Quantum Imaging

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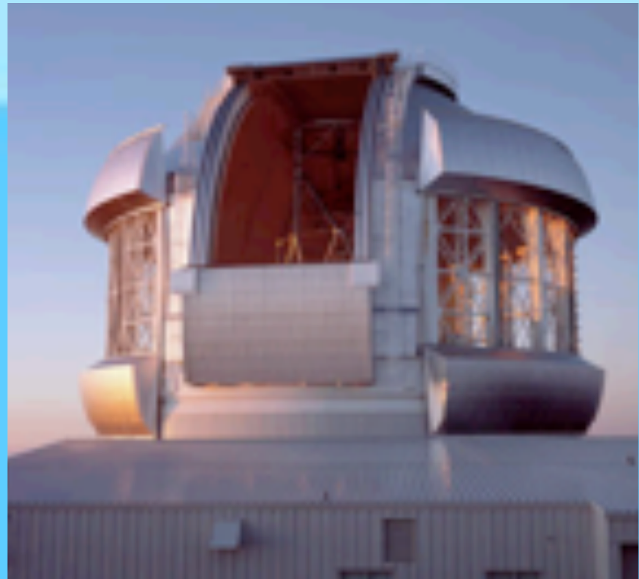
Resolution ?

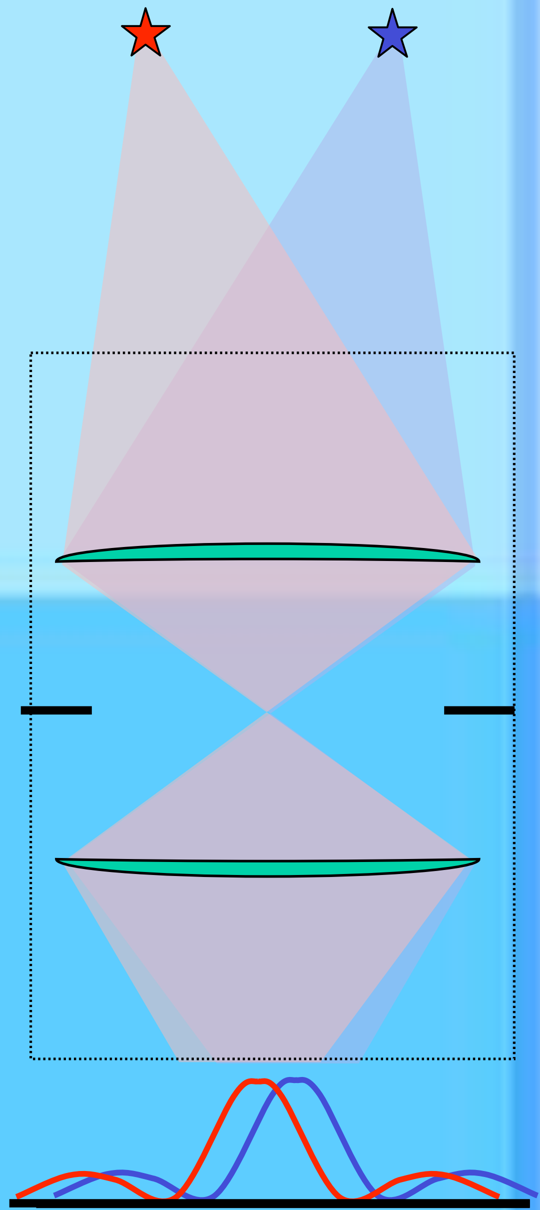
- Ability to separate details

XIXth century : Lord Rayleigh

how accurately can I separate two objects in space ?

Gemini north dome, Hawaii



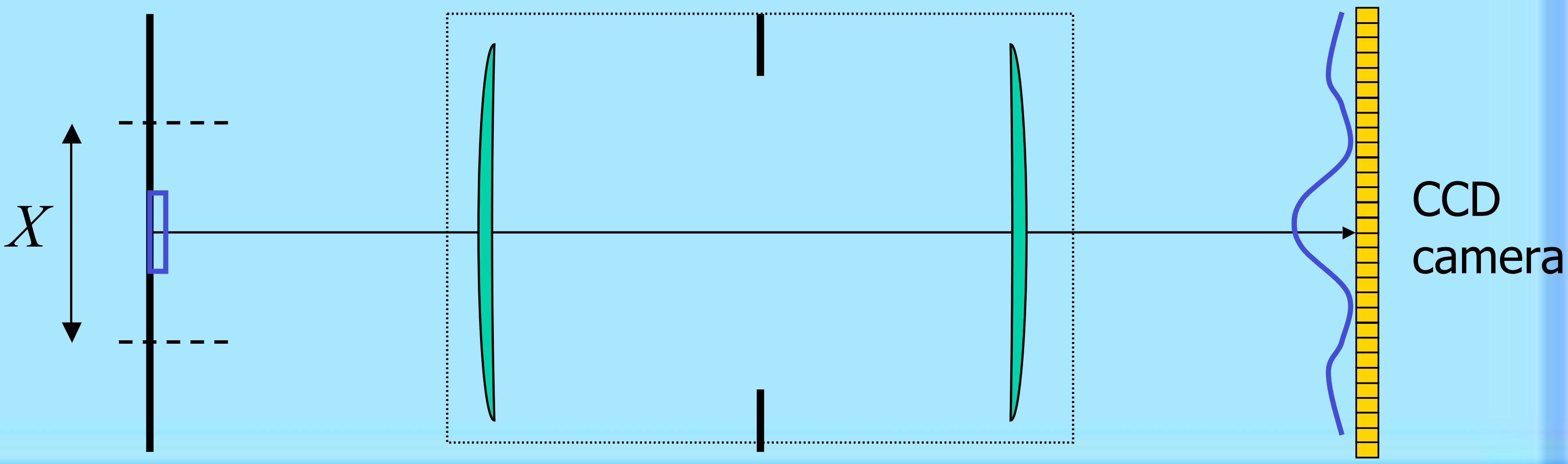


Resolution is limited by spot size

Diffraction theory : resolution limited by the wavelength ... λ

Resolution ?

XXI[st] century : image sensor : diode arrays, CCD cameras, ...



Object plane
a(y)

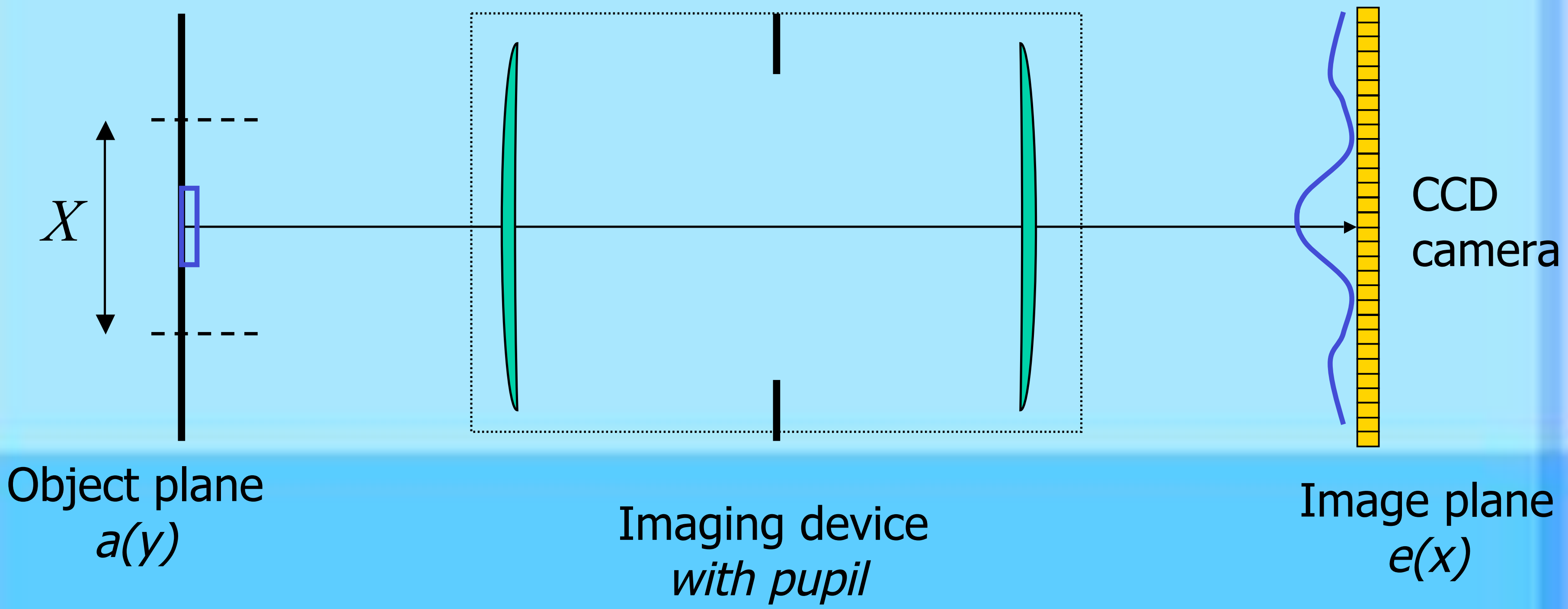
Imaging device
with pupil

Image plane
e(x)

There exist eigenmodes of the system (*prolate spheroidal functions*), $f_k(x)$ with eigenvalues t_k (transmission coefficient).

The knowledge of these functions, together with *e(x),* allows the 'perfect' reconstruction of the object

Limits to resolution



Quality of the detectors : size, number of pixel, response,…
Classical noise (vibrations, thermal noise,…) ⇨ **Technical limits**

Quantum nature of light (quantum noise) ⇨ **Fundamental limit**

Optical resolution vs. information extraction

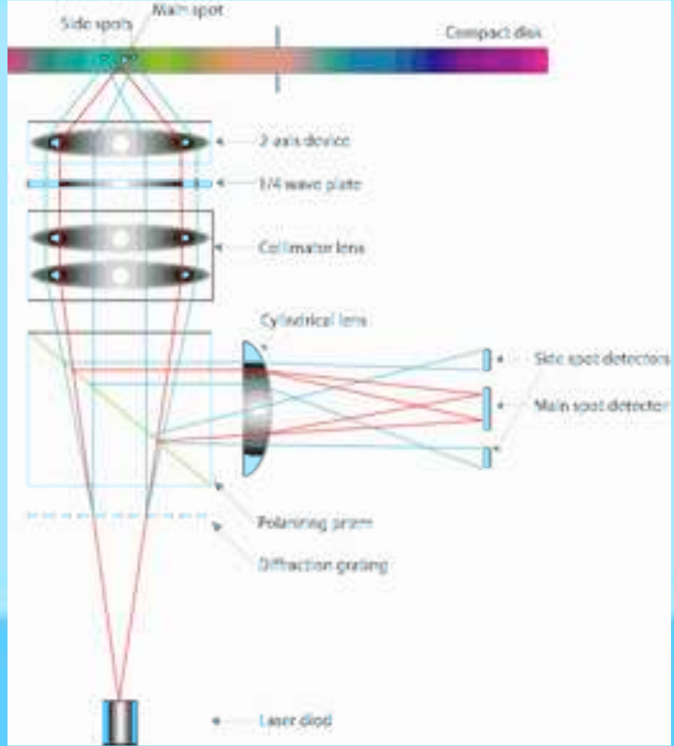
Optical resolution

No a-priori information on the image : smallest details measurable.

- In many practical cases : the Rayleigh criteria.
- Crossing the standard quantum limits requires very multimode quantum light, i.e. many resources.

Optical resolution vs. information extraction

Information extraction



Optical read out

A lot of a-priori information : presence and/or modification of a given pattern.

- Quantum limit easier to reached : orders of magnitude smaller than the Rayleigh criteria.
- We will show that crossing the standard quantum limit requires a limited amount of resources.

