Ultracold disordered and complex quantum gases: a review

Wanderin' quantum optics theory
(From Hannover to Barcelona)

ICFO
Institut
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Fotòniques

Some “truths” about complex systems

- Complex systems are characterized by structurally simple, but **non-linear interactions**. Often incorporate **disorder**
- Complex systems (in particular **disordered systems**) often have very many „relevant“ states (energy minima, attractors, etc.)
- Complex systems exhibit often **long range** correlations in space and time (in particular when interactions themselves are long range)
- Complex system often incorporate **fractal** structures, **hierarchical** or **ultrametric** structures
- **Quantum** complex systems are **notoriously** (i.e. much more than non-complex) **difficult** to simulate !

Outline

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- Various methods of introducing a controlled disorder
- Weakly interacting Bose gases in random potential: toward Anderson localisation.
- Anderson and Bose glasses in optical lattices: toward strongly correlated systems
- Disordered ultracold Fermi gases: a case study of Fermi-Bose mixtures
- Quantum information and disordered and complex gases: generation of entanglement
- Disordered induced order: breaking a continuous symmetry
- Ultracold lattice spinor gases: with and without disorder

Disordered and frustrated quantum lattice gases

1. **Four ways to create random (but controlled) on-site potential**

2. **Using optical super-lattices:**
   - Add a disordered lattice(s) created by speckle radiation pattern to the main lattice (in traps PRL's by Florence, Orsay, Hannover…)
   - Add a lattice(s) with incommensurable period (quasi-disorder)
     - papers by us, Roth and Burnett, see also T. Schulte et al. PRL. **95**, 170411 (2005)

3. **Quenching auxiliary atoms as random scatterers:**
   - Place auxiliary atoms in a lattice and ramp potential wells up non-adiabatically. For small filling factors, the atoms will be localized at random positions. Super-impose this system of random scatterers with the main lattice – see recent paper of Y. Castin group

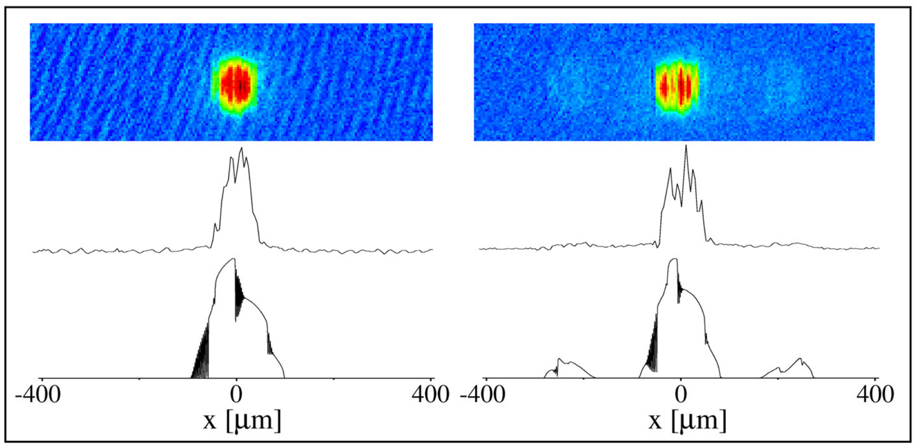
4. **Employing Feschbach resonances in random magnetic fields:**
   - Disordered interactions - see H. Gimperlein et al., cond-mat/0506572

+ Frustarted non-radom!!!

Routes toward Anderson localisation: interplay between disorder and interactions in trapped gases

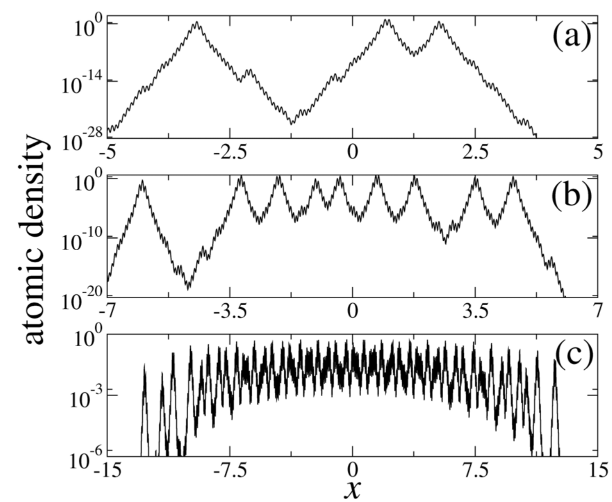
Experiment by T. Schulte et al.

- speckles too "large"
- interactions too "strong"



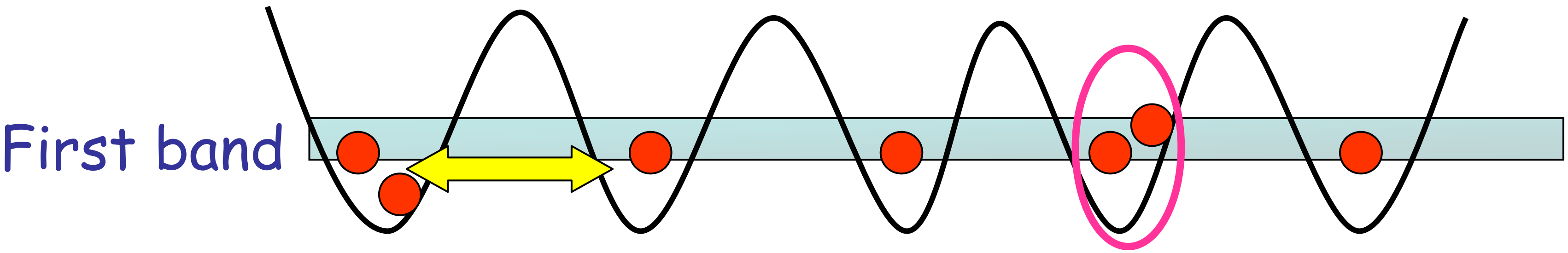
Theory by T. Schulte et al.

- "quasidisorder"



But, observe 11th Commandement:
You shall not block, or obscure the laser access

Before talking about disorder in lattices, let us define order: an optical lattice with atoms loaded on it.



