Stability of an atom laser in the presence of pumping and feedback

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AUSTRALIAN RESEARCH COUNCIL
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QUANTUM ATOM OPTICS

Continuous atom lasers:

Why pump an atom laser?

- Continuous source
- Modal stability
- Gain-narrowing (linewidth reduction)

Atom laser models

- Semiclassical (Gross-Pitaevskii equation)
  - No quantum statistics
  - Tractable with full spatial structure
- Quantum
  - Only tractable models are single (or few) mode

An ideal atom laser in an ideal world

When the atom laser is in single mode operation:

- Semiclassical model predicts infinitely narrow output
- Quantum model necessary to determine linewidth
- Fortunately, single mode quantum model is tractable

$$\hat{H} = \hbar\omega_0 \hat{b}_0^\dagger \hat{b}_0 + U_0 \hat{b}_0^\dagger \hat{b}_0^\dagger \hat{b}_0 \hat{b}_0 + \int d\mathbf{p}\, \hbar\omega_\mathbf{p}\, \hat{c}_\mathbf{p}^\dagger \hat{c}_\mathbf{p}$$

$$+ \hbar \int d\mathbf{p} \left(\chi(\mathbf{p}) \hat{b}_0^\dagger \, \hat{c}_\mathbf{p} + \chi^*(\mathbf{p}) \hat{b}_0 \, \hat{c}_\mathbf{p}^\dagger \right) + \hat{H}_{\mathrm{pump}}(\hat{b}_0, \hat{b}_0^\dagger)$$

output modes

lasing mode

$$\hat{c}_\mathbf{p}(t) = \hat{c}_\mathbf{p}(0)\, e^{-i\omega_\mathbf{p} t} - i\,\chi^*(\mathbf{p}) \int_0^t e^{-i\omega_\mathbf{p}(t-u)} \hat{b}_0(u)$$

An ideal atom laser in an ideal world

The spectrum of the output atom laser flux

$$\frac{\partial \langle \hat{c}_{\mathbf{p}}^{\dagger} \hat{c}_{\mathbf{p}} \rangle}{\partial t} = \langle \hat{c}_{\mathbf{p}}^{\dagger}(0) \hat{c}_{\mathbf{p}}(0) \rangle + \langle \hat{B}_{lah}\, \hat{c}_{\mathbf{p}}(0) \rangle + \langle \hat{c}_{\mathbf{p}}^{\dagger}(0)\, \hat{J}_{unk} \rangle$$

$$+2 |\chi(\mathbf{p})|^2\, \Re e \left(\int_0^t du\, e^{-i\omega_{\mathbf{p}}(t-u)} \langle \hat{b}_0^{\dagger}(t) \hat{b}_0(u) \rangle \right)$$

Two time correlation: $g(\tau) \equiv \lim_{t\to\infty} \langle \hat{b}_0^{\dagger}(t+\tau) \hat{b}_0(t) \rangle$

For a system in a coherent state: $g(\tau) \approx e^{i(\omega_0+\Delta)\tau}$

Monoenergetic output

An ideal atom laser in an ideal world

The spectrum of the output atom laser flux

$$\frac{\partial \left\langle \hat{c}^{\dagger}_{\mathbf{p}} \hat{c}_{\mathbf{p}} \right\rangle}{\partial t} = \left\langle \hat{c}^{\dagger}_{\mathbf{p}}(0) \hat{c}_{\mathbf{p}}(0) \right\rangle + \left\langle \hat{B}_{lah}\, \hat{c}_{\mathbf{p}}(0) \right\rangle + \left\langle \hat{c}^{\dagger}_{\mathbf{p}}(0)\, \hat{J}_{unk} \right\rangle$$

$$+2\left|\chi(\mathbf{p})\right|^2 \Re e\left(\int_0^t du\, e^{-i\omega_{\mathbf{p}}(t-u)} \left\langle \hat{b}^{\dagger}_0(t) \hat{b}_0(u) \right\rangle \right)$$

Two time correlation: $g(\tau) \equiv \lim_{t\to\infty} \left\langle \hat{b}^{\dagger}_0(t+\tau) \hat{b}_0(t) \right\rangle$

For a highly pumped system with a wise choice of pumping mechanism: $g(\tau) \approx e^{i(\omega_0+\Delta)\tau} e^{-r|\tau| / 4(\bar{n}+n_s)^2}$

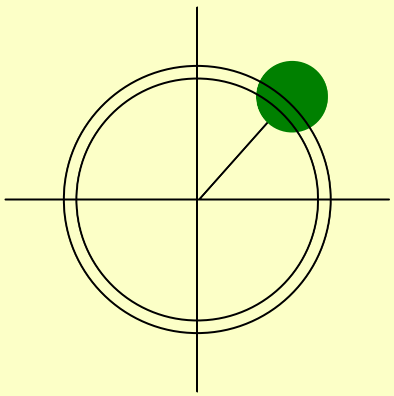
r pumping rate

$\bar{n}$ mean lasing mode number

n_s spontaneous loss

Gain narrowing

Output flux $\propto r \propto \bar{n}$ Output linewidth $\propto \dfrac{r}{4\left(\bar{n}+n_s\right)^2} \propto \dfrac{\gamma_{unpumped}}{\bar{n}}$