Outline

- Single mode versus multimode light
- Quantum limits to resolution
- Few modes approach : the quantum laser pointer
- Many modes approach : multimode cavities

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Modal decomposition of light

Paraxial approximation

A beam of light is the result of the excitation of an infinite set of harmonic oscillators.
The electric field distribution can be expanded over a transverse mode basis :

- plane waves basis : very suitable for calculation

$$E(\vec{r},t) = \sum_{\vec{k}} \alpha(\vec{k}) e^{i(\vec{k}.\vec{r}-\omega(\vec{k})t)}$$

However, for the propagation of a beam of light, we make several approximations :
- the light is monochromatic : $\omega(k)$¼ ω_0
- the direction of propagation is well defined : k ¼ k_z

$$E(\vec{\rho},z,t) = \mathcal{E}(\vec{\rho},z) e^{-i\omega_0(t-\frac{z}{c})} \text{ with } \vec{\rho} = (x,y)$$

Where $\mathcal{E}(\vec{\rho},z)$ is the slowly varying envelope of the fields that satisfies the propagation equation in the vacuum, projected onto the polarisation axis :

$$\frac{\partial^2 \mathcal{E}(\vec{\rho},z)}{\partial x^2} + \frac{\partial^2 \mathcal{E}(\vec{\rho},z)}{\partial y^2} + 2ik\frac{\partial \mathcal{E}(\vec{\rho},z)}{\partial z} = 0$$

Modal decomposition of light

Transverse modes basis

$\mathcal{E}(\vec{\rho}, z)$ can be expanded on a transverse modes basis such as : $\{u_i(\vec{\rho}, z)\}_i$

$$\int u_i^*(z, \vec{\rho}) u_j(z, \vec{\rho}) d^2\rho = \delta_{ij}$$ orthonormality

$$\sum_i u_i^*(z, \vec{\rho}) u_i(z, \vec{\rho}\,') = \delta(\vec{\rho} - \vec{\rho}\,')$$ completeness

There is then a unique set of coefficient α_i such as :

$$\mathcal{E}(\vec{\rho}, z) = \sum_i \alpha_i u_i(\vec{\rho}, z)$$

α_i: field amplitude in mode i

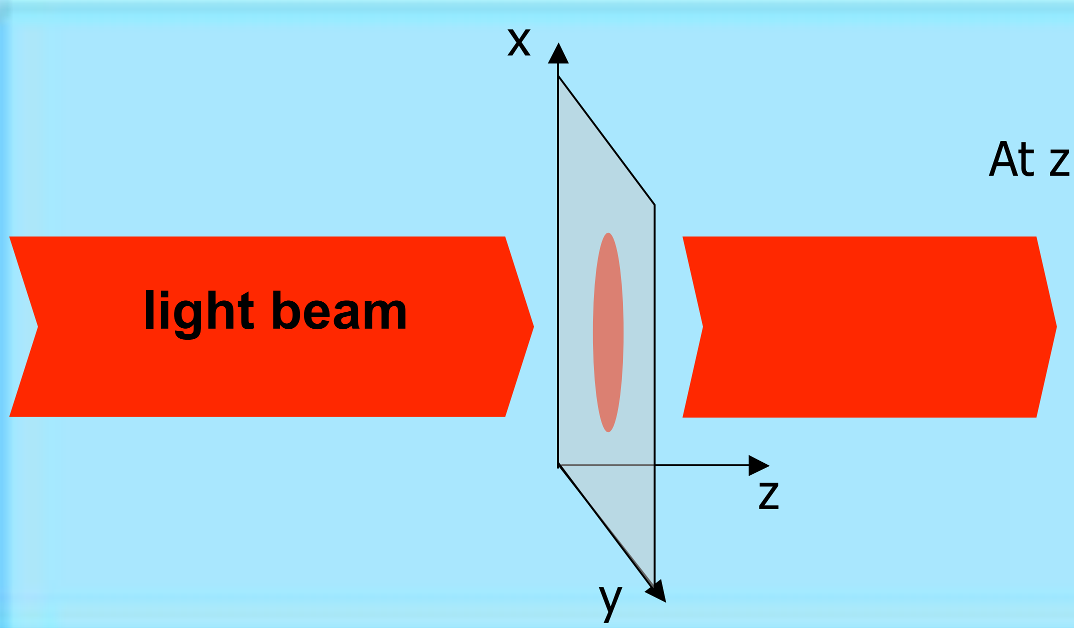
$u_i(\vec{\rho}, z)$: shape of mode i

It contains all the image information

Remark :

As the modes have to satisfy the propagation equation, their knowledge at z=0 is enough.

Modal decomposition of light



At z=0, $\mathcal{E}(\vec{\rho}) = \sum_i \alpha_i u_i(\vec{\rho})$

$$\mathcal{E}(\vec{\rho}, z) = \sum_i \alpha_i u_i(\vec{\rho}, z)$$

Examples

- Pixel basis : $u_i(\vec{\rho}, z = 0) \approx \delta(\vec{\rho} - \vec{\rho}_i)$

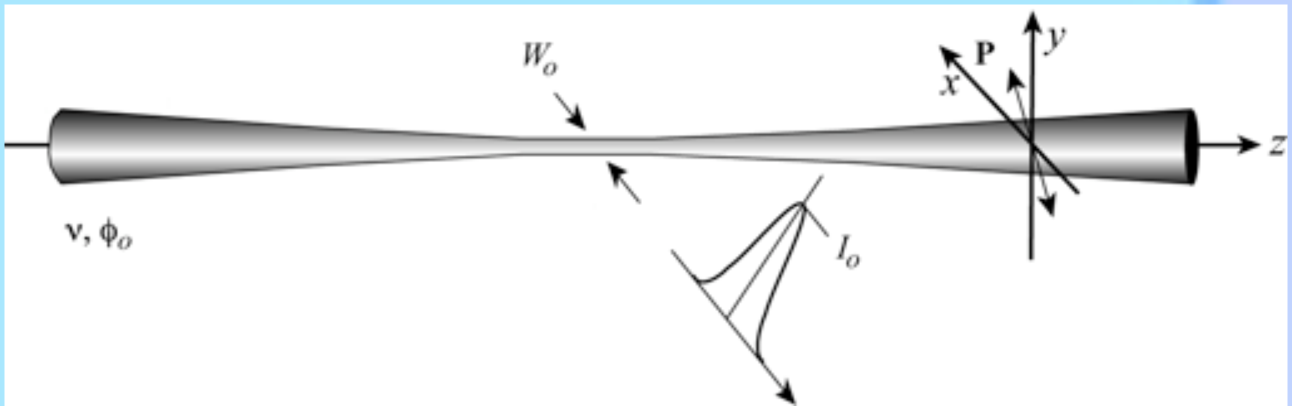
Advantages : very natural to describe random images
convenient for numerical simulation

Drawbacks : mode diffraction is very important
predicting the field shape under propagation is difficult

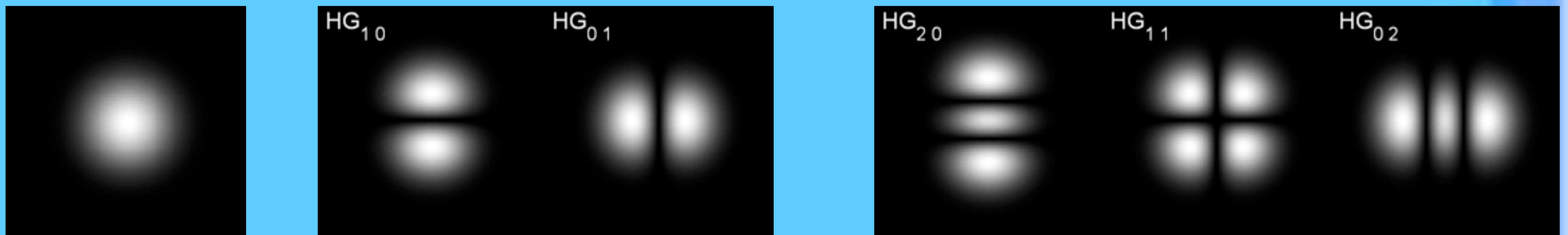
Gaussian modes

Gaussian modes basis : eigen modes of the propagation

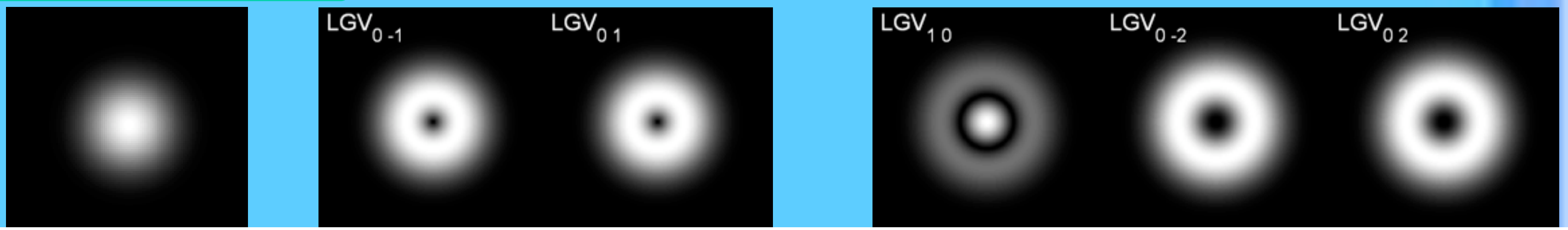
These modes have a transverse shape that remain constant under propagation.
They are adapted for light coming out of a cavity (such as laser beams).



Hermite-Gauss modes



Laguerre-Gauss modes



Single mode vs. multimode classical light ?

Possible to compute the number of modes ?

It depends on the choice of the basis !

For a field coming out of a cavity, one will naturally choose the Hermite Gauss or Laguerre Gauss basis.

Single mode basis

We have a given image : $\mathcal{E}(\vec{\rho}, z)$

We choose the first mode such as : $u_0(\vec{\rho}, z) = \frac{\mathcal{E}(\vec{\rho},z)}{|\mathcal{E}(\vec{\rho},z)|}$

It is always possible to choose the other modes to satisfy the completeness and orthonormality conditions $\{u_i(\vec{\rho}, z)\}_i$

In that basis : $\mathcal{E}(\vec{\rho}, z) = \alpha_0 u_0(\vec{\rho}, z)$

No intrinsic definition of multimode at the classical level

Quantum description of the field

Each mode is treated as a single harmonic oscillator

We associate to each mode a set of creation and annihilation operator

$$u_i \quad \longrightarrow \quad \hat{a}_i, \ \hat{a}_i^\dagger$$

It allows to define the number of photon in each mode

The electric field operator

$$\hat{\mathcal{E}}(\vec{\rho}, z) = \sum_i \hat{a}_i(z) u_i(\vec{\rho}, z)$$

$$\hat{a}_i = < \hat{a}_i > + \delta\hat{a}_i$$

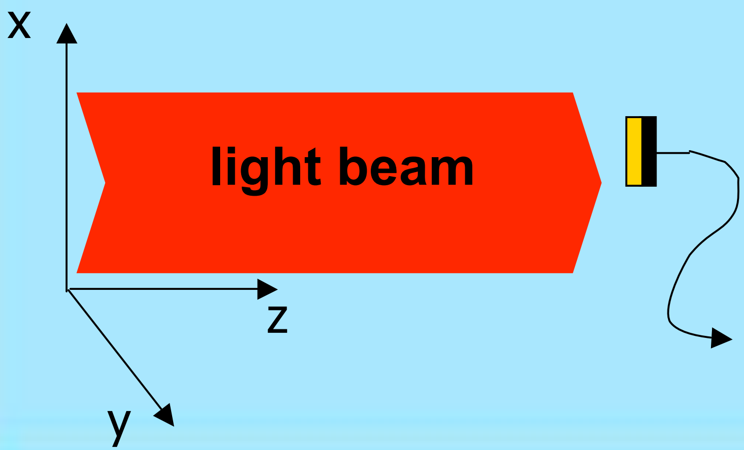
classical value α_i

quantum fluctuations

$$\mathcal{E}(\vec{\rho}, z) = \sum_i \alpha_i u_i(\vec{\rho}, z)$$

Signal given by a detector

Detector : signal proportional to the number of photons



Detector covering a transverse area D

$$\hat{N}(D) \propto \int_D \hat{\mathcal{E}}^\dagger(\vec{\rho})\hat{\mathcal{E}}(\vec{\rho})d^2\rho$$

Signal and noise

The signal is given by the mean number of photon

$$< \hat{N}(D) >$$

The noise is the variance of the number of photons

$$V(\hat{N}) = \Delta\hat{N}^2 = < \hat{N}(D)^2 > - < \hat{N}(D) >^2$$

Single mode quantum field

Known classical image

$$< \hat{\mathcal{E}}(\vec{\rho}, z) >= \mathcal{E}(\vec{\rho}, z) = \alpha_0 u_0(\vec{\rho}, z)$$

Electric field operator

$$\hat{\mathcal{E}}(\vec{\rho}, z) = \sum_i \hat{a}_i(z) u_i(\vec{\rho}, z)$$

Annihilation operators

$$< \hat{a}_0 >= \alpha_0$$
$$< \hat{a}_i >_{i \neq 0} = 0$$

Single mode field

The field state in all the modes except the first one is a coherent vacuum

It then corresponds to the single mode quantum optics studied in the lecture of Hans Bachor.

It exists a proper definition of single mode at the quantum level

It is based on the quantum fluctuations

The same can be done for a statistical superposition of modes

Outline

- Single mode versus multimode light
- Quantum limits to resolution
- Few modes approach : the quantum laser pointer
- Many modes approach : multimode cavities

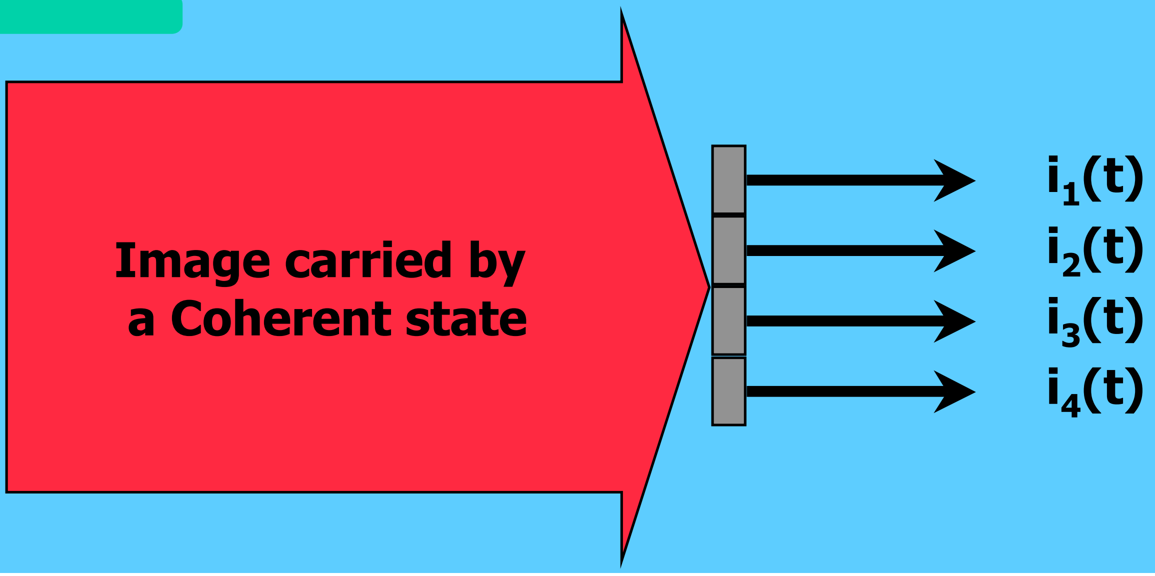
Quantum limits to resolution

Light used in the experiment is single-mode coherent light

modes		Quantum state
$u_0(\vec{r})$	—	Coherent state $\|\alpha_0>$
$u_1(\vec{r})$	—	vacuum
$\vdots$		$\vdots$
$u_n(\vec{r})$	—	vacuum

} Classical light !