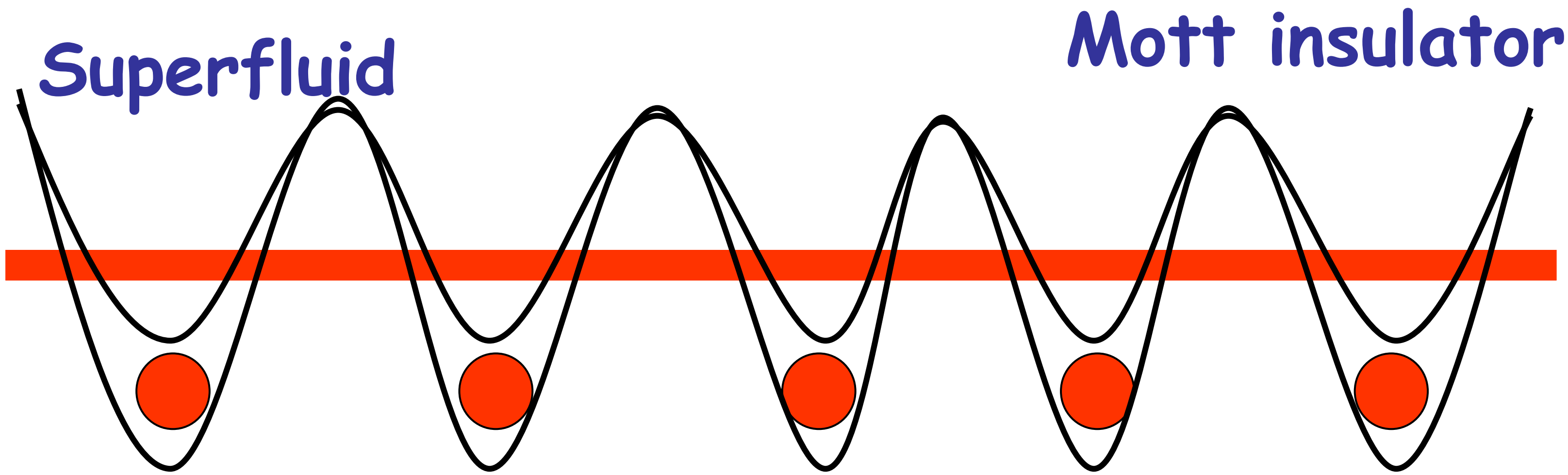
Tunneling **On site interactions**

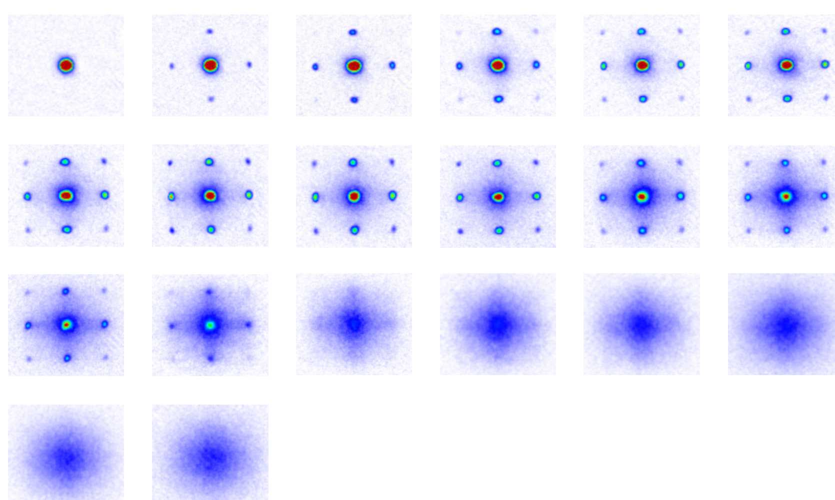
Bose-Hubbard model

$$H = \frac{1}{2} U \sum_i n_i(n_i - 1) - \frac{1}{2} J \sum_{<ij>} b_i^+ b_j + h.c. + \mu \sum n_i$$

Bose gas in an optical lattice

Idea: D. Jaksch, C. Bruder, J.I. Cirac, C.W. Gardiner and P. Zoller

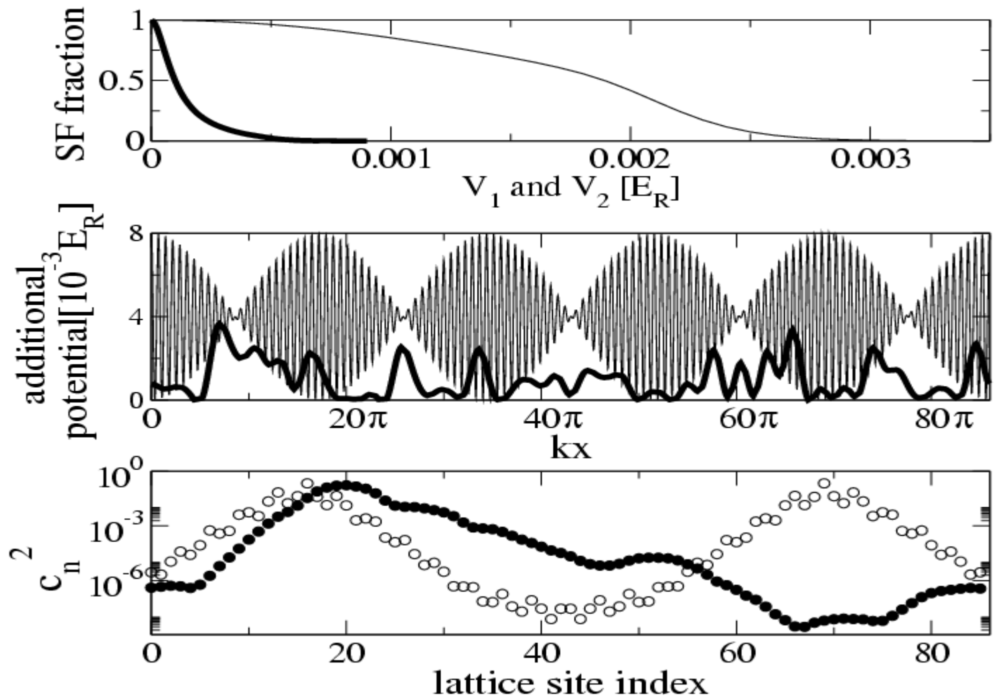




By courtesy of M. Greiner, I. Bloch, O. Mandel, and T. Hänsch

Creating Anderson glass in a disordered optical lattice

$$H = -\Sigma_{<ij>} (Jb^{\dagger}_i b_j + h.c.) + \Sigma_i h_i b^{\dagger}_i b_i + \Sigma_i U n_i (n_i - 1)/2$$



Description:

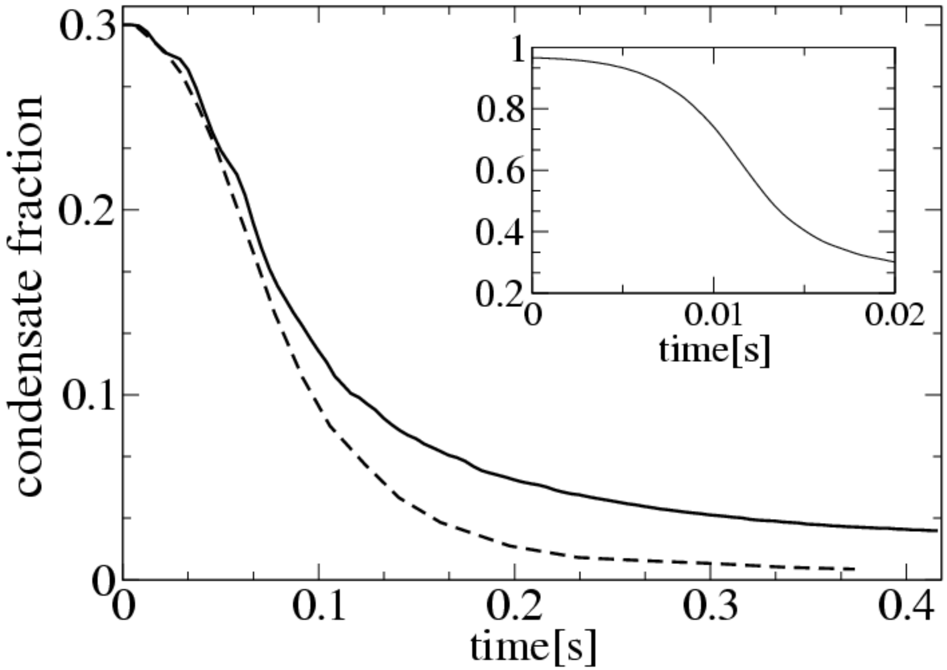
i) **Bose-Hubbard model with random on-site energies**
ii) **negligible on-site interactions**
iii) **„boost" method to calculate the SF fraction**
iv) **localization of the condensate wave functions**