Phase diffusion increases with interactions

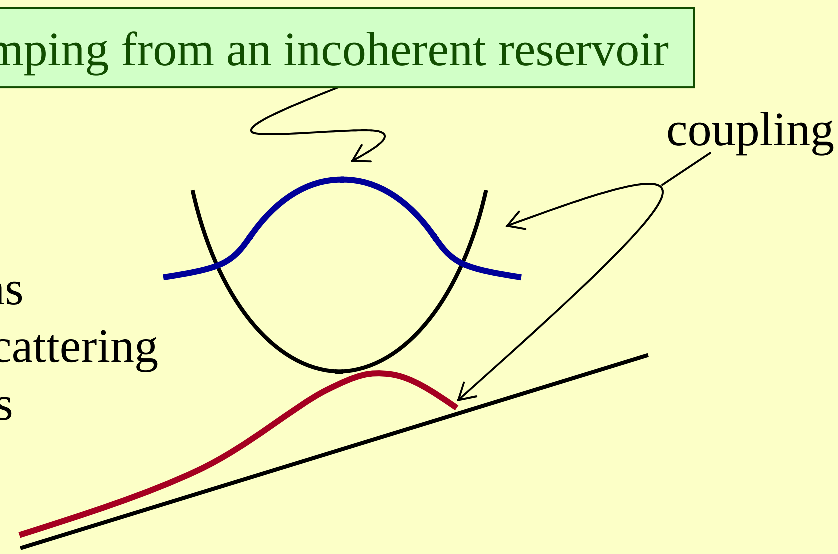
- Controllable by feedback

H.M.Wiseman and L.K.Thomsen, *PRL* ***86***, *1143 (2001)*

This model, and the narrow linewidth require:

- Well chosen pumping mechanism
- Stable, single mode operation
- Low interactions

Semiclassical atom laser model:



- BEC-BEC, beam-beam and BEC-beam interactions
- BEC-BEC, beam-beam and BEC-beam inelastic scattering
- Gravity, trapping potentials, background gas losses
- Momentum kick during output coupling
- Spatial coupling and pumping

$$i\hbar\frac{\partial}{\partial t}\psi_t(\mathbf{x}) = \left(-\frac{\hbar^2}{2m}\nabla^2 + V_{trap}(\mathbf{x}) + U_{tt}|\psi_t|^2 + U_{tu}|\psi_u|^2 - i\hbar\gamma_t^{(1)} - i\hbar\gamma_{tt}^{(2)}|\psi_t|^2 - i\hbar\gamma_{tu}^{(2)}|\psi_u|^2 + i\kappa\rho\right)\psi_t + \hbar\Omega e^{i\mathbf{k.x}}\psi_u$$

$$i\hbar\frac{\partial}{\partial t}\psi_u(\mathbf{x}) = \left(-\frac{\hbar^2}{2m}\nabla^2 + V_{gravity}(\mathbf{x}) + U_{uu}|\psi_u|^2 + U_{tu}|\psi_t|^2 - i\hbar\gamma_u^{(1)} - i\hbar\gamma_{uu}^{(2)}|\psi_u|^2 - i\hbar\gamma_{tu}^{(2)}|\psi_t|^2\right)\psi_u + \hbar\Omega e^{-i\mathbf{k.x}}\psi_t$$

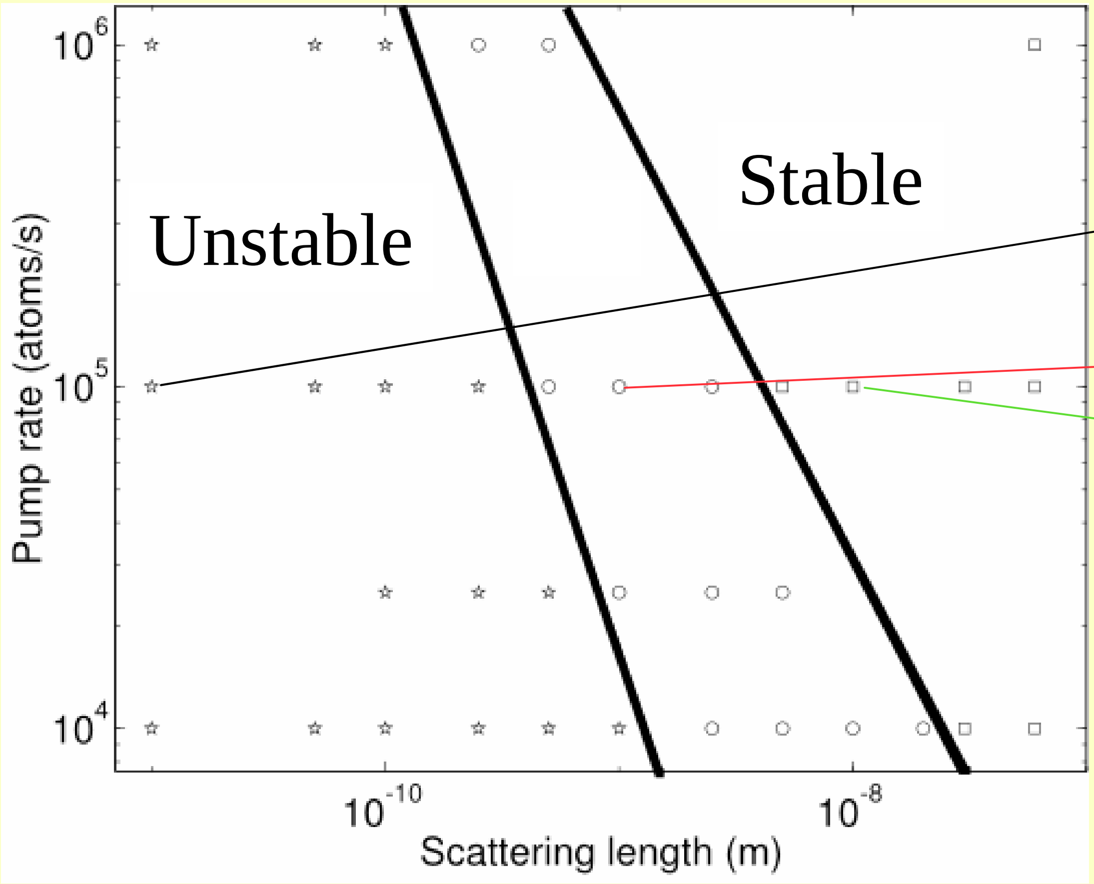
$$\frac{\partial}{\partial t}\rho(\mathbf{x}) = r - \gamma_{res}\rho - \kappa(\mathbf{x})|\psi_t|^2\rho + \lambda\nabla^2\rho$$