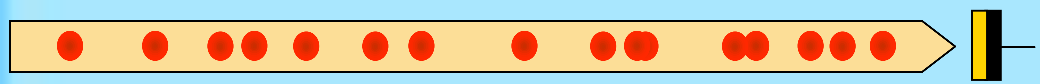
Measurement performed

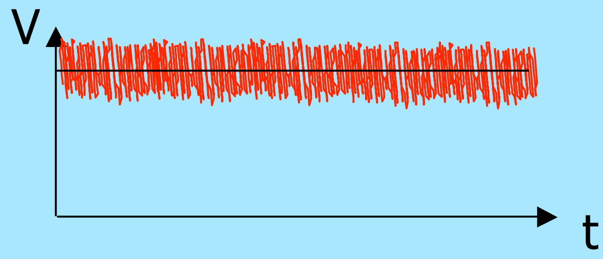


Photon picture of coherent single mode light

Usual quantum optics description

Continuous wave regime (1mW ~ 10^{17}photons/s)



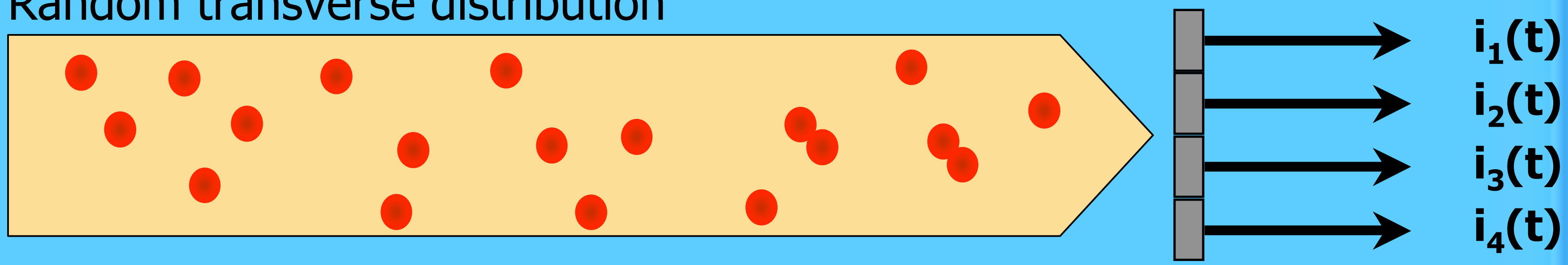


Photon number : **Poisson statistic** (also called white noise)

$$\Delta N = \sqrt{N}$$ Shot noise

Spatial quantum optics description

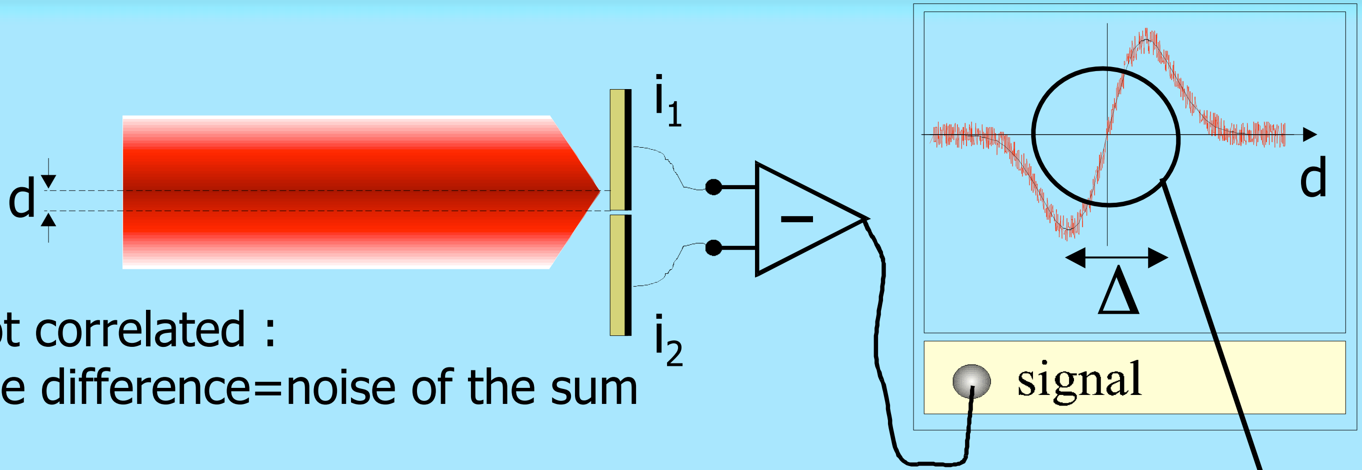
Random transverse distribution



Each detectors sees Poissonian noise

$$\Delta N_i = \sqrt{N_i}$$ Local Shot noise

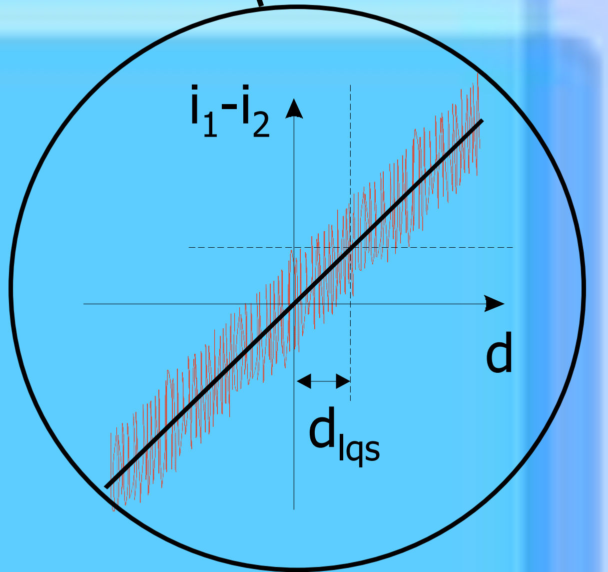
Two pixels case



i_1 and i_2 not correlated :
Noise of the difference=noise of the sum

Smallest displacement detectable

$$d_{lqs} = \frac{\Delta}{\sqrt{N}} << \lambda$$



Signal scales with N

Noise scales with $\sqrt{N}$

Example :

- Beam of 1mW
- Δ=200µm
- Integration time of 10µs

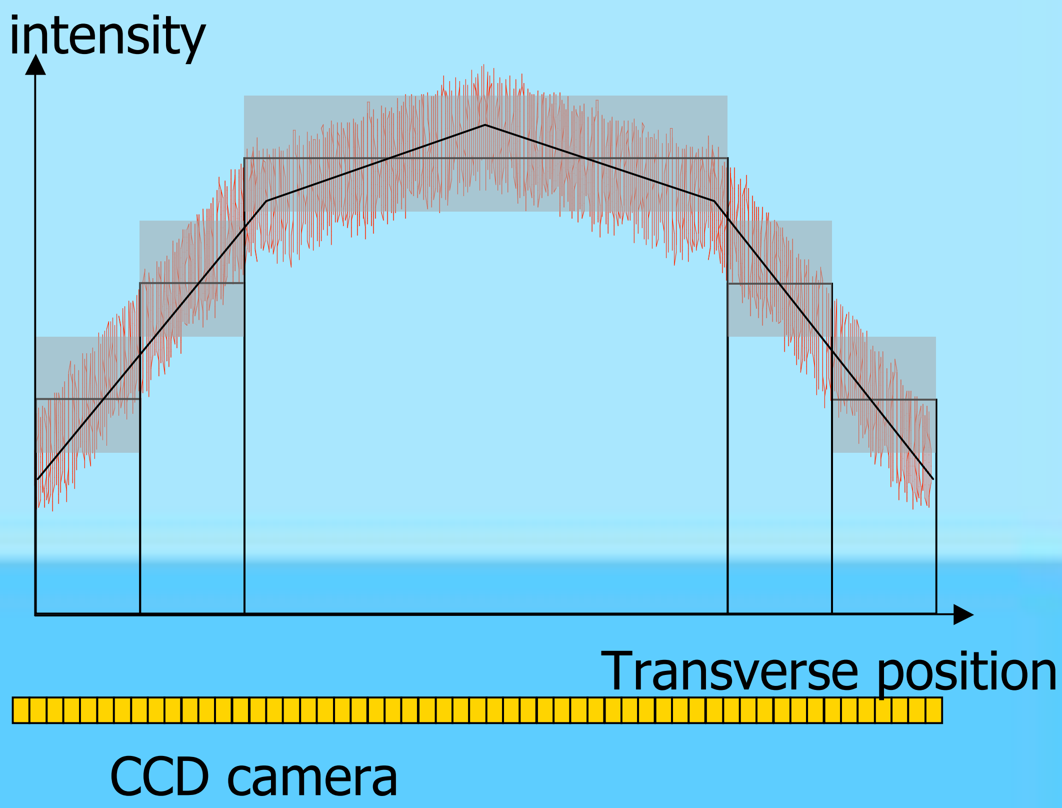
$$d_{lqs} = 5Å$$

Dimensionless quantity $\frac{d}{\Delta}\sqrt{\ } = \frac{1}{\sqrt{N}} = 4,3.10^{-6}$

Image characterisation

Relevant quantity :

$N(\rho)$ = N photons per m²



$$S_{sql} \approx \sqrt{2}\,\frac{L_{\text{var}}}{\sqrt{N(\rho)}}$$

$$L_{\text{var}} \approx \frac{N(\rho)}{\left\|\vec{\nabla} N(\rho)\right\|}$$

Standard quantum limit to resolution

1 dimension

$$d_{sql} = \frac{\Delta}{\sqrt{N}}$$

2 dimensions

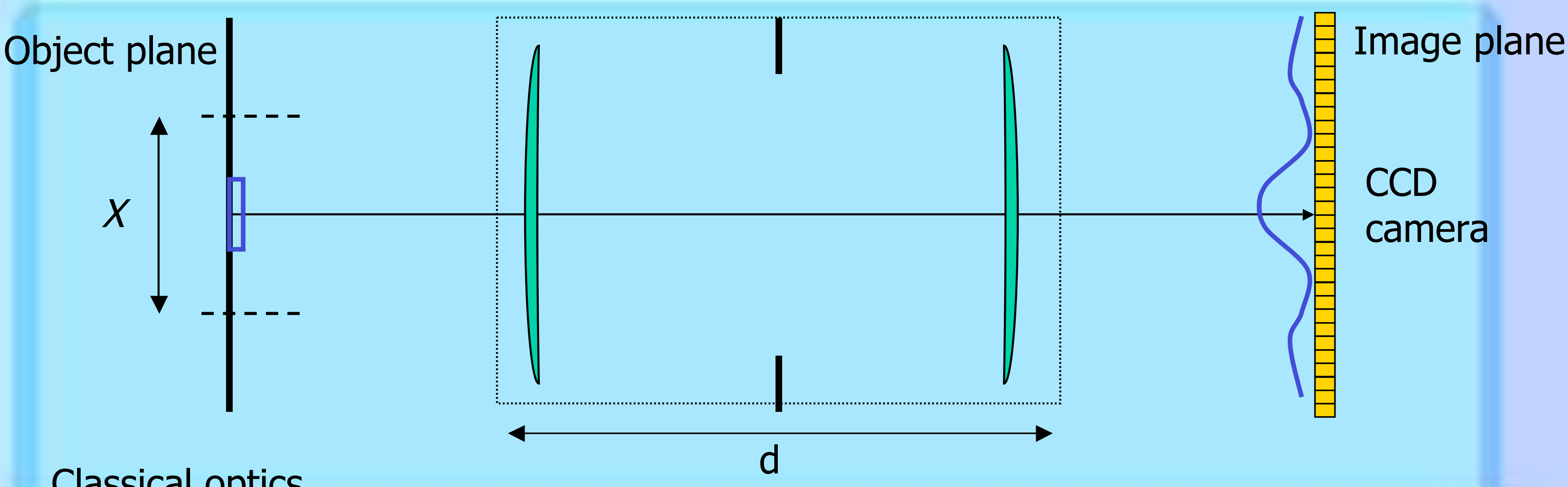
$$S_{sql} \approx \sqrt{2}\,\frac{L_{\mathrm{var}}}{\sqrt{N(\rho)}}$$