B. Damski, J. Zakrzewski, L.Santos, P. Zoller, and M. Lewenstein,

Phys. Rev. Lett. **91**, 080403 (2003)

Bose glass in a disordered optical lattice

$$H = - \Sigma_{<ij>} (J(t) b^{\dagger}_i b_j + h.c.)$$
$$+ \Sigma_i h_i(t) b^{\dagger}_i b_i + \Sigma_i U(t) n_i (n_i - 1)/2$$



Description:

i) time-dependent Bose-Hubbard model with random on-site energies
ii) growth of the disorder
iii) „boost“ method to calculate the SF fraction
iv) rapid decrease of the SF and the condensate fraction

Bose-Fermi mixtures = spinless interacting fermions in random optical lattices: From Fermi glass to fermionic spin glass and quantum percolation

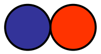
A. Sanpera, A. Kantian, L. Sanchez-Palencia, J. Zakrzewski, and M. Lewenstein

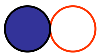
cond-mat/0402375, Phys. Rev. Lett. **93**, 040401 (2004)

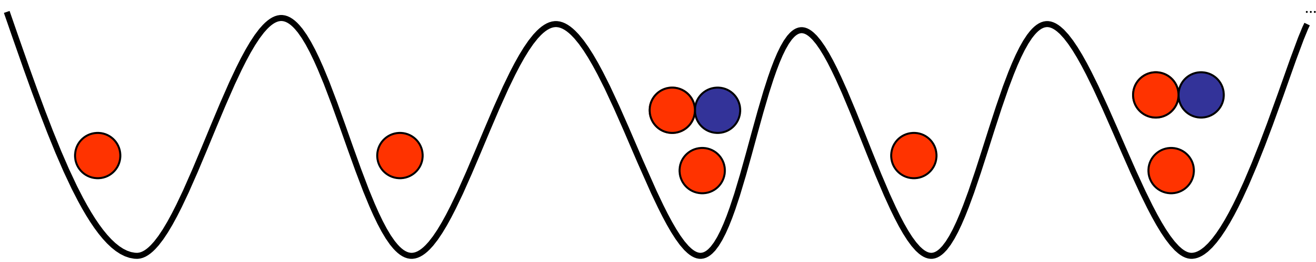
V. Ahufinger, B. Damski, A. Kantian, L. Sanchez-Palencia, A. Sanpera, and M. Lewenstein

(a review of AMO disordered systems – cond-mat/0508042, Phys. Rev. A**72**, 063616 (2005))

In the limit of weak tunneling of fermions/bosons these systems are described in terms of **composite fermions** consisting of

Fermion + boson $m=1, n=1$ 

Fermion + bosonic hole $m=1, n=0$ 



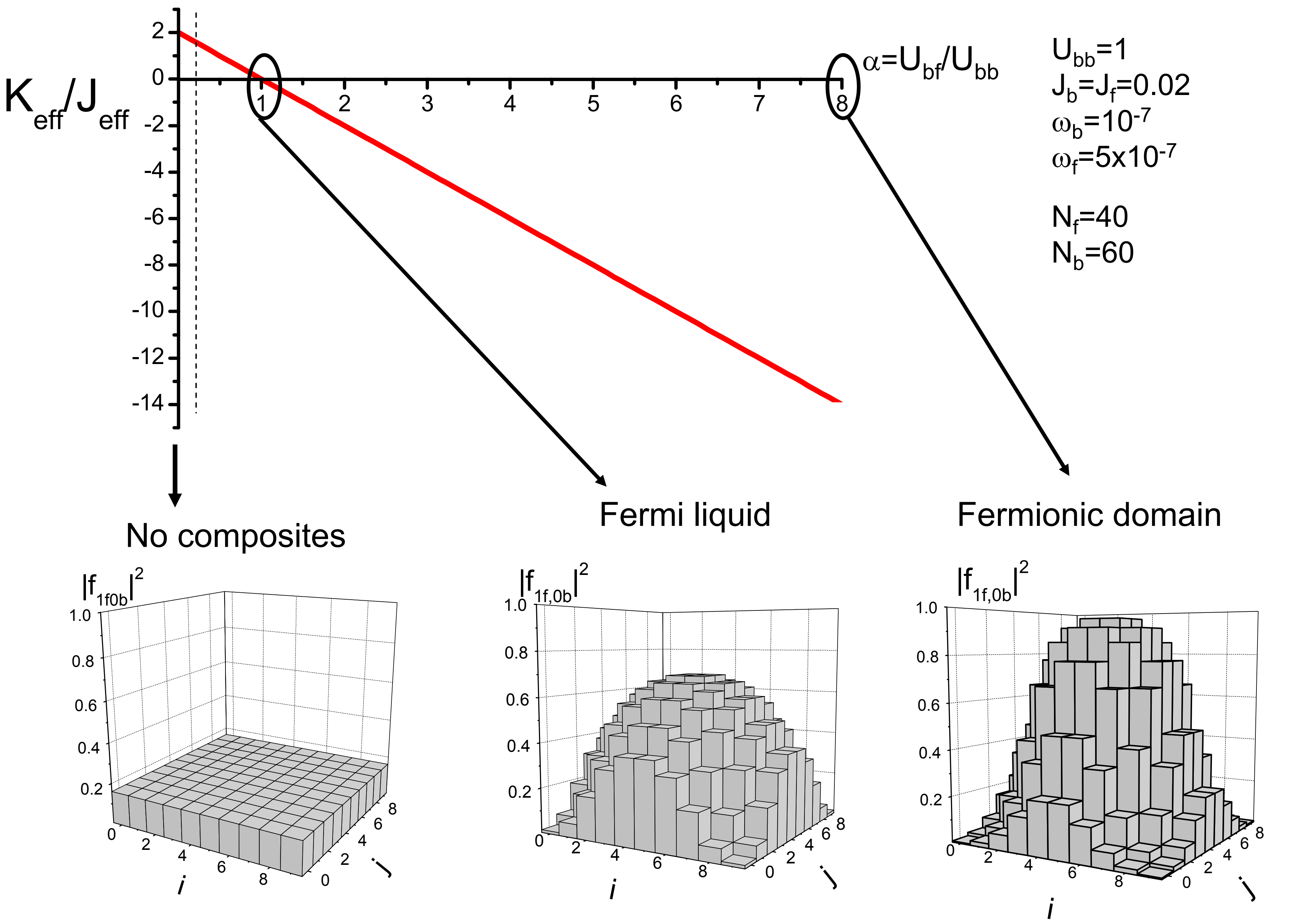
- Low tunneling $J << U_{bf}, U_{bb}$
- Low Temperature
- **Effective** Fermi-Hubbard Hamiltonian (second order perturbation theory)

$$H_{eff} = \sum_{<ij>} \left(-J_{ij} \left[F_i^{+} F_j + h.c. \right] + K_{ij} M_i M_j \right) + \sum_i \tilde{\mu}_i M_i$$