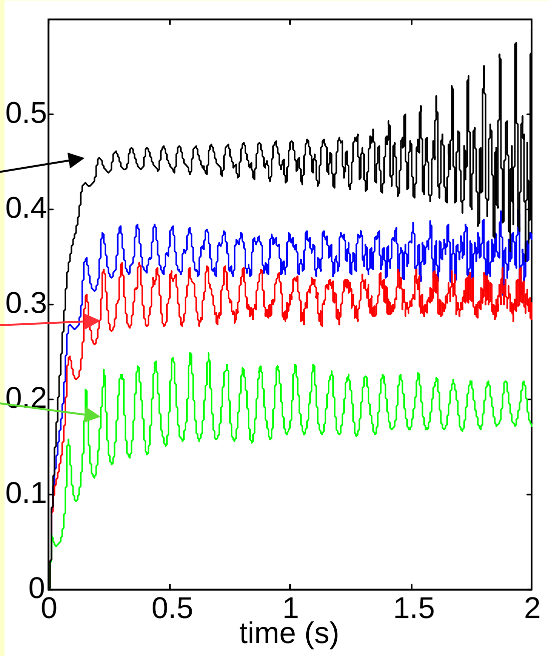
width σ

No interactions, strong pumping

Stability with spatially independent pumping: ($\sigma = \infty$)

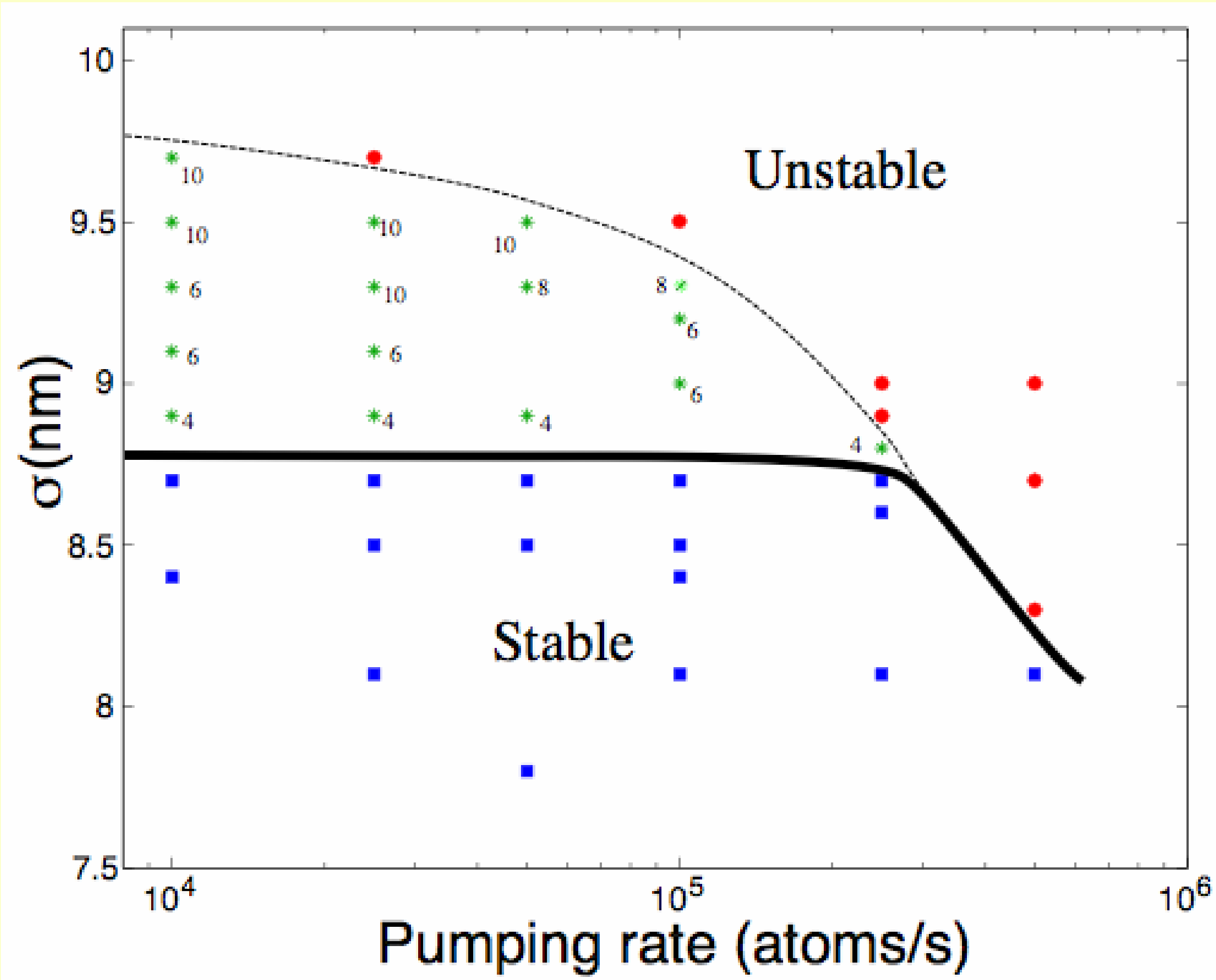
Stability depends on scattering length and pumping rate