Image characteristic length

Shot noise or photon noise

Improve the sensitivity : Increase the beam intensity
Reduce the photon noise

Standard quantum limit in object reconstruction



Classical optics

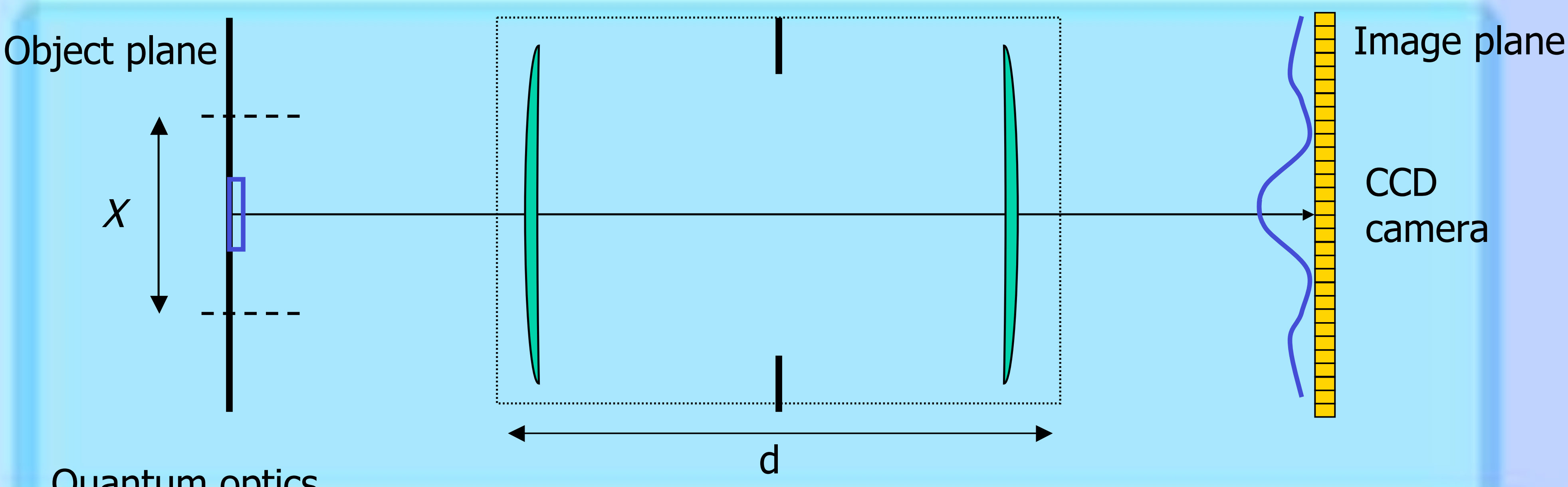
there exists eigenstates f_n of the imaging device, with transmission t_n

$$E_{object} = f_n \quad \Longrightarrow \quad E_{image} = t_n f_n$$

$$E_{image} = \sum_n c_n f_n \quad \Longrightarrow \quad E_{object} = \sum_n \frac{c_n}{t_n} f_n$$

if the coefficients c_n are perfectly known,
object shape can be reconstructed **without limitation due to diffraction**

Standard quantum limit in object reconstruction



Quantum optics

$$E_{object} = \sum_n \frac{c_n}{t_n} f_n$$

Cannot be known for $n > n_{max}$

n_{max} depends on Shannon number $dX/\lambda f$ of the set-up

The knowledge of coefficients C_n is not perfect.

One can show that $\left(\Delta_{c_n}\right)^2$ do not depend on n.

n small : $t_n \approx 1$

n large : $t_n << 1$

Standard quantum limit in image reconstruction $\delta x_{\min} \approx X / n_{\max}$

"Superresolution" very difficult in practice

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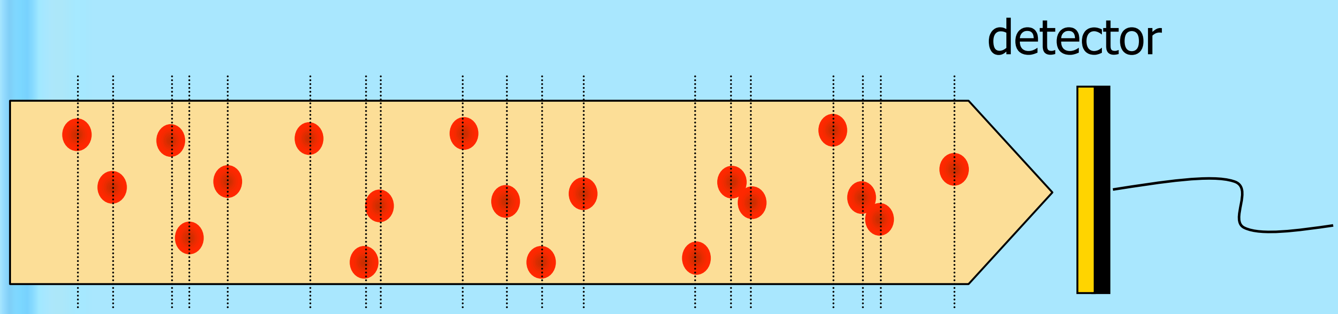
- Few modes approach : the quantum laser pointer
- Many modes approach : multimode cavities

Use single mode squeezed light ?

Single-mode coherent light

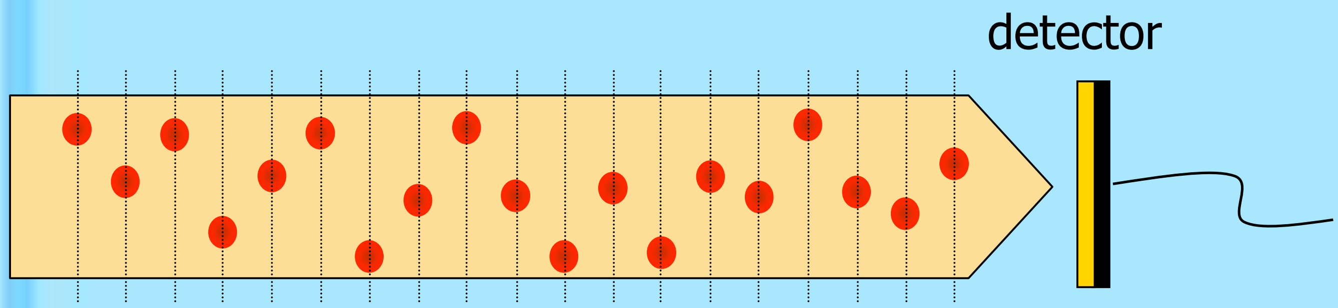


Detection noise :
shot noise

Photons randomly distributed in time and space

Use single mode squeezed light ?

Single-mode squeezed light

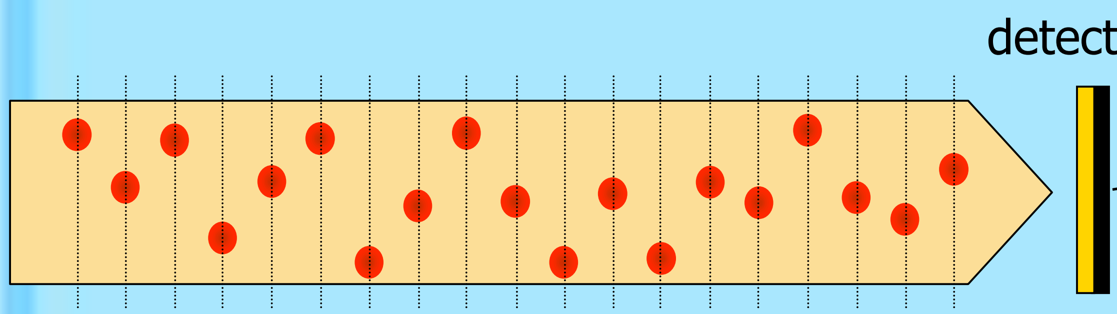


Photons ordered in time : detection noise below the shot noise

Photons ordered in time but randomly distributed in space

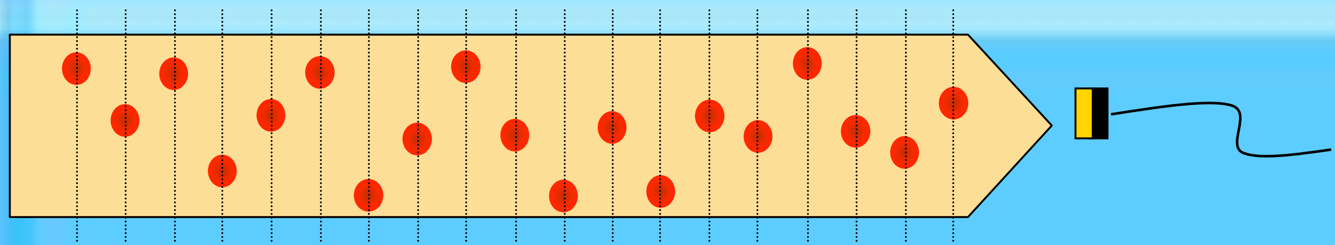
Use single mode squeezed light ?

Single-mode squeezed light



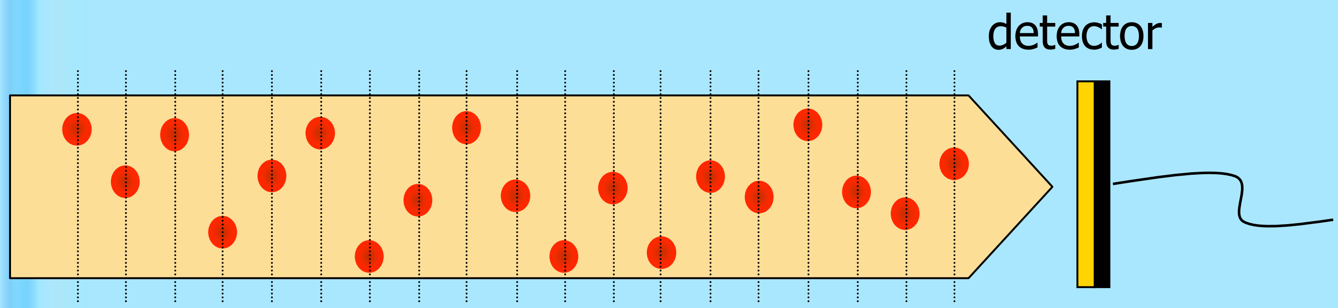
Photons ordered in time : detection noise below the shot noise

Partial detection



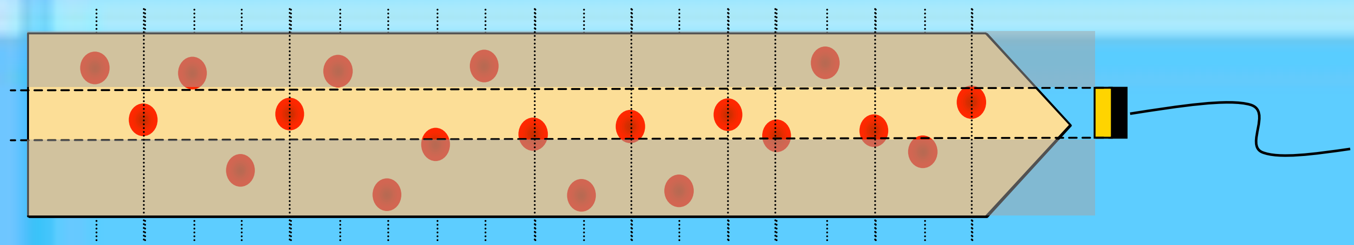
Use single mode squeezed light ?

Single-mode squeezed light

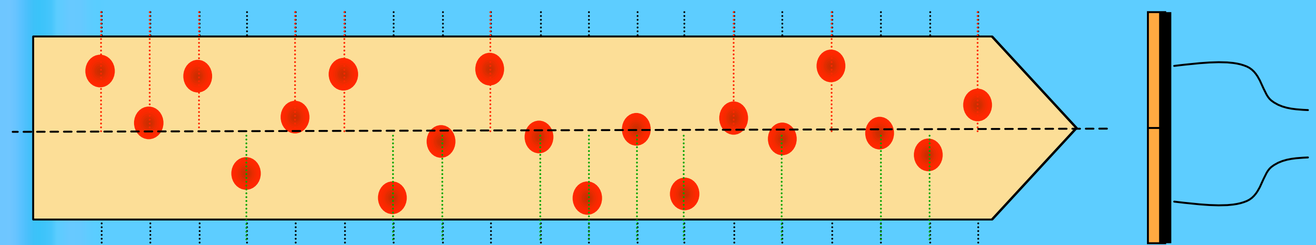


Photons ordered in time : detection noise below the shot noise

Partial detection



Partial detection is equivalent to a loss : no spatial order.



Partial measurement

No spatial squeezing

Multimode quantum light

Electric field operator

$$\hat{\mathcal{E}}(\vec{\rho}, z) = \sum_i \hat{a}_i(z) u_i(\vec{\rho}, z)$$

Known classical image

$$< \hat{\mathcal{E}}(\vec{\rho}, z) >= \mathcal{E}(\vec{\rho}, z) = \alpha_0 u_0(\vec{\rho}, z)$$

Annihilation operators

$$< \hat{a}_0 >= \alpha_0$$
$$< \hat{a}_i >_{i \neq 0} = 0$$

Multimode light ?

one of the other modes $u_1, u_2, u_3, \ldots$ is not in a coherent vacuum state

For instance : squeezed vacuum

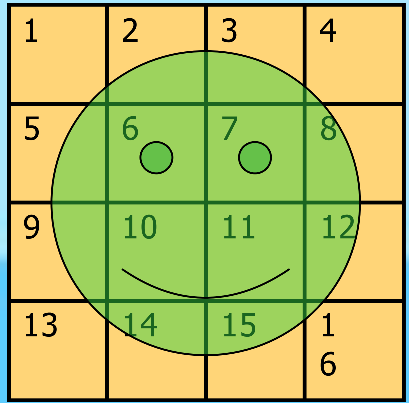
Can be applied to any physical dimension.

Which mode for which measurement ?