HOPPING of COMPOSITES

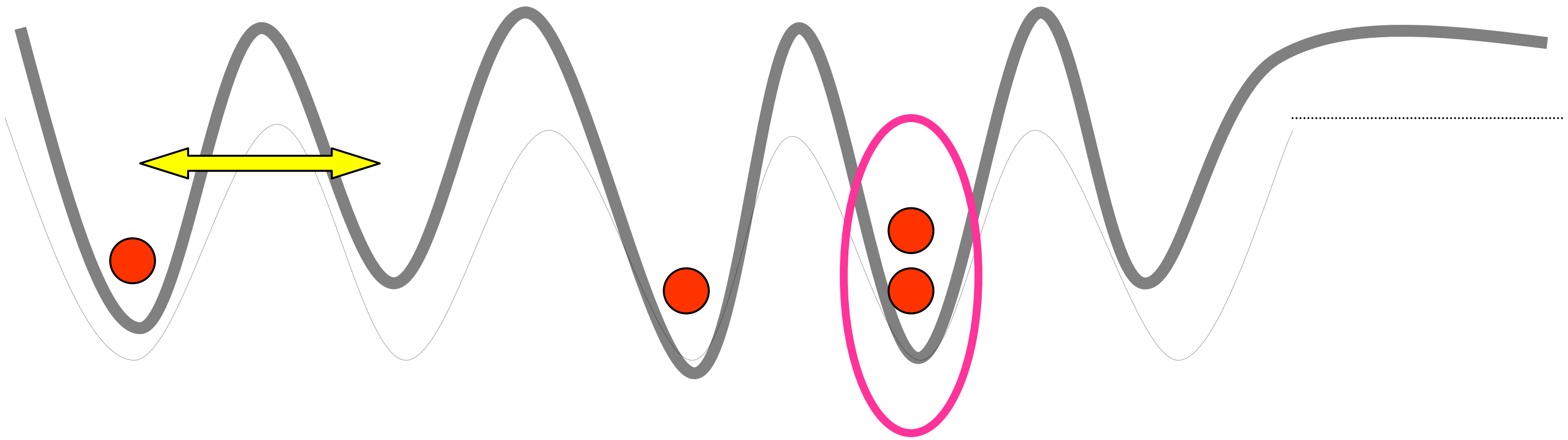
INTERACTIONS between COMPOSITES

DISORDER



K_{eff}/J_{eff}
$\alpha=U_{bf}/U_{bb}$
$U_{bb}=1$
$J_b=J_f=0.02$
$\omega_b=10^{-7}$
$\omega_f=5\times10^{-7}$
$N_f=40$
$N_b=60$
No composites
Fermi liquid
Fermionic domain
$|f_{1f0b}|^2$
$|f_{1f,0b}|^2$
$|f_{1f,0b}|^2$
i
j

Disorder: Speckle radiation or supperlattices or…

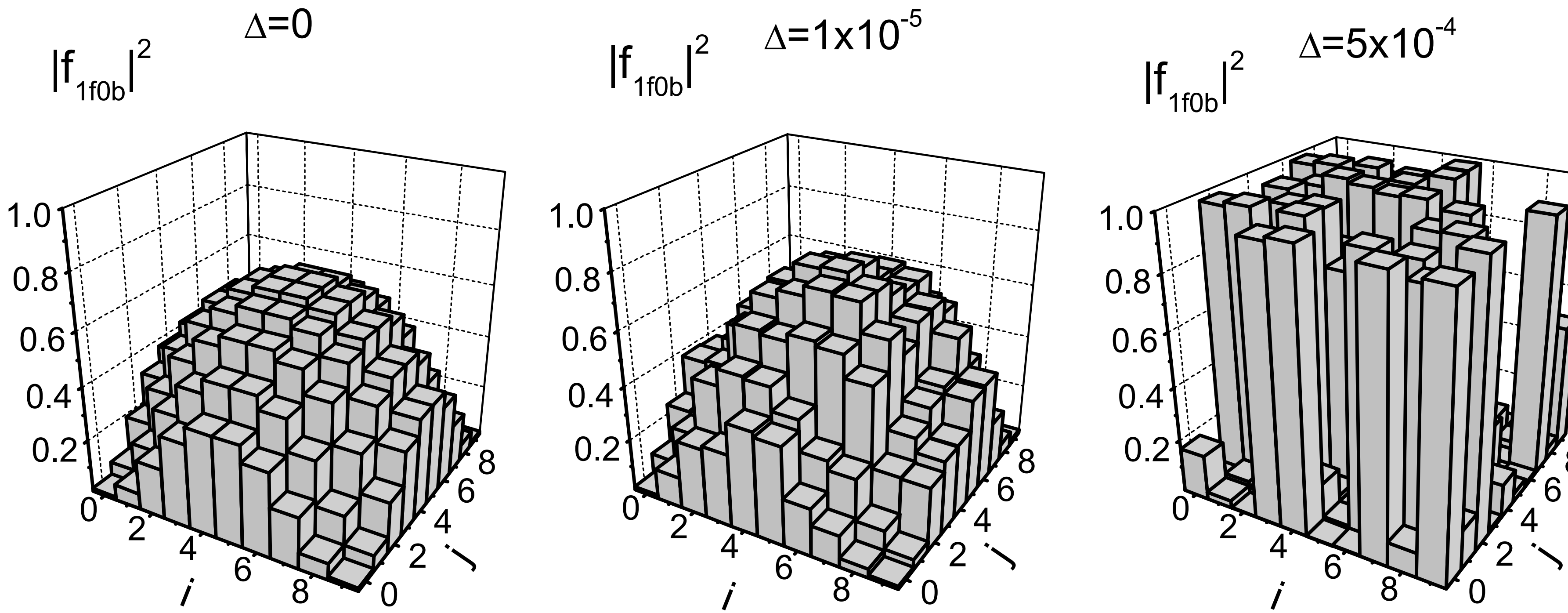


Tunneling

On site interactions

Damski et al. PRL 2003

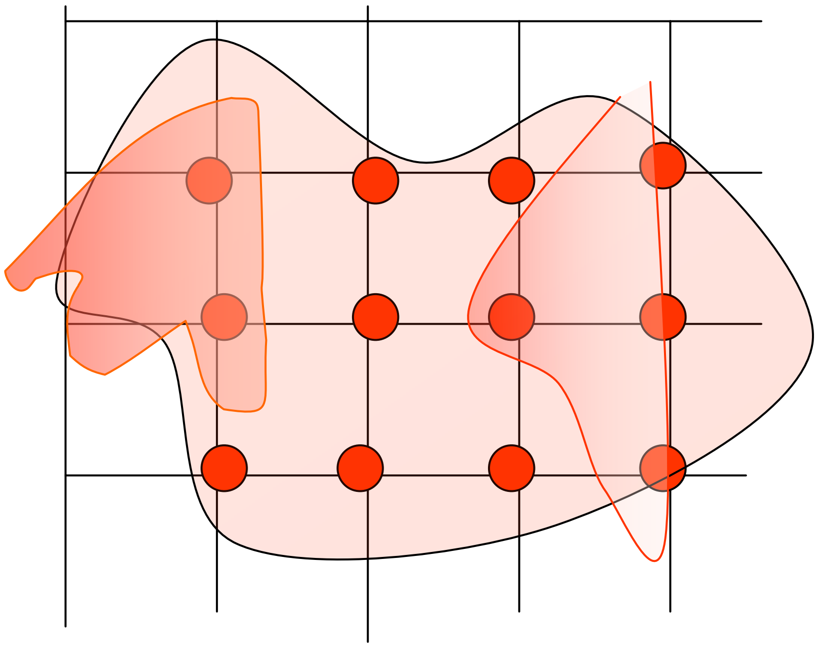
Growing adiabatically small disorder



Fermi liquid → Fermi glass

DEPENDING ON THE **DISORDER** WE WILL FIND (at low Temperatures):