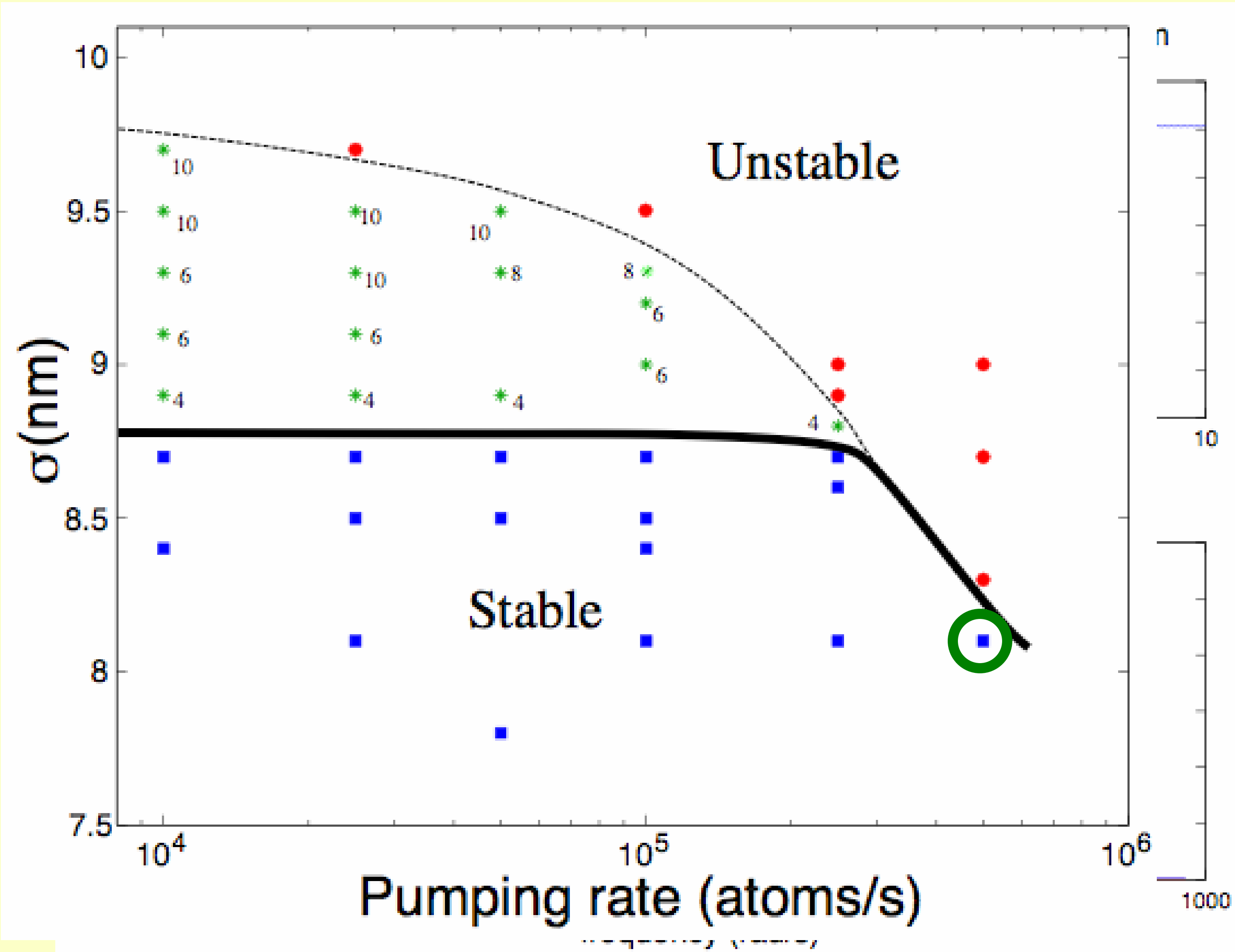
PRL **88**, 170403 (2002)

Stability phase diagram (a = 0)