Linear measurement of an image

Pixel-like configuration

- Image incident on a CCD camera



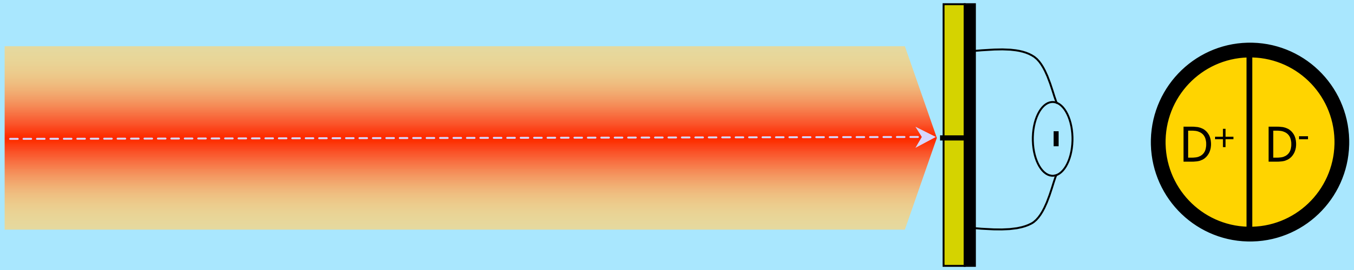
- Linear measurement
  - Intensity on each detector : ***N(D$_i$)***
  - Gain on each detector : σ_i
  - One measurement defined by :

$$N(\{\sigma_i\}) = \sum_i \sigma_i N(D_i)$$

- Image is known
  Measurement : a function of the gains

Difference measurement

Two identical signals from the light source



zero mean signal

$$\hat{N} = \hat{N}(D^+) - \hat{N}(D^-)$$

$$\left\langle \hat{N} \right\rangle = 0$$

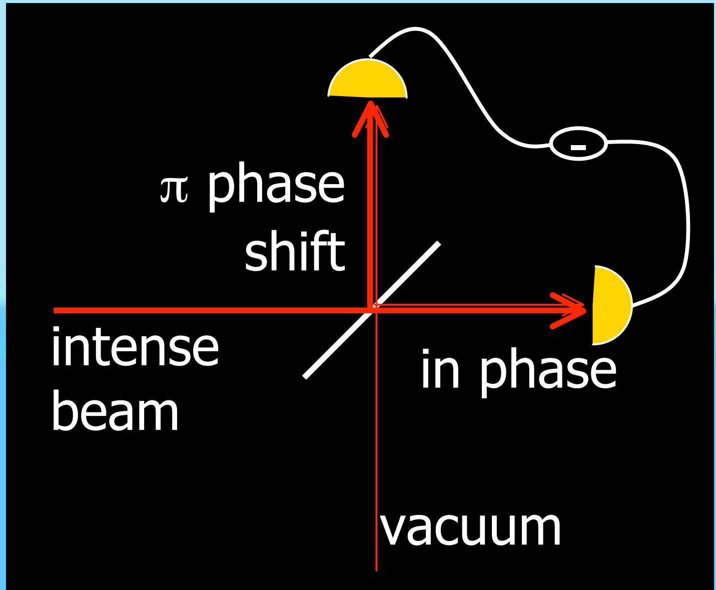
With a classical field : $\mathcal{E}(\vec{\rho}) = \alpha_0 u_0(\vec{\rho})$ cancellation of the common mode noises

there is no noise in the measurement

However, with a quantum description : $\hat{\mathcal{E}}(\vec{\rho}) = \sum_i \hat{a}_i u_i(\vec{\rho})$
there is quantum noise !

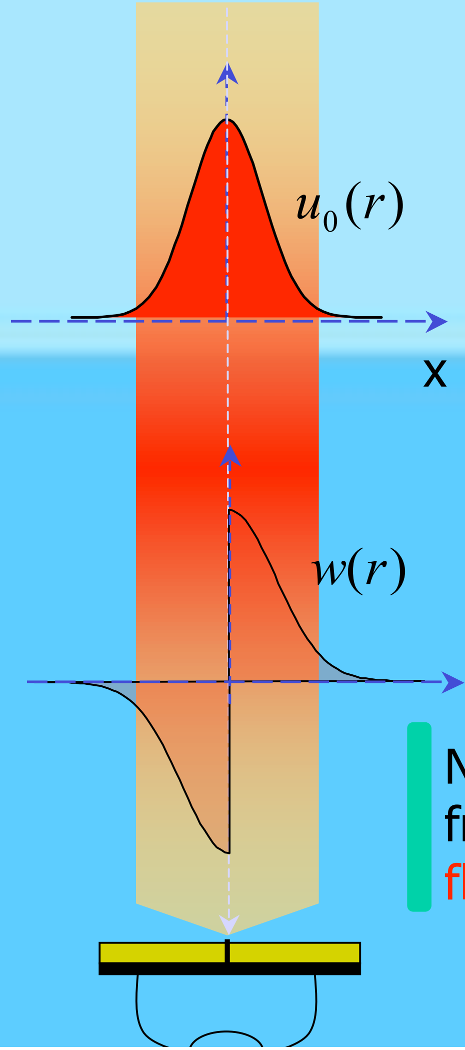
Noise in a difference measurement

Balanced detection



Noise come from the vacuum port

Split detection

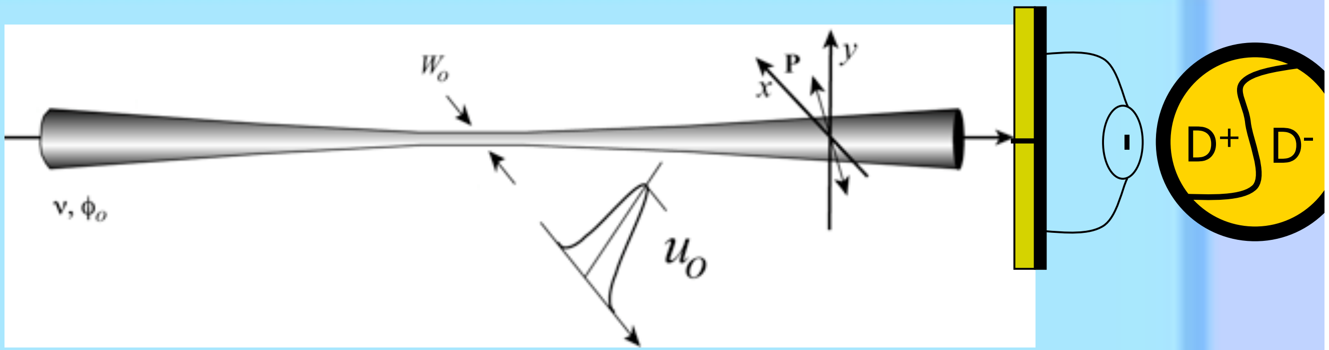


Noise come from the flipped mode

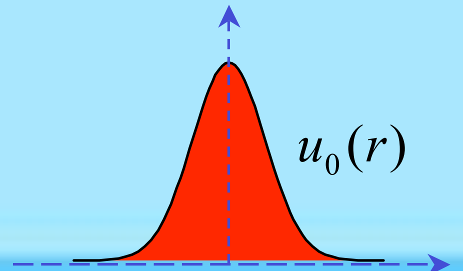
Noise in a difference measurement

Image description

$$< \hat{\mathcal{E}}(\vec{\rho}) >= \alpha_0 u_0(\vec{\rho})$$



where u_0 is, for instance, a Gaussian mode

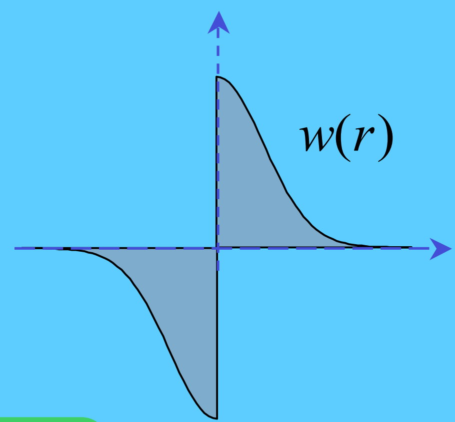


Origin of the noise

Noise originates from the flipped mode

$$w(\vec{r}) = u_0(\vec{r}) \quad if \ \vec{r} \in D^+$$

$$w(\vec{r}) = -u_0(\vec{r}) \quad if \ \vec{r} \in D^-$$



w is orthogonal to u_0 !!

Variance of the noise

$$V(\hat{N}) = N V(\hat{X}^1_w)$$

Amplitude noise of mode w

Noise in a difference measurement

NOISE

$$V(\hat{N}) = NV(\hat{X}_w^1)$$

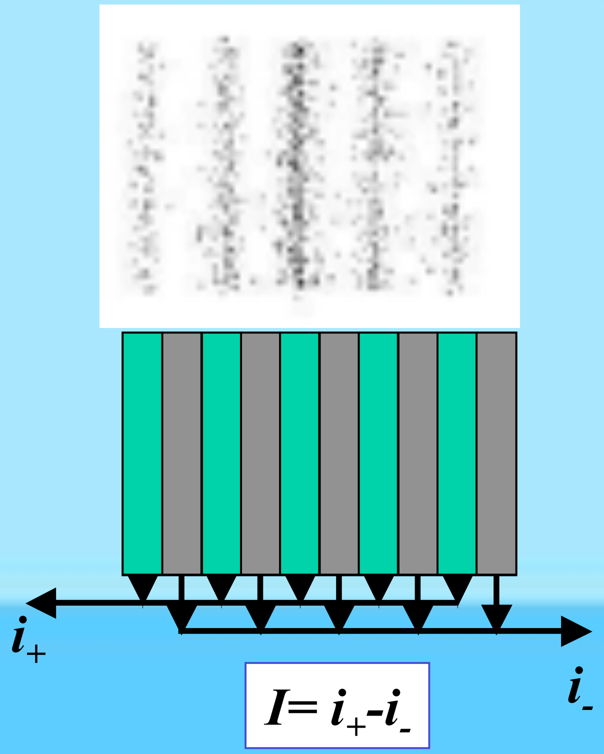
- Noise on a difference measurement : from a single mode, the flipped mode.

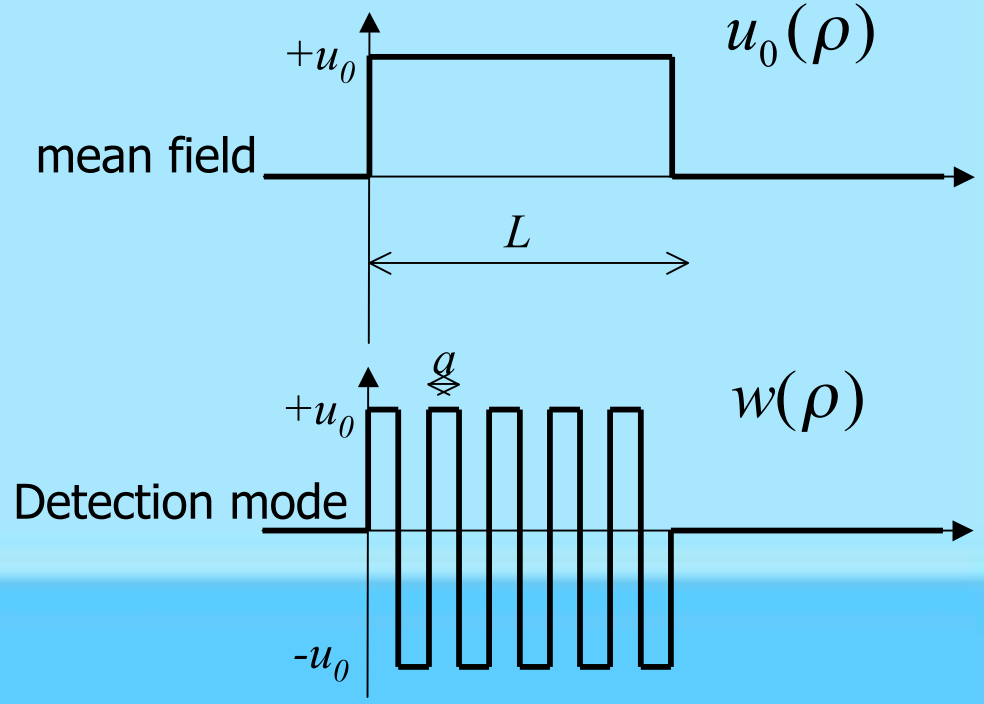
- Reduce the noise in that measurement : necessary and sufficient to inject vacuum squeezing in that mode

Transverse modes description

modes	Spatially squeezed light
$u_0(\vec{r})$	any state of mean value α_0
$u_1(\vec{r}) = w(\vec{r})$	squeezed vacuum
$u_2(\vec{r})$	vacuum
$\vdots$	$\vdots$

Noise in a general measurement





Variance of the noise

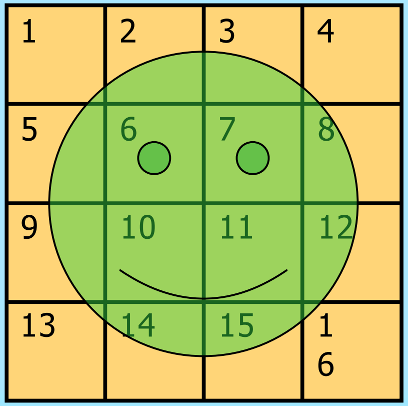
$$V(\hat{N}) = NV(\hat{X}_w^1)$$

Transverse modes description

Same as for the differential measurement.

Noise in a general measurement

General measurement



$$\hat{N}(\{\sigma_i\}) = \sum_i \sigma_i \hat{N}(D_i)$$

Mean field mode : any shape
What is the detection mode ?