Fermi glass phase
Mott insulators
Domain insulators
„Dirty" superfluids
Quantum percolation
Spin glasses!
Checkboard phases
Density wave phases



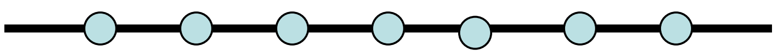
DISORDERED AND COMPLEX ULTRACOLD GASES AND QUANTUM INFORMATION

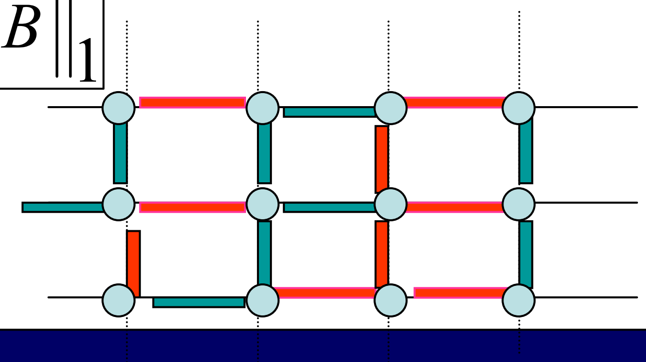
1) A. Sanpera A, A. Kantian A, L. Sanchez-Palencia, J. Zakrzewski, and M. Lewenstein, Atomic Fermi-Bose mixtures in inhomogeneous and random lattices: From Fermi glass to quantum spin glass and quantum percolation, Phys. Rev. Lett. **93**, 040401 (2004).
2) A. Sen De, U. Sen, M. Lewenstein, V. Ahufinger, M. Pons, and A. Sanpera, Disordered complex systems using cold gases and trapped ions, quant-phys/0508018, Proceedings of the 17th International Conference on Laser Spectroscopy (World Scientific, Singapore 2005),), Eds. E.A. Hinds, A. Ferguson, and E. Riis, p.156-166.
3) A. Sen De, U. Sen, V. Ahufinger, H.J. Briegel, A. Sanpera, and M. Lewenstein, Quantum Information Processing in Disordered and Complex Quantum Systems, quant-ph/0507172, submitted to Phys. Rev. A.
4) M. Pons, V. Ahufinger, C. Wunderlich, A. Sanpera, and M. Lewenstein, Trapped ion chain as a neural network, cond-mat/0512606, submitted to Phys. Rev. Lett.

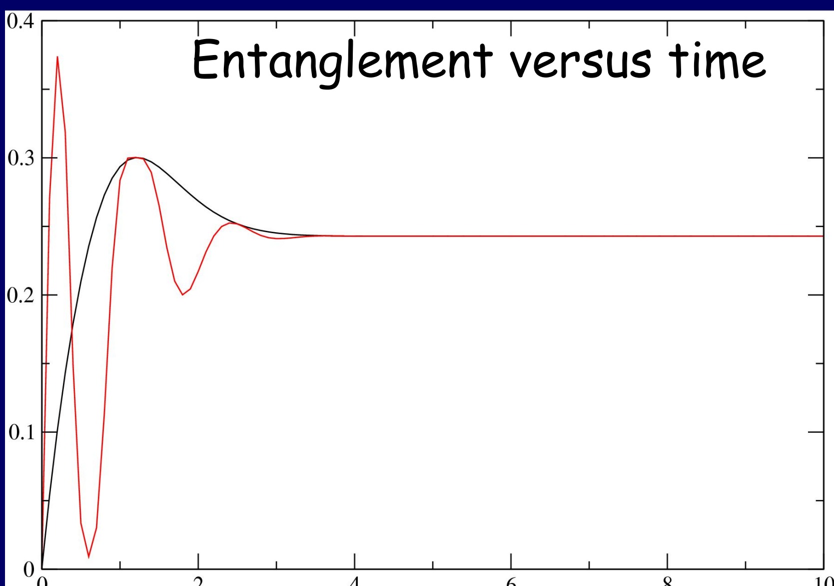
ENTANGLEMENT IN SPIN GLASSES IN 1D AND 2D LATTICES

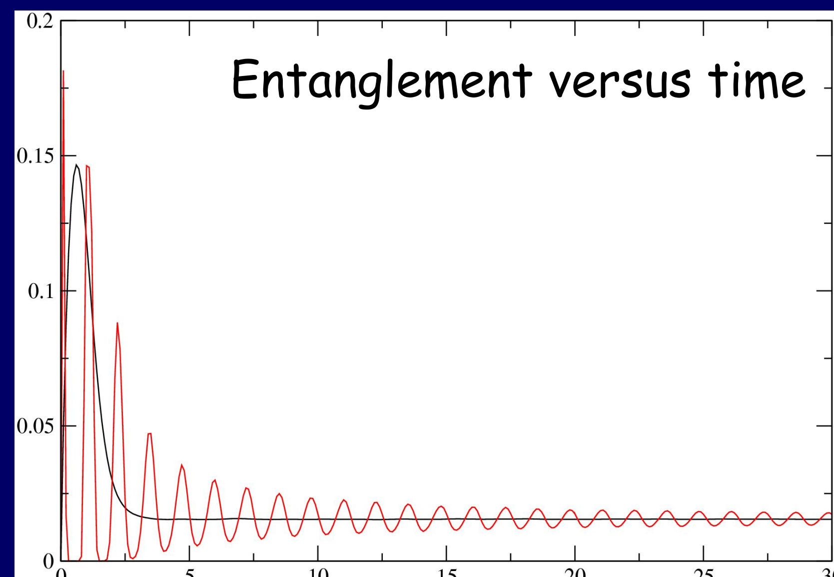
$$H_{E-A} = -\sum_{\langle ij \rangle} J_{ij}\sigma_i\sigma_j + h\sum_i \sigma_i \qquad \rho(t,\{J_{ij}\}) = \exp\{-iH_{E-A}t\}|\Psi\rangle\langle\Psi|\exp\{+iH_{E-A}t\}$$