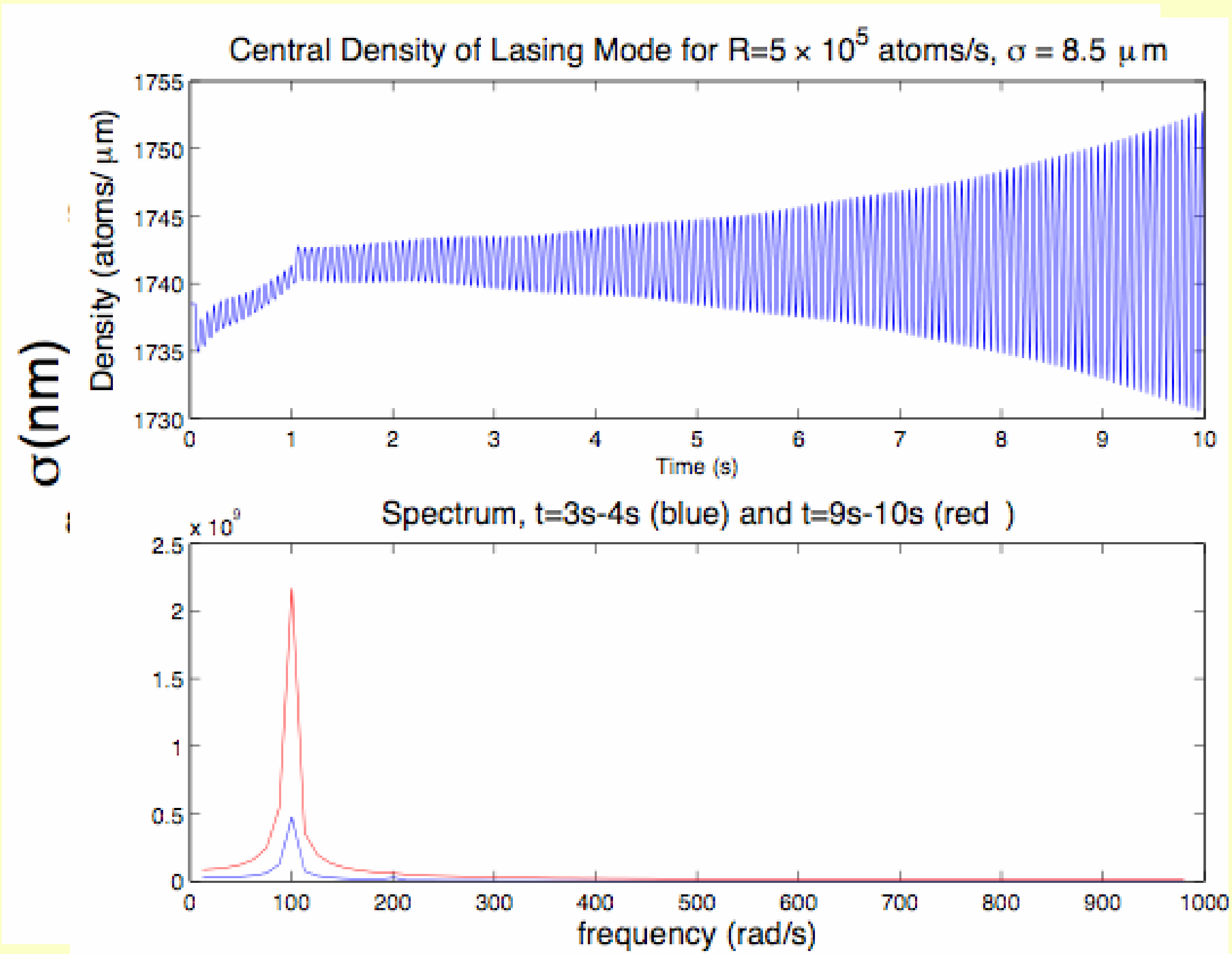


PRA **68**, 023607 (2003)