Detection mode

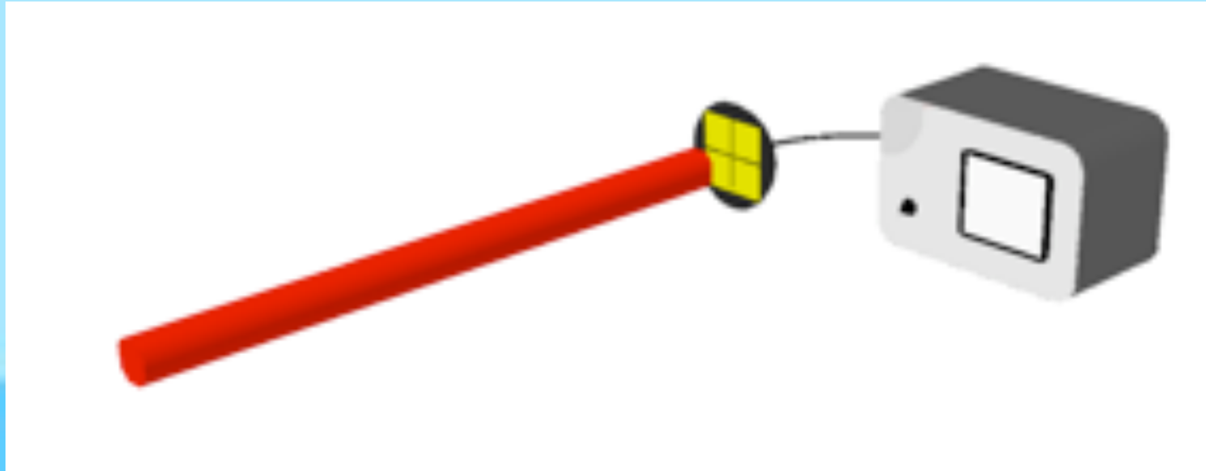
It exists a detection mode *w* such as $if\ \vec{\rho} \in D_i, w(\vec{\rho}) = \frac{1}{f} \sigma_i u_0(\vec{\rho})$

Variance of the noise

$$V(\hat{N}) = f^2 N V(\hat{X}_w^1)$$

Application to the laser pointer

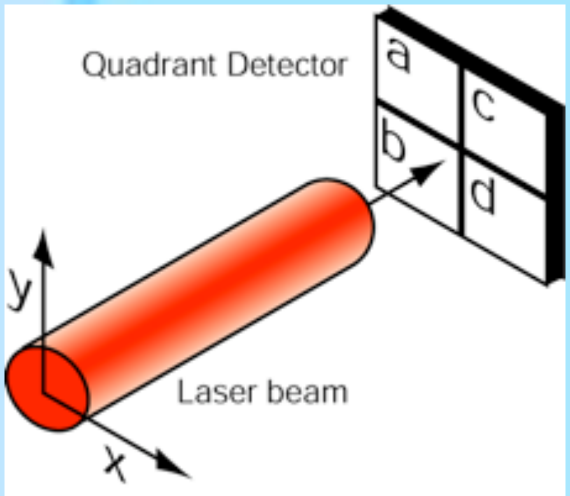
Measurement of a light beam with a quadrant detector



- Position / orientation of the beam
- Quantum limit of the measurements

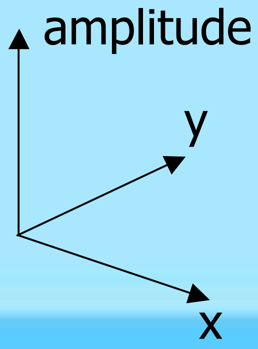
Used in many physical apparatus such as : atomic force microscope
laser guided devices