$$\boxed{E_{LN}(\rho_{AB}) = \log_2 \left\| \rho_{AB}^{T_A} \right\|_1}$$





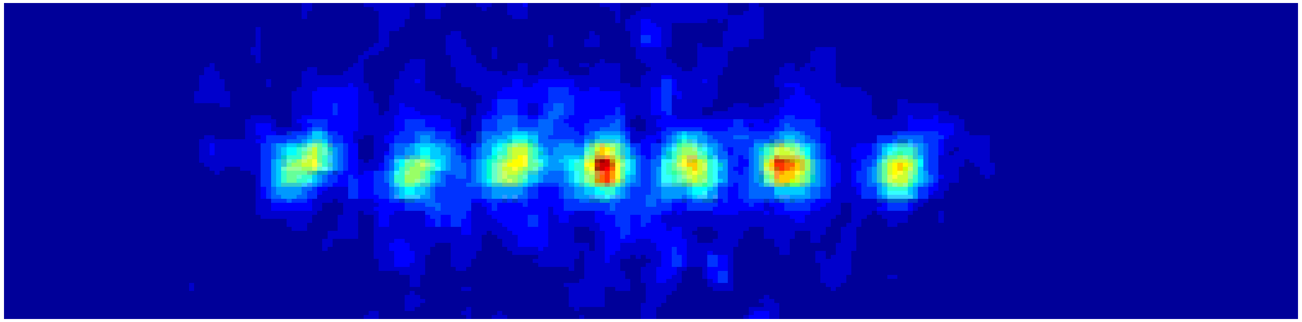




COMPLEX SYSTEMS and LONG RANGE INTERACTIONS: QIP IN NEURAL NETWORKS

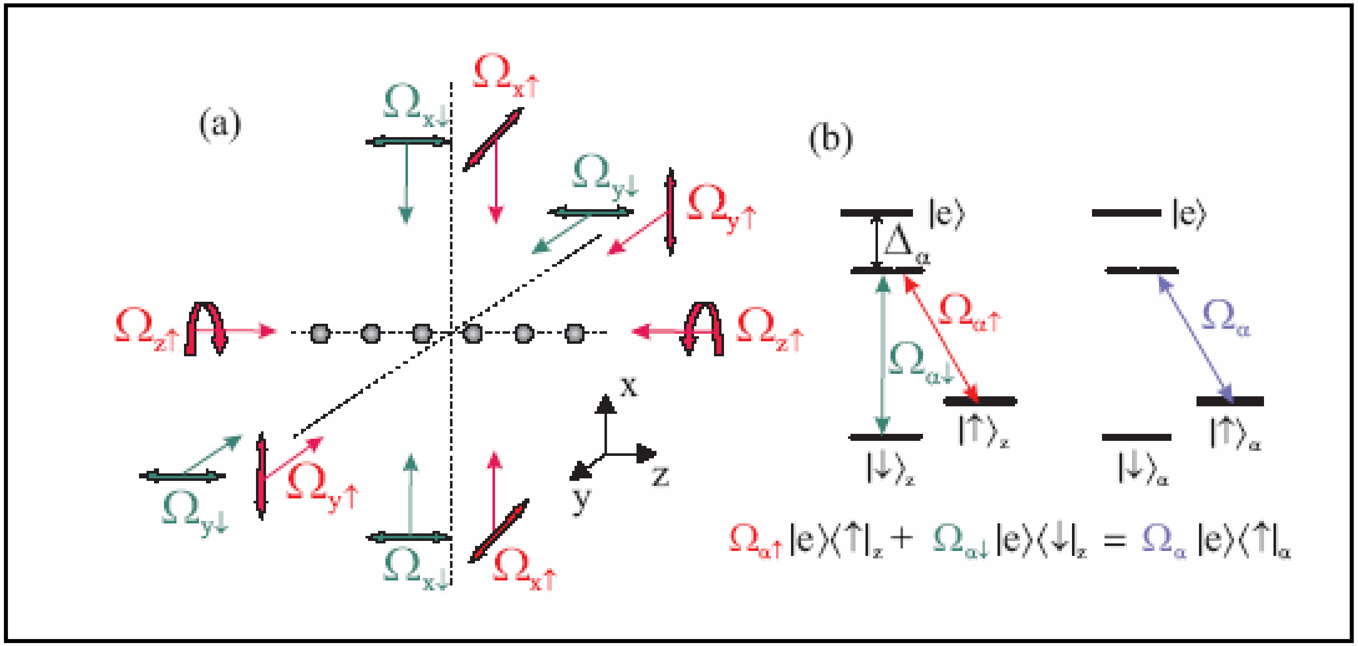
M. Pons, V. Ahufinger, C. Wunderlich, A. Sanpera,
and M.Lewenstein (cond-mat/0512606)

Trapped ions with engineered interactions:
Spin chains with
long range couplings and neural networks



R. Blatt home page group in Innsbruck (thanks!)

Trapped ion chain as a neural network



$$H = \sum_{\alpha,n} \hbar\omega_{\alpha,n} a^{+}_{\alpha,n} a_{\alpha,n} - 2\sum_{\alpha,i} F_\alpha q_{\alpha,i} \left|\uparrow\right\rangle\left\langle\uparrow\right|_{\alpha,i} + \sum_{\alpha,i} B^\alpha \sigma_i^\alpha$$

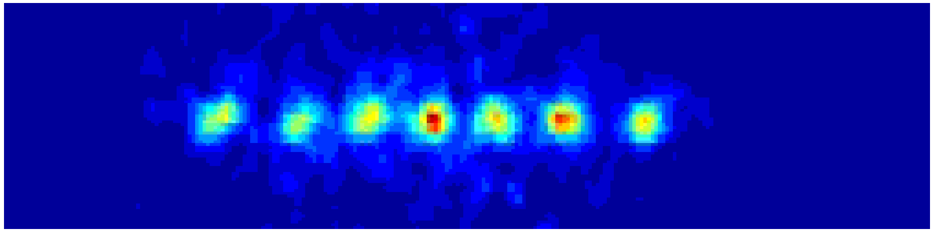
Canonical transformation

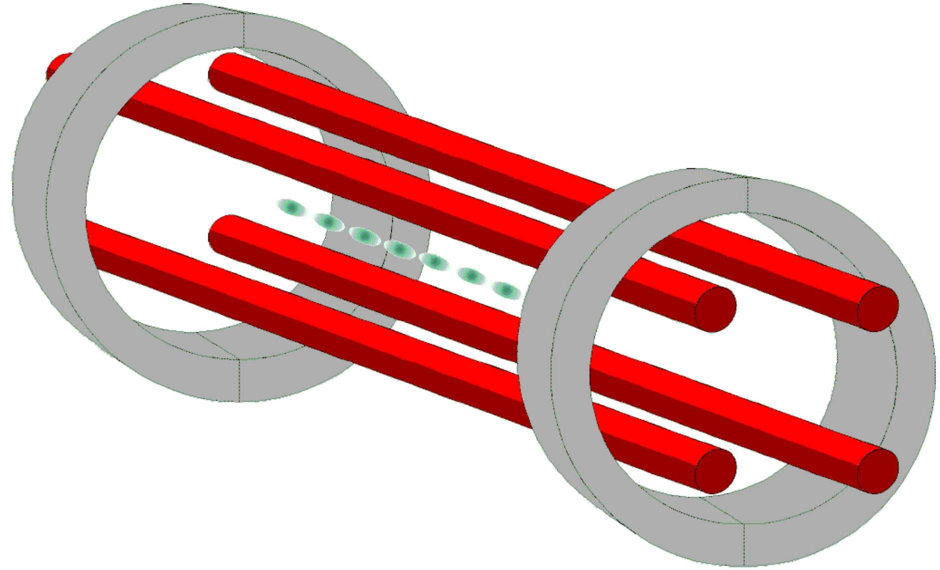
$$\overline{H} = \frac{1}{2}\sum_{\alpha,i,j} J_{ij}\sigma_i^\alpha \sigma_j^\alpha + \sum_{\alpha,i} B'^\alpha \sigma_i^\alpha$$

