de Broglie matter wave.**

These matter waves are best thought of as a macroscopic **quantum order parameter,** describing a **new kind of condensed matter,** like a solid or a liquid. Using external fields, these condensates can move, change shape, oscillate, and scatter off each other, producing spectacular **interference** patterns. They allow one to see **quantum effects** with our eyes!

However, what is a **BEC** in an **interacting system of Bosons**? Not so obvious! **Useful to give a little history of BEC here.**

A Brief history of BEC 1938-1960

London (1938) first suggested that the **transition** at T = 2.17K in liquid He^4 was due to the formation of a BEC of He^4 atoms.

Tisza (1938) then suggested that the spectacular **superfluidity** effects first observed in 1938 were related to the coherent motion of this condensate. It took time to "put clothes" on this concept.

Landau (1941) developed a very successful **two-fluid theory** of superfluid He^4 based on **elementary excitations** of the many-body system. However, Landau's beautiful theory made no mention of BEC (or even mentioned **atoms**!).This was puzzling for years!

In the period 1950 -1960, many-body theorists agreed that a Bose **condensate** was the **microscopic basis** of the superfluidity in liquid He^4 and of the Landau phenomenological theory.

BEC is a broken symmetry: modern theory of the superfluid phase of matter

A good theoretical formalism always tries to capture the **key physical phenomena** involved in a new phase of matter, ie superfluidity. Bose condensation in an interacting system of N atoms involves a **finite fraction** (N_c/N) of atoms occupying the same "single particle" state $\Phi(r,t)$

$\Phi(r,t) = (n_c)^{1/2}\, e^{iS}$: amplitude and phase variables

The condensate **density** is $n_c(r, t) = |\Phi(r, t)|^2$

The superfluid **velocity** is $$v_s(r,t) = \frac{\hbar}{m} \nabla S(r,t)$$

It is the **phase** of $\Phi(r,t)$ which is the **origin** of all the wavelike and superfluid properties of the condensate. Note that this motion is **irrotational** (curl $v_s = 0$).

The dynamics of the condensate matter wave is that of **superfluid**, with **no dissipation** since we are dealing with “millions” of atoms all in one **single quantum state**. This is the **same physics** as in superconductors and in superfluid ^{4}He. What is the meaning of $\Phi(r,t)$??

Crucial theoretical ideas came from

Bogoliubov (1947,1963), Penrose and Onsager (1956)
Beliaev (1958), Nozieres and Gavoret (1964)
Hohenberg and Martin (1965), Anderson($\sim$ 1960).

These developed quantum field theory using Green’s functions to **define** and deal with a Bose condensate in a **systematic** way. It can deal with systems with strong interactions and at finite temperatures(**large depletion**).

Beliaev (1958) gave the **general formulation** of BEC by identifying the macroscopic wavefunction with the broken symmetry value of the quantum field operator,

$$\Phi(r,t) = \langle \psi(r,t) \rangle_{anomalous}$$

This anomalous average **picks** (symmetry-breaking) a **specific phase** of the superfluid phase, at the price of not dealing with states with a definite value of N. This is done by adding a **small perturbation** which creats and destroys atoms. Note that relation to the **many particle wavefunction** $\Psi(r_1, r_2, r_3, \ldots, r_N)$ is non-trivial, but this is **not** needed.

This gives a systematic way of **separating** out the **condensate part** in an interacting system of Bosons,

$$\psi(r) = \Phi(r) + \tilde{\psi}(r)$$

= **condensate part** + **non-condensate part**

Superfluidity in liquid He^4 and in Bose gases

First application of BEC concept was to **liquid ^{4}He** in 1938 by Fritz London, where the superfluid transition is at $T_c = 2.17K$.

However, in **liquid ^{4}He** with **strong interactions**, even at $T = 0$, only **9%** of the ^{4}He atoms are in the condensate.

The analogy between superfluid He^4 and a dilute gas only appears at **finite temperatures**, where there are a lot of atoms **thermally excited out** of the condensate. If the collisions are strong enough to produce local equilibrium , one can derive the famous **Landau two-fluid equations.**

Superfluid component => condensate atoms
Normal fluid component => non-condensate atoms

Research on **Fermi Superfluids** is published in

1. **Physical Review A**, in **matter wave** section.
2. **Physical Review Letters**, first section (**general physics**).
3. Important papers also appear in **Nature** and **Science.**
4. Nice reviews of recent results are given in the *Search and Discovery* Section of **Physics Today.** Good for non-experts.

For more details on my own research, see
http://www.physics.utoronto.ca/~griffin

My work on Fermi superfluids is done in collaboration with
Prof. Yoji Ohashi , University of Tsukuba, Japan.

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More information on ultracold matter:

1. Two excellent textbooks have been published:

   BOSE-EINSTEIN CONDENSATION IN DILUTE GASES, by **Pethick and Smith** (Cambridge , 2002).

   BOSE-EINSTEIN CONDENSATION, by **Pitaevski and Stringari** (Oxford, 2003).

2. Varenna Lectures, **BOSE-EINSTEIN CONDENSATION IN ATOMIC GASES, ed by Inguscio, Stringari and Wieman,** (IOS Press, Amsterdam, 1999). Excellent review articles.

3. Classic book on **interacting Fermi and Bose gases**:

   QUANTUM THEORY OF MANY-PARTICLE SYSTEMS by **A. L. Fetter and J. D.Walecka** (Available as a Dover book)