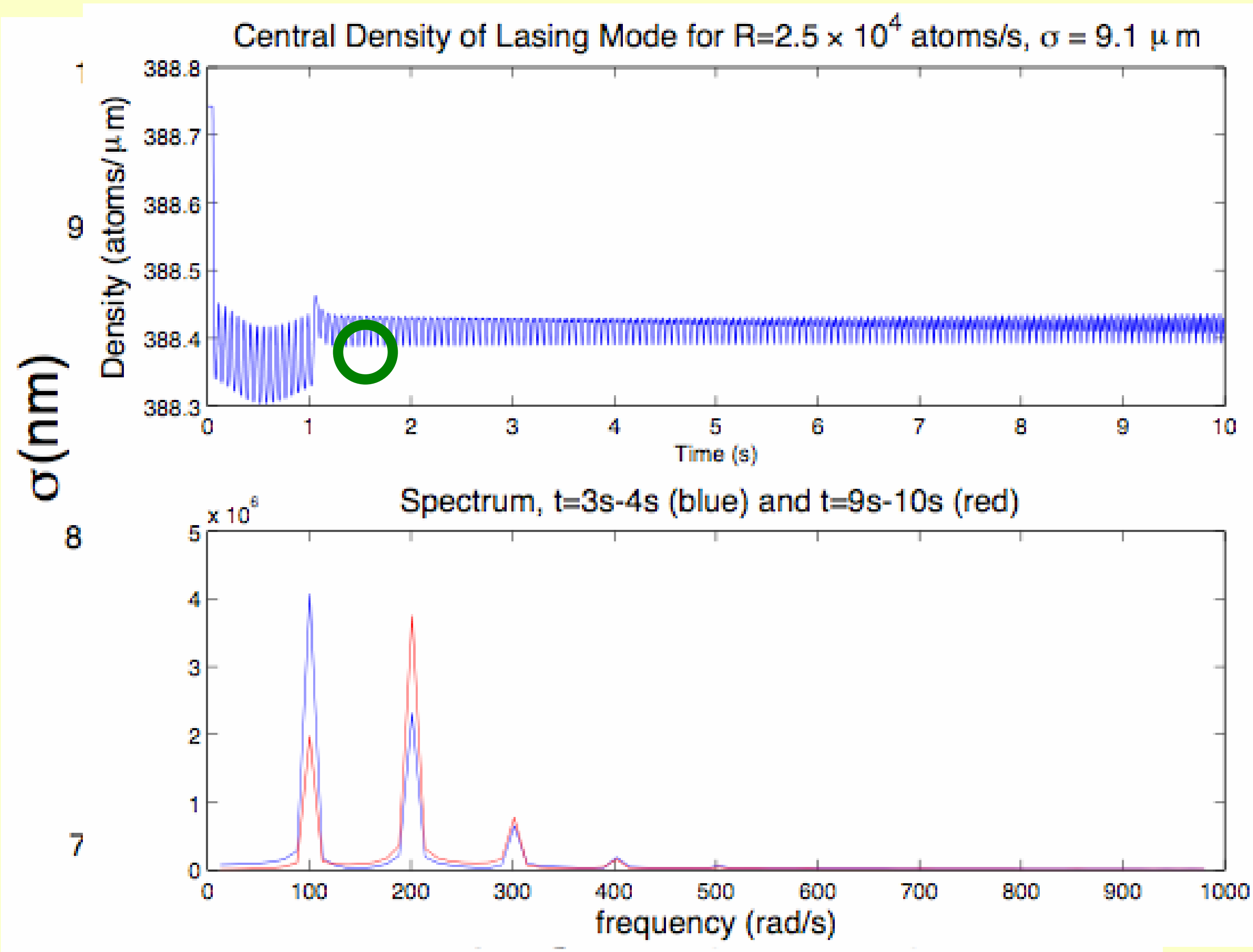
Frequency analysis of stable points



Frequency analysis of the unstable points



…so what were the green points?



Conclusions of the semiclassical model:

Only three important parameters

Stability increases with:

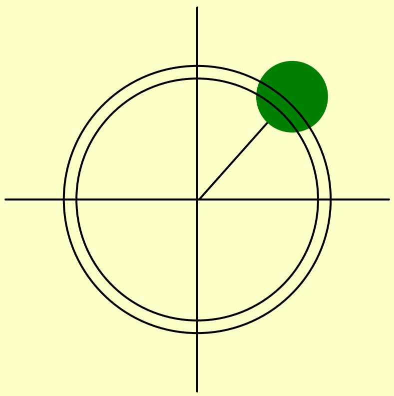
- spatially narrow pumping mechanism
- increasing pump rate
- increasing interactions

The first modifies the gain profile

The second two modify the loss profile

Gain narrowing (reprise)

Output flux $\propto r \propto \bar{n}$ Output linewidth $\propto \dfrac{r}{4\left(\bar{n}+n_s\right)^2} \propto \dfrac{\gamma_{unpumped}}{\bar{n}}$



Phase diffusion increases with interactions

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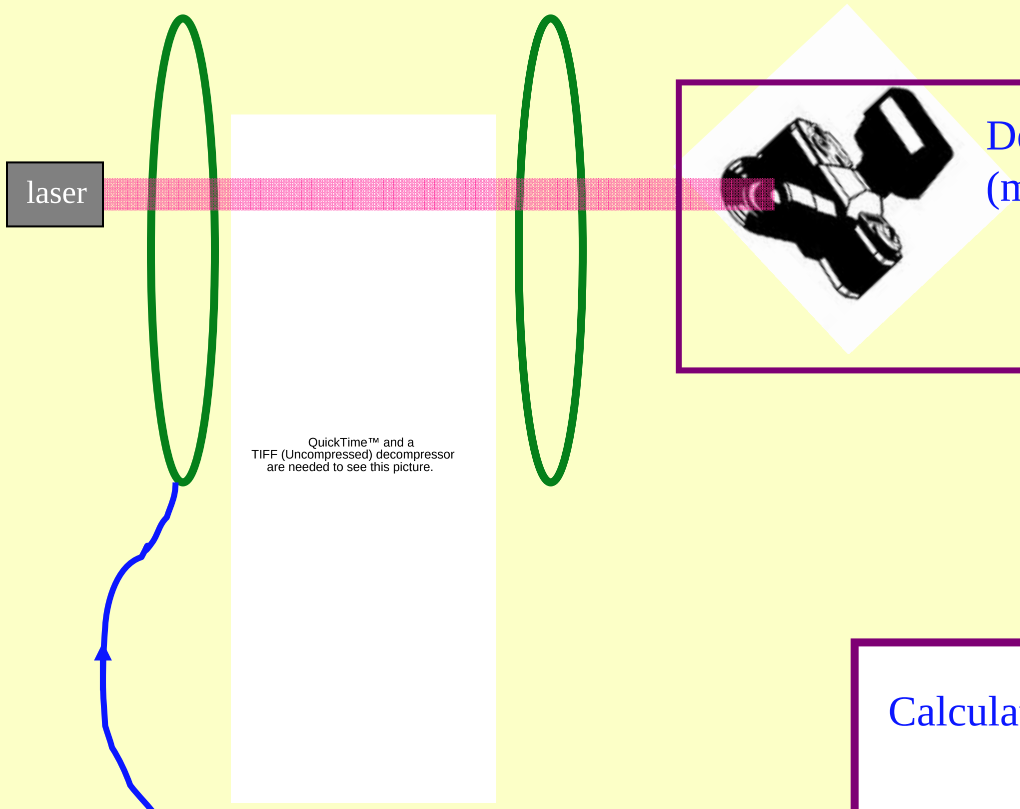
H.M.Wiseman and L.K.Thomsen, *PRL* ***86***, *1143 (2001)*

This model, and the narrow linewidth require:

- Well chosen pumping mechanism
- Stable, single mode operation
- Low interactions

Requires high interactions!

Feedback:



Detector
(measures condensate density)

$$\rho(\mathbf{r}) = |\psi(\mathbf{r})|^2$$

Calculate moments of atomic density, eg.

Centre of mass position: $\langle x \rangle = \int x \rho(x) dx$

Condensate width: $\langle x^2 \rangle = \int x^2 \rho(x) dx$ etc.