Small displacements measurement

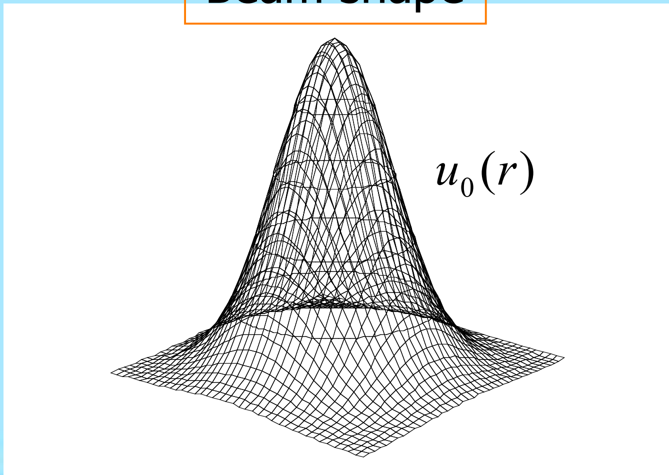


$$I_x = (I_a + I_b) - (I_c + I_d)$$

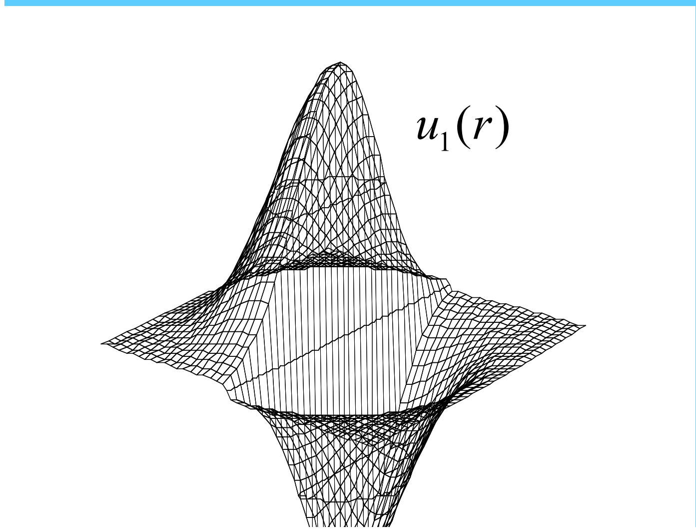
$$I_y = (I_a + I_c) - (I_b + I_d)$$



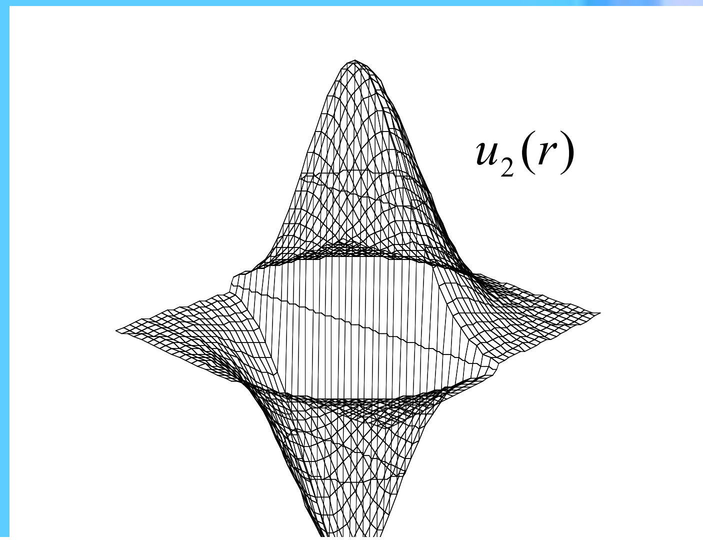
Beam shape



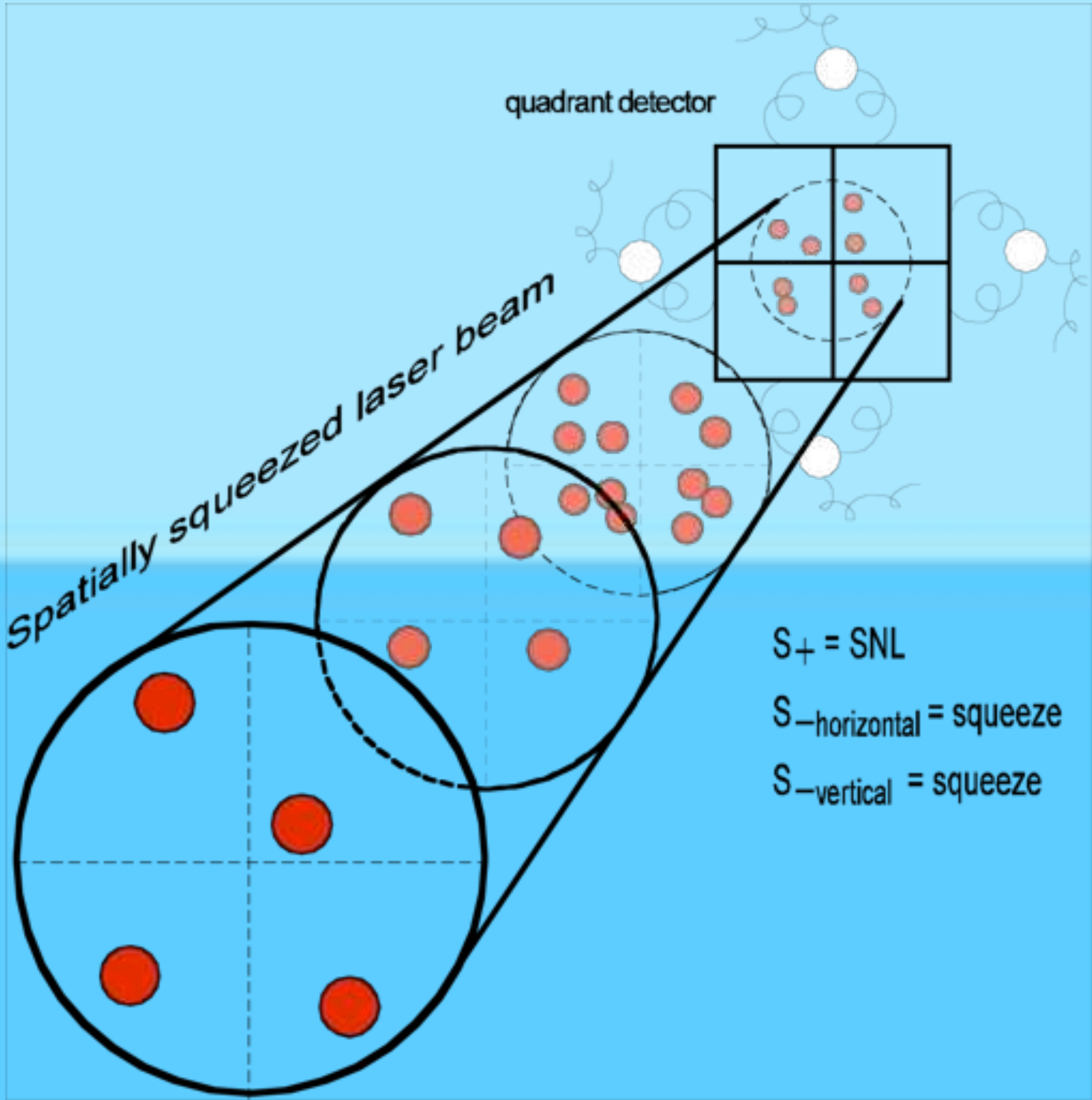
x flipped mode



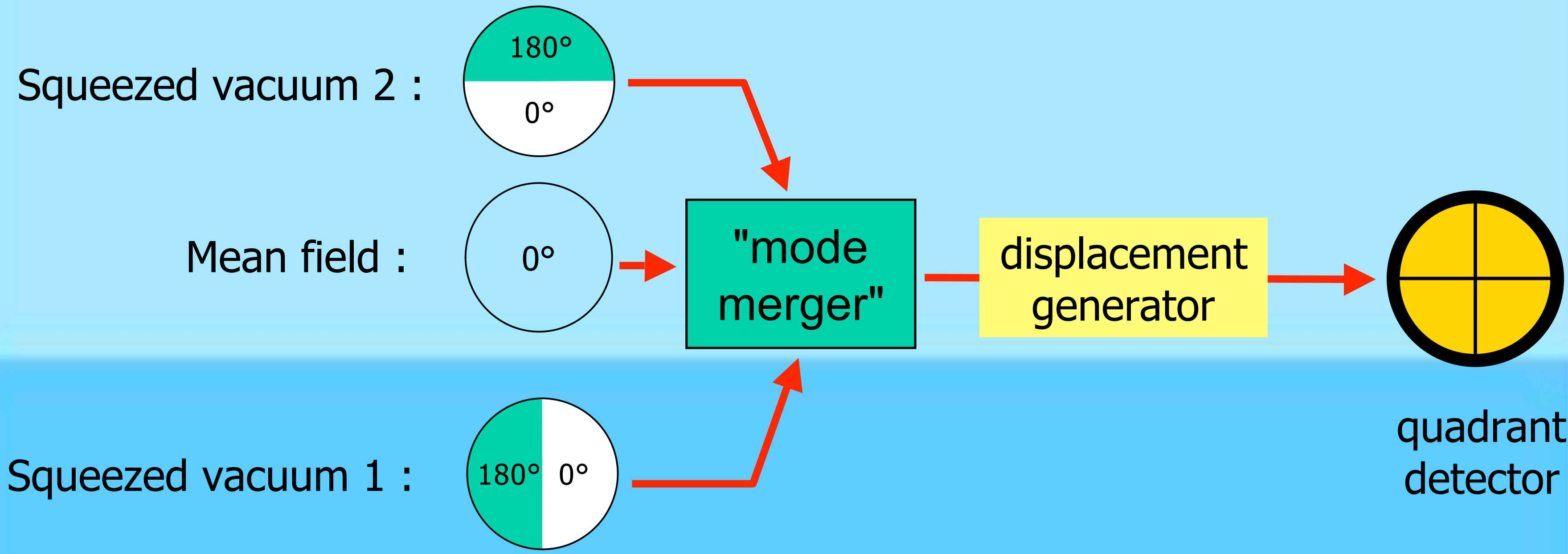
y flipped mode



Small displacements measurement



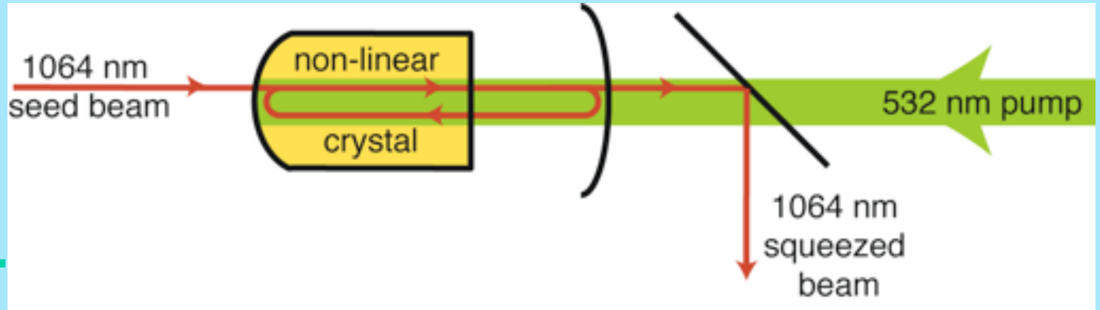
Experimental implementation



Experimental Setup

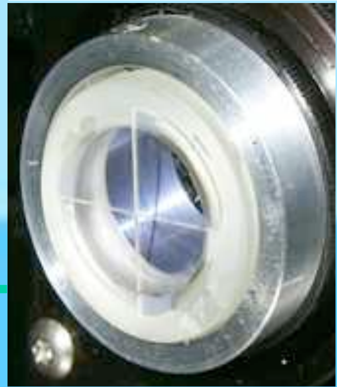
Squeezed vacuum

Below threshold Optical Parametric Amplifier



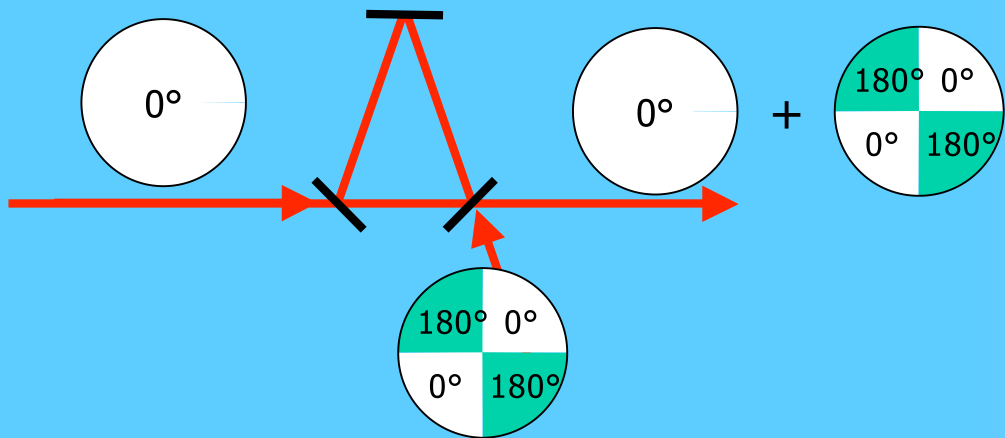
Transverse modes

"Cut and Paste" waveplate



Mode merger

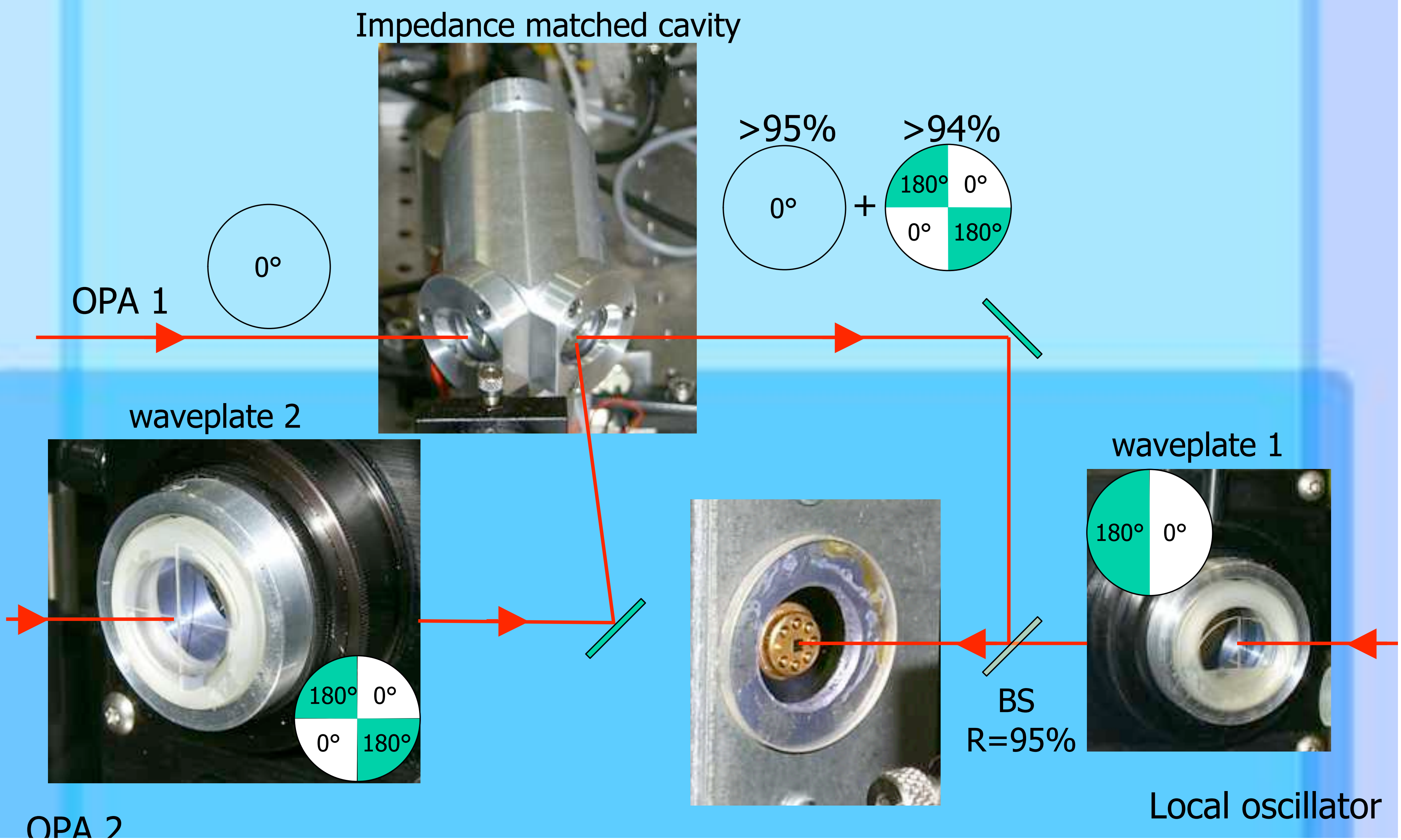
Impedance matched cavity



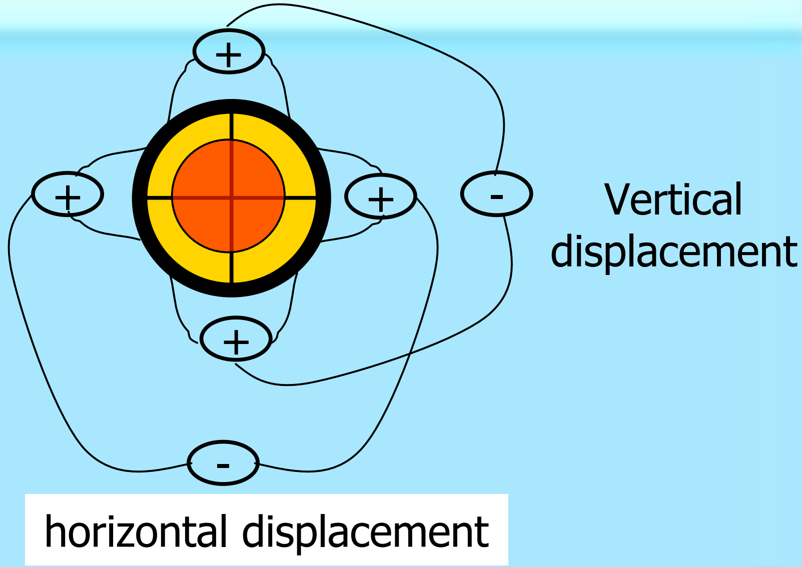
Displacement generator

Physical system !

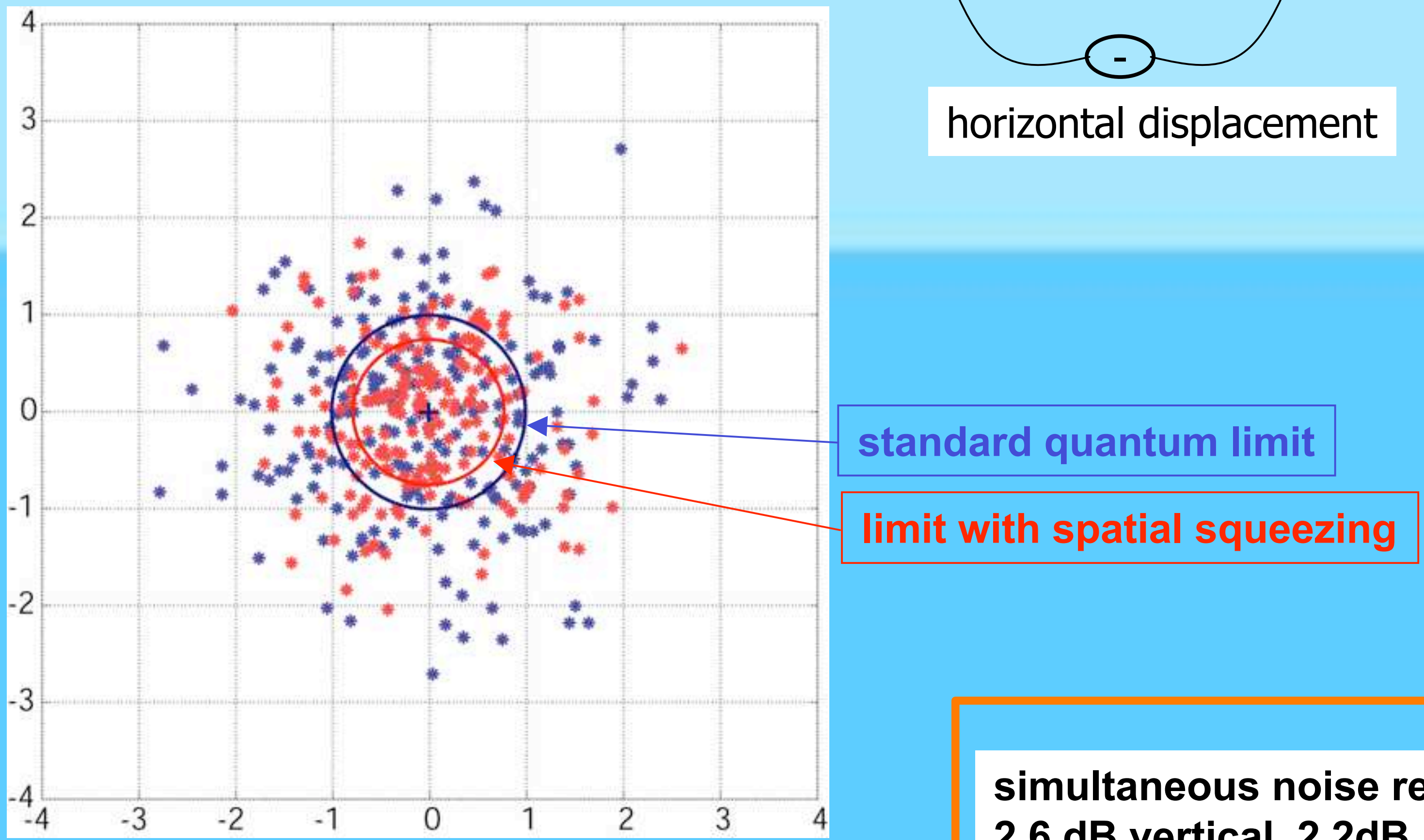
Small displacements measurement



Spatial quantum noise



Relative vertical position



Relative horizontal position

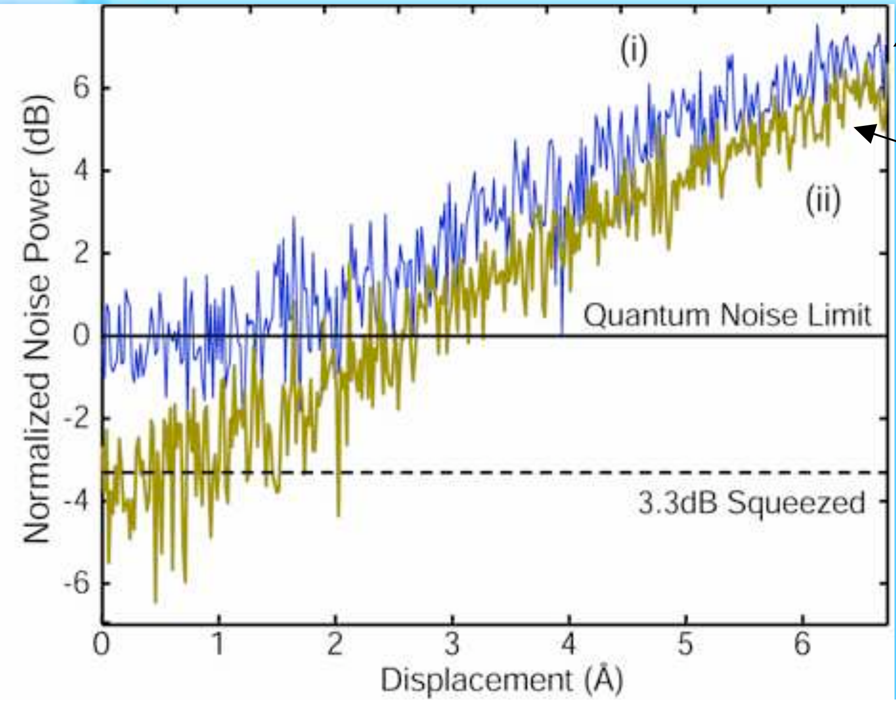
simultaneous noise reduction : 2.6 dB vertical, 2.2dB horizontal

Small displacements measurement

Oscillation at 4.5 MHz : mirror on a piezo-electric crystal.