„Neural network“

F. Mintert and Ch. Wunderlich, PRL 87, 257904 (2001); D.Porras and J.I. Cirac, PRL **92**,207901 (2004)

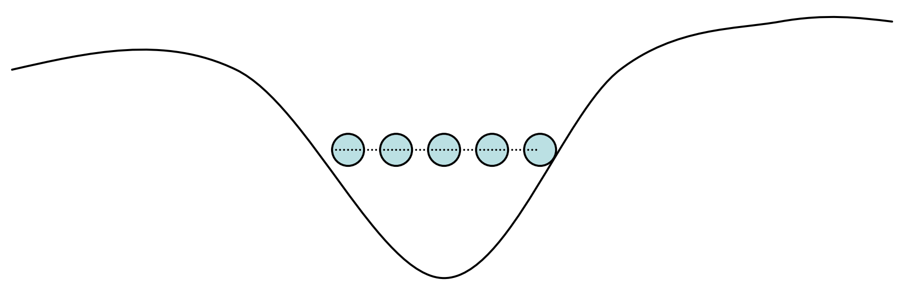
Ions trap Innsbruck



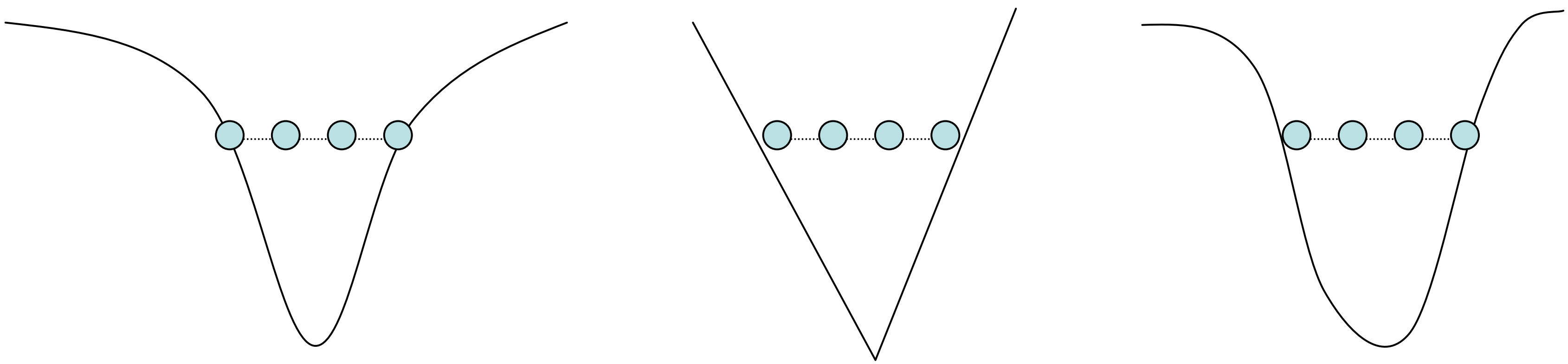


$$J_{ij} = -\sum_n \frac{F_\alpha^2}{m\omega_{\alpha,n}^2} \mathrm{M}_{i,n}^\alpha \mathrm{M}_{j,n}^\alpha$$

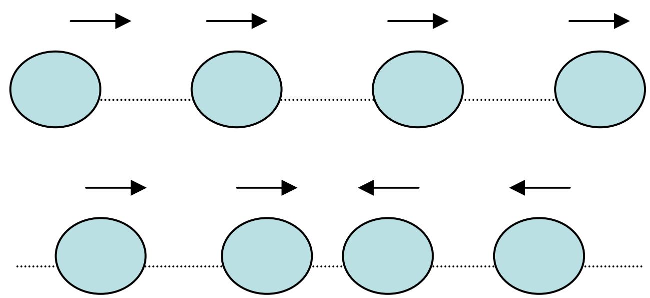
Eigenvalues Eigenvectors of
the phonon modes

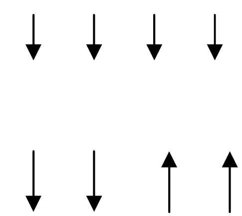


1D ion chains



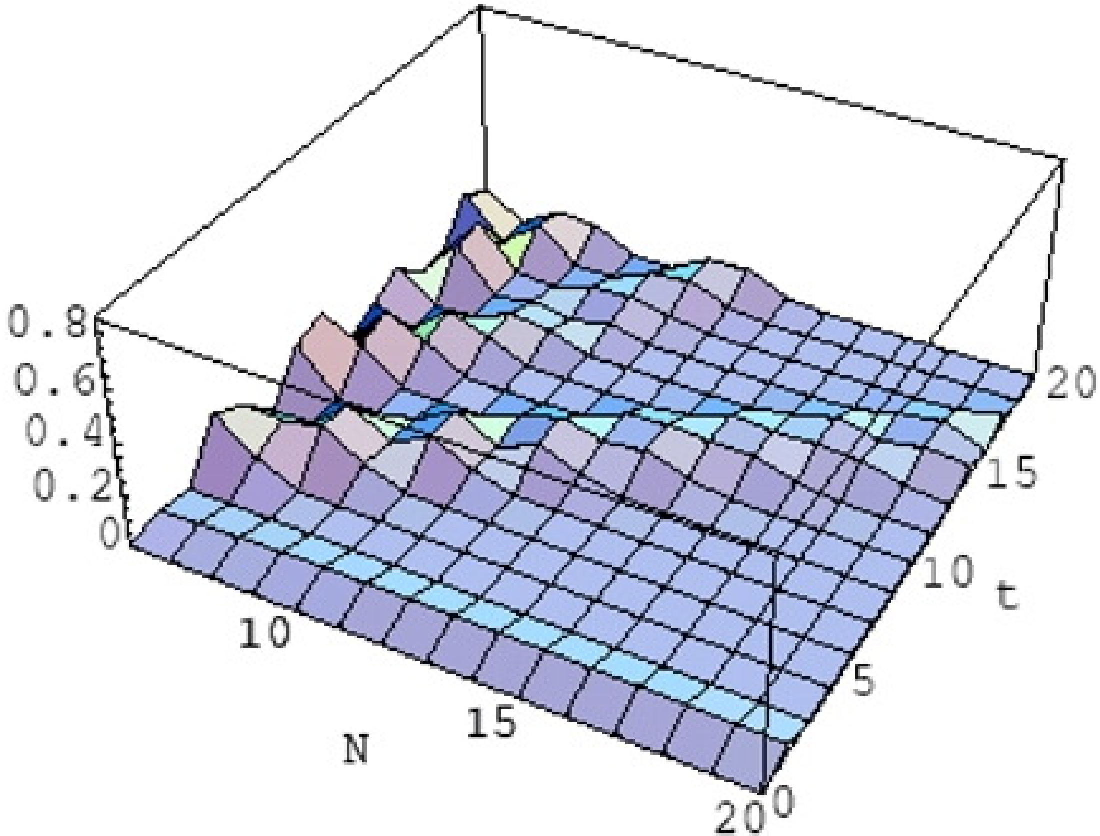
From phonons….   to spin





NEURAL NETWORK HOPFIELD MODEL !

$$E_{LN}(\rho_{AB}) = \log_2 \left\| \rho_{AB}^{T_A} \right\|_1$$



Dynamics of two-particle entanglement in a spin system with long-range interactions plotted against the number of spins (X-axis) and time (Y-axis). The results exhibit a revival of entanglement after certain time. The time of revival grows with number of spins.