Control potential

Semiclassical model of spatial feedback:

$$i\hbar \frac{\partial \psi(\mathbf{r},t)}{\partial t} = \left(\hat{H}_0 + \hat{H}_f\right)\psi(\mathbf{r},t)$$

$$\hat{H}_0 = -\frac{\hbar^2}{2m}\nabla^2 + V_0(\mathbf{r}) + U_0|\psi|^2$$

uncontrolled trap potential

time-dependent amplitudes

$$\hat{H}_f = \sum_i a_i(t)\, f_i(\mathbf{r}) + \sum_j b_j(t)\, g_j(\mathbf{r})|\psi|^2$$

external potentials

spatially dependent
interaction strengths

What is our feedback goal?

Energy itself is not a good metric (feedback dependent)

$$\boxed{E_0(\psi) = \left\langle \frac{-\hbar^2}{2m}\nabla^2 + V_0 \right\rangle + \frac{1}{2}\left\langle U_0 |\psi|^2 \right\rangle}$$

$$\frac{dE_0}{dt} = \frac{i}{\hbar}\langle [\hat{H},\ \hat{T} + V_0] \rangle + \frac{U_0}{2}\frac{d}{dt}\int |\psi|^4 d^3\mathbf{r}$$

$$= \frac{-i\hbar}{2m}\int \sum_i a_i f_i(\mathbf{r})(\psi^* \nabla^2 \psi - \psi \nabla^2 \psi^*) d^3\mathbf{r}$$

$$- \frac{i\hbar}{2m}\int \sum_j b_j g_j(\mathbf{r}) |\psi|^2 (\psi^* \nabla^2 \psi - \psi \nabla^2 \psi^*) d^3\mathbf{r}$$

$$= -\sum_i a_i(t)\frac{d\langle f_i(\mathbf{r})\rangle}{dt} - \frac{1}{2}\sum_j b_j(t)\frac{d\langle g_j(\mathbf{r})|\psi|^2\rangle}{dt}$$

The feedback scheme

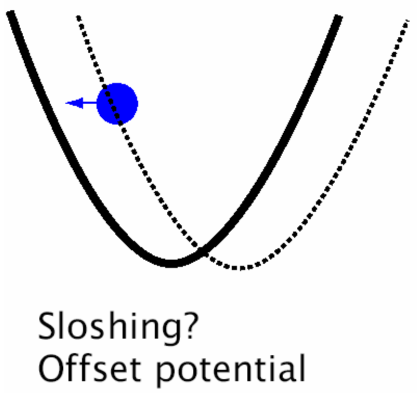
Choose $$a_i(t) = c_i \frac{d}{dt}\langle f_i(\mathbf{r})\rangle \qquad b_i(t) = u_i \frac{d}{dt}\langle g_i(\mathbf{r})|\psi|^2\rangle$$

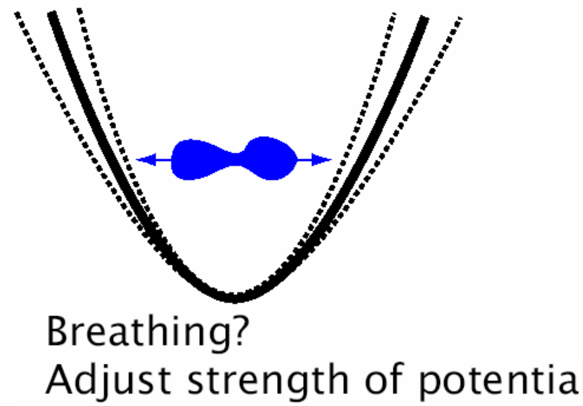
Where c_i and u_i are positive constants so that

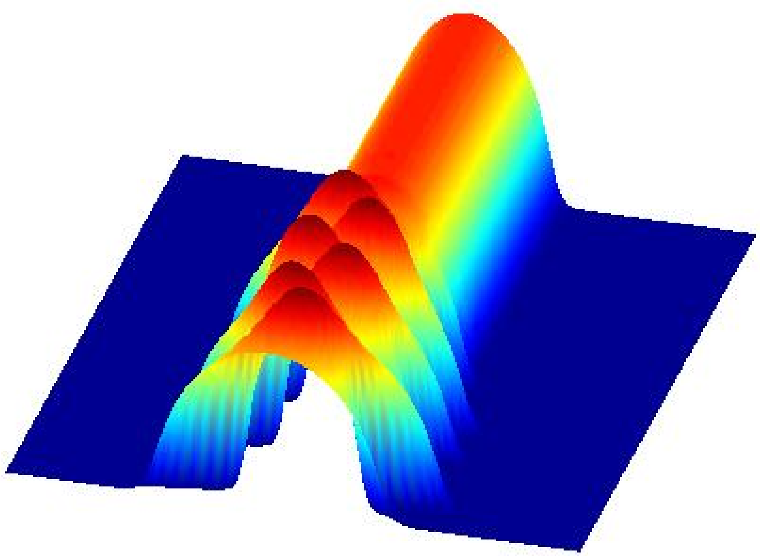
$$\boxed{\frac{dE_0}{dt} = -\sum_i c_i \left(\frac{d\langle f_i(\mathbf{r})\rangle}{dt}\right)^2 - \frac{1}{2}\sum_j u_j \left(\frac{d\langle g_j(\mathbf{r})|\psi|^2\rangle}{dt}\right)^2}$$

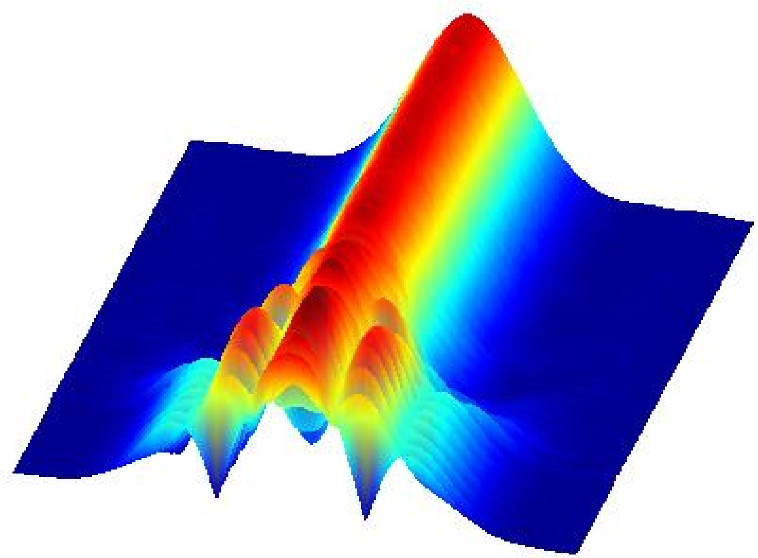
e.g. $$\boxed{\frac{dE_0}{dt} = -c_1\left(\frac{d\langle x\rangle}{dt}\right)^2 - c_2\left(\frac{d\langle x^2\rangle}{dt}\right)^2 - u_1\left(\frac{d\langle|\psi|^2\rangle}{dt}\right)^2}$$

Feedback:


Sloshing?
Offset potential

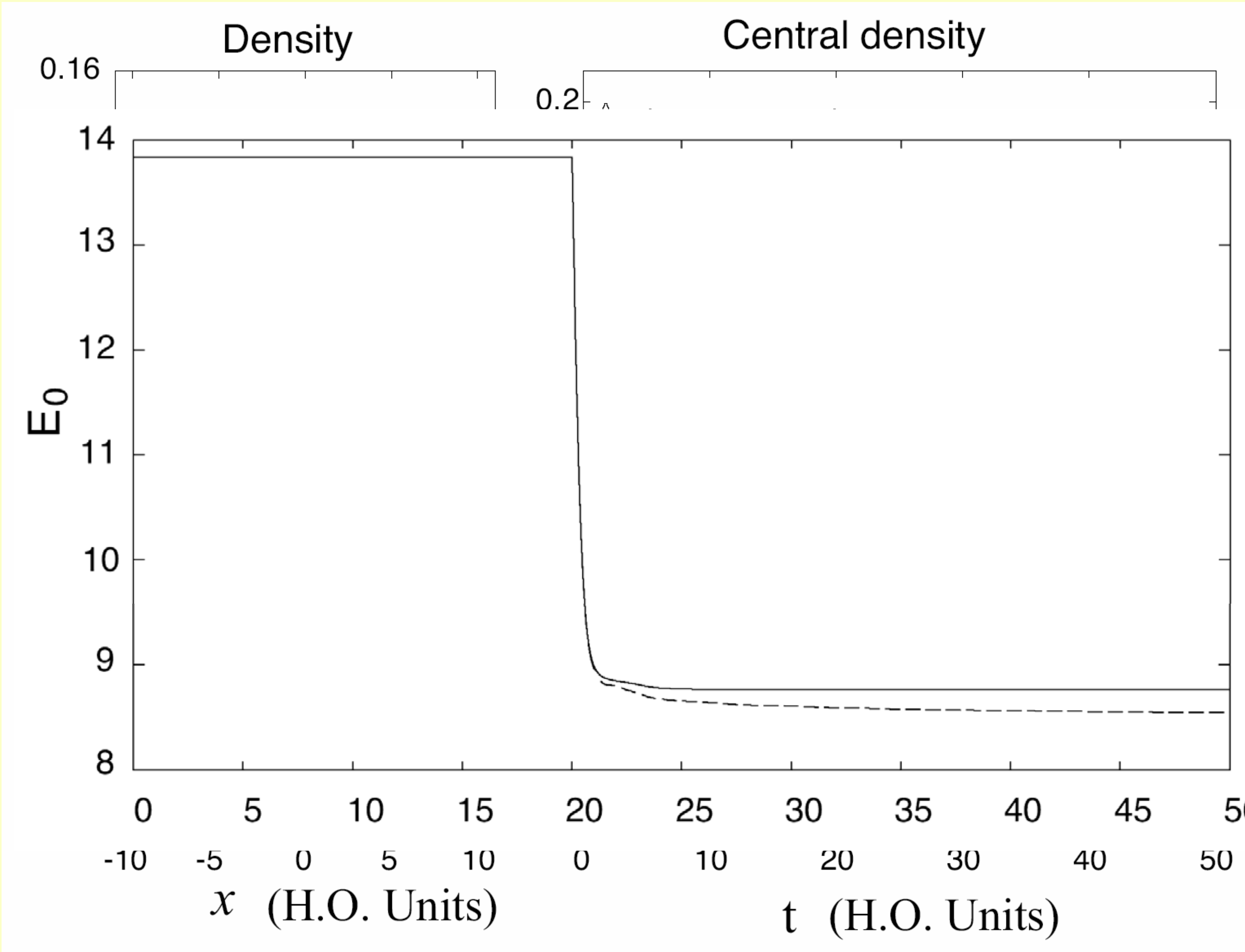

Breathing?
Adjust strength of potential





Feedback to