Oscillation amplitude is linearly increased with time.

Signal measured



Coherent state

Spatially squeezed state

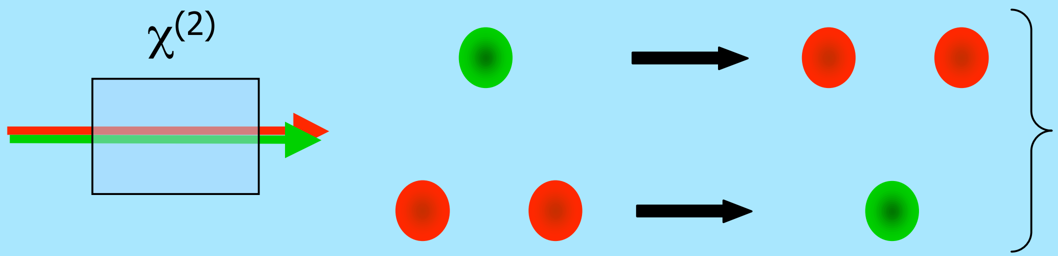
Outline

- Single mode versus multimode light
- Quantum limits to resolution
- Few modes approach : the quantum laser pointer
- Many modes approach : multimode cavities

The parametric process

Second order non-linearity

$\chi^{(2)}$ medium : mix two different wavelengths.

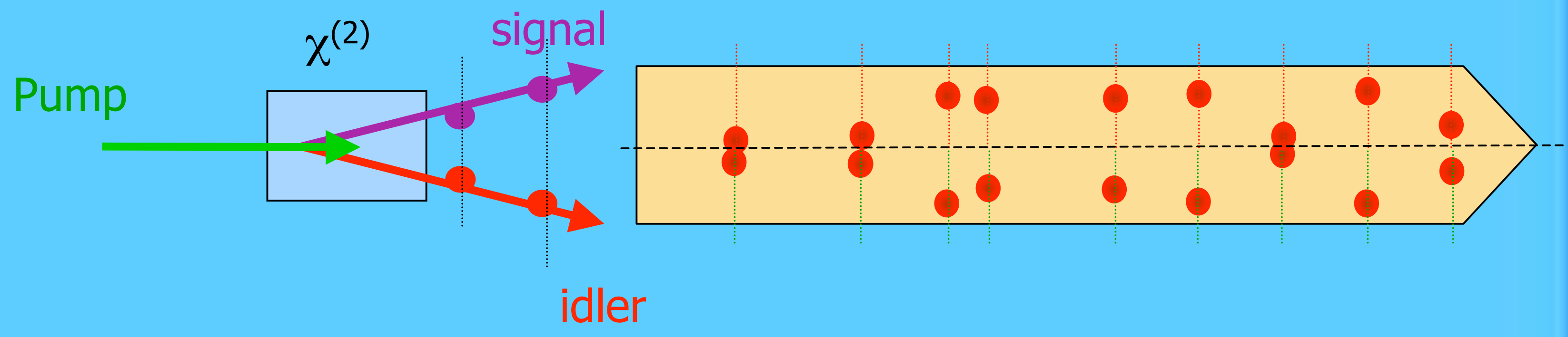


Such as $\omega_1 = \omega_2 + \omega_2$

energy conservation

Parametric down conversion

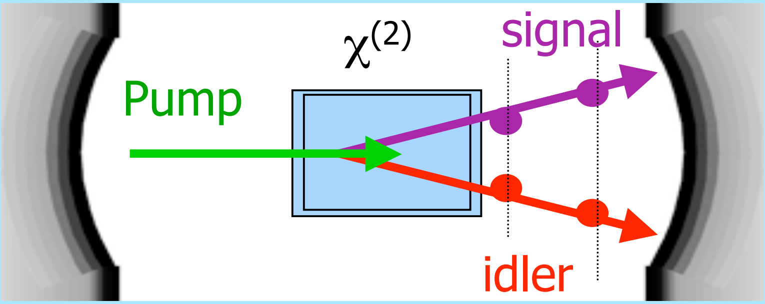
Photons correlated in time and position



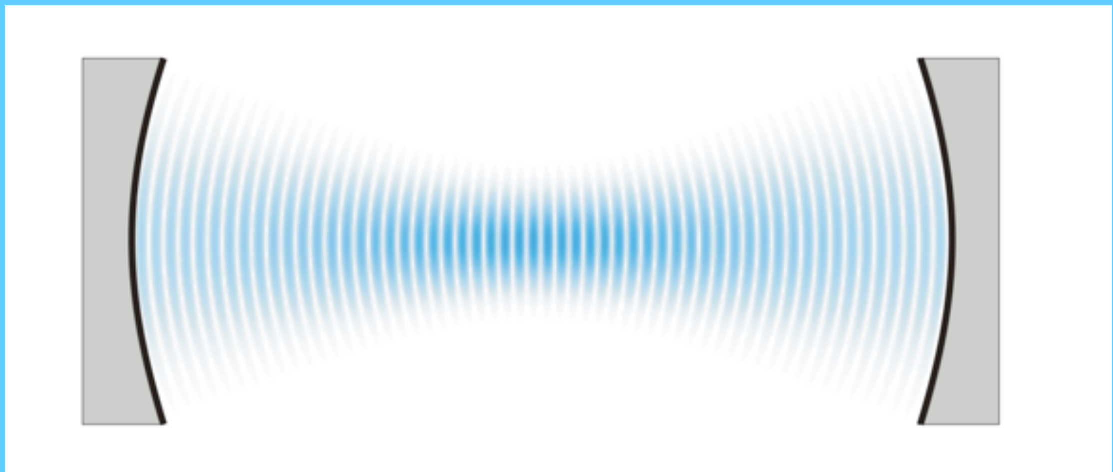
Used in many single photon experiments to create spatial entanglement

Single mode cavities

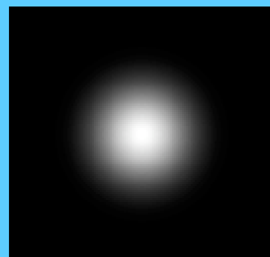
Cavity to increase the non linearity



Cavity select a spatial mode



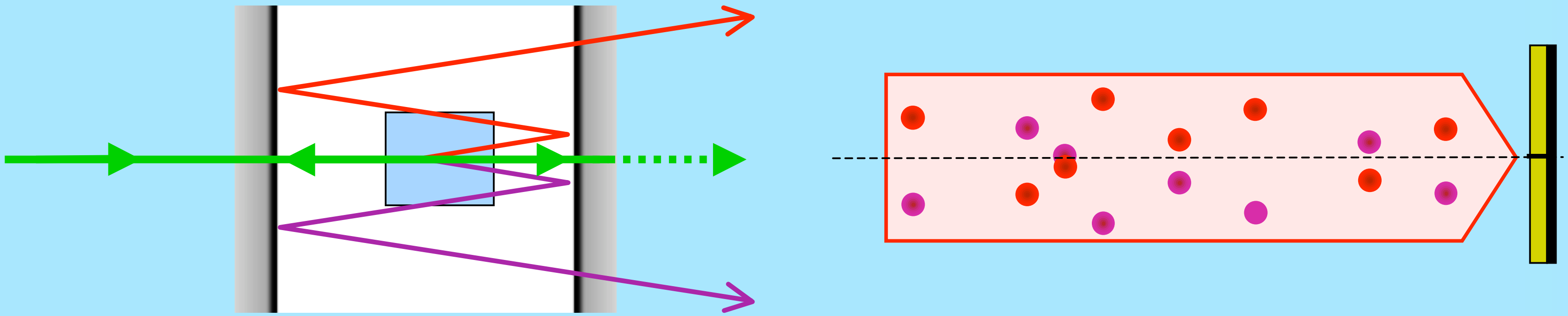
The output is one Gaussian mode



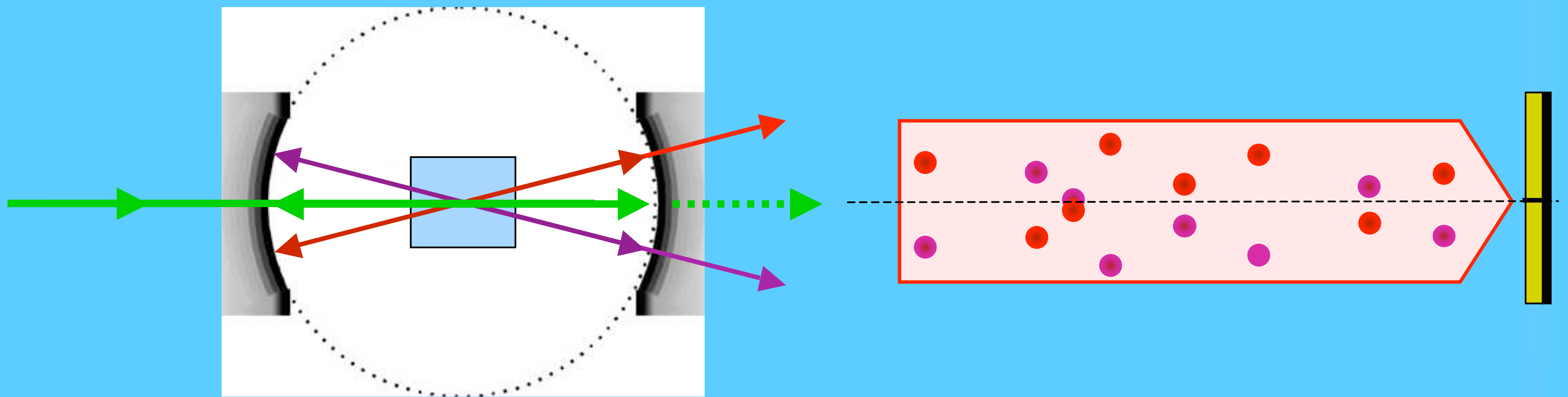
The spatial order is lost !

Multimode cavities

Planar cavity

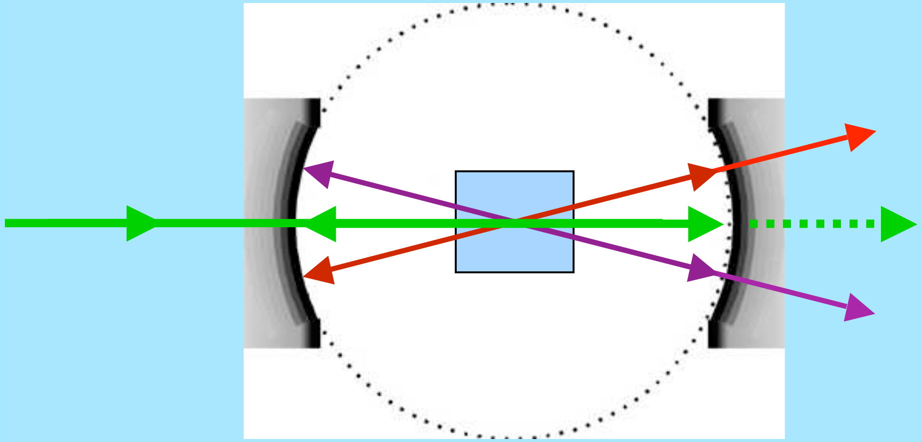


Spherical cavity



What can these cavities do ?

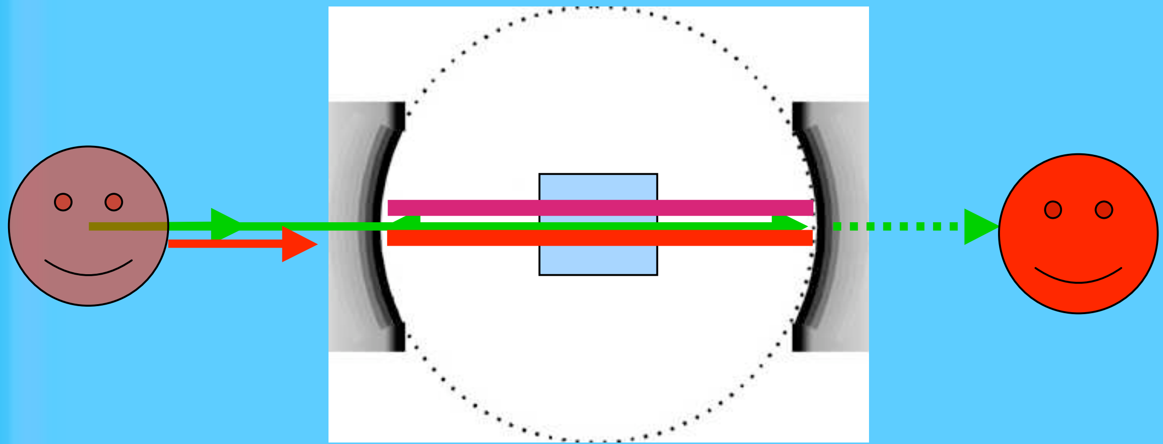
Generate multimode quantum states



Generate multimode squeezed light :

Squeeze all the transverse modes simultaneously.
Generate spatial entanglement.
...

Noiseless amplification of images



The noise properties of the amplified image are better than what can be achieved with a classical amplifier.

Conclusion

- Quantum noise, and not diffraction, gives the ultimate limit to resolution
- The spatial dimension of light brings a lot of degree of freedom : many new quantum states are accessible.
- It is possible to improve several measurement performed on the same beam using appropriately designed spatially squeezed light.
- The future is toward improvement of practical apparatus (like optical resolution) on the one hand, and generation of highly multimode light on the other hand

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Ping Koy Lam

Warwick Bowen
Nicolai Grosse
Magnus Hsu

Result of a very active collaboration between France and Australia.
Cotutelle PhD students are welcome !

Some references

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