Spin models in random fields: Disorder induced order

J. Wehr, A. Niederberger, L. Sanchez-Palencia, A. Sen (De), U. Sen, and M. Lewenstein,
in preparation

Large effects by arbitrarily small disorder

Classical Ising spin model in random magnetic fields:

- Arbitrarily small random field (with the probability distribution respecting the Ising Z_2 symmetry) destroys spontaneous magnetization in the Ising spin model in 2D (i.e. at the lower critical dimension) at any temperature T.

- In XY spin model in 2D, according to Mermin-Wagner theorem there is no magnetisation at any finite T. Random, symmetrically distributed field of arbitrarily small strength in X direction breaks the continuous O(2) (U(1)) symmetry of the XY model, and prevents, obviously, magnetisation in the X direction. The model attains magnetisation in Y direction at T=0, and, amazingly, at finite temperatures!!! Disorder induced order!!!
- How does quantum effects (quantum fluctuations, transverse fields) change these pictures?

Large effects by arbitrarily small disorder

Spin models in random magnetic fields:

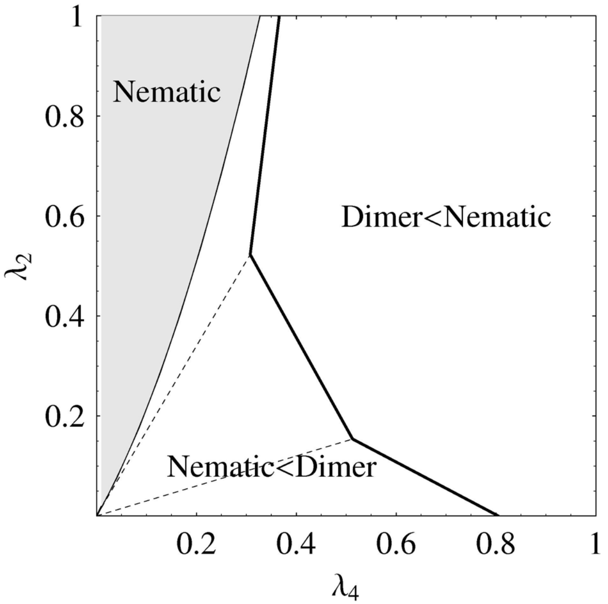
- Classical ferro- (antiferro-) magnetic Heisenberg model in 3D has spontaneous magnetisation (Néel order). Arbitrarily small random field in the Z direction prevents ordering in this direction and breaks the spin rotational symmetry. The model becomes "more like" XY model!!! The critical temperature grows by 50%!!! Disorder induced order!!!
- Classical ferro- (antiferro-) magnetic XY model in 3D has spontaneous magnetisation (Néel order). Arbitrarily small random field in the X direction prevents ordering in this direction and breaks the spin rotational symmetry. The model becomes "more like" Ising model!!! The critical temperature grows by 100%!!! Disorder induced order!!!
- How does quantum effects (quantum fluctuations, transverse fields) change these pictures?

Spinor F=2 gases in optical lattice

(work in progress, Ł. Zawitkowski, K. Eckert, A. Sanpera and M. Lewenstein)

- Strong coupling (Mott) limit
- 1, 2, 3 particles per lattice site…
- Effective Hamiltonian

$$H=\sum_{<i,j>}\lambda_S P_S + \text{disorder}=$$
$$\sum_{<i,j>} \text{Polynom (Heisenberg)}+ \text{disorder}$$



1 atom per site

Future challenge: E. Polzik idea – carry over light-atom interface to spinor gases!

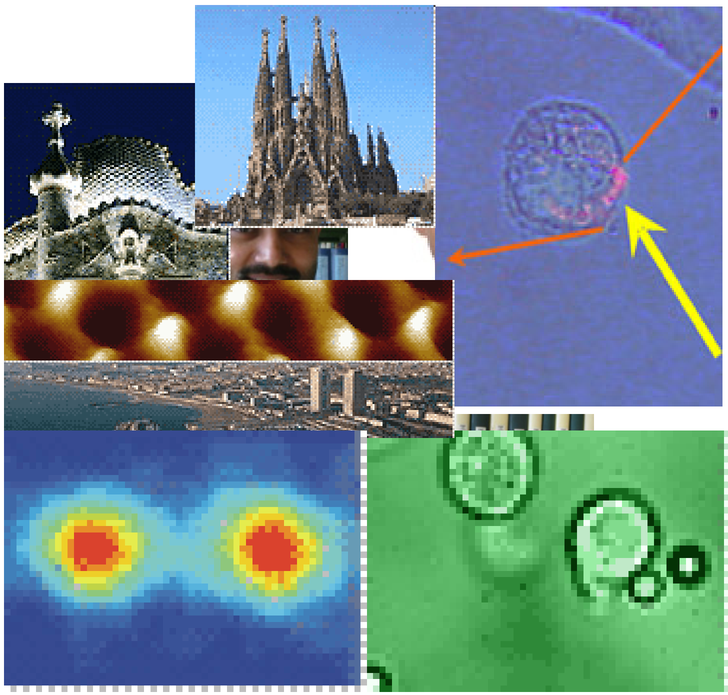
CONCLUSIONS :

- *There are more interesting things on earth and heaven that are dreamt of by our philosophers!!!*

Wow!!!

Theoretical Quantum Optics

at ICFO



+Chiara Menotti, Christian Trefzger, Jonas Larson, Sibille Braumgart, Mikke Leskinen and, last but not least, the Hannover gang of four

Cold atoms and cold gases:

- Weakly interacting Bose and Fermi gases (solitons, vortices, phase fluctuations, atom optics, quantum engineering)
- Dipolar Bose and Fermi gases
- Collective cooling, CW atom laser, quantum master equation
- Strongly correlated systems in AMO physics

Quantum Information:

- Quantification and classification of entanglement
- Quantum cryptography and communications
- Implementations in quantum optics

Matter in strong laser fields:

- High harmonics generation, above threshold ionization, multielectron ionization
- Attophysics
- Analogies: Super-intense laser-atom physics and nonlinear atom optics

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Question: Can AMO physics help?

1. Can cold atoms or ions be used to model complex systems? YES!

- Bose gas in a disordered optical lattice: From Anderson to Bose glass
- Fermi-Bose mixtures in random lattices: From Fermi glass to fermionic spin glass and quantum percolation
- Trapped ions with engineered interactions: Spin chains with long range interactions and neural networks

2. Can cold atoms and ions be used as quantum simulators of complex systems?

YES!

3. Can cold atoms and ions be used for quantum information processing in complex systems?

YES?

SUMMARY OF: QIP WITH DISORDERED SYSTEMS

1. Can one generate entanglement in trapped ion systems of Ising spin chains with long range couplings? YES!

- We prepare the system in the product state $\psi(0) = |+\rangle|+\rangle|+\rangle\ldots$, where $|+\rangle$ is an eigenstate of σ_x
- We then engineer the couplings and apply for certain time, so that $\psi(t) = \exp(-iH_{Ising}t)\psi(0)$

2. Can one generate entanglement in atomic spin glasses? (short range Edwards – Anderson model) YES!

- We apply the same procedure as above, but engineer the SG couplings and apply for certain time, so that $\psi(t) = \exp(-iH_{SG}t)\psi(0)$

3. Can quantum information be processed in atomic complex systems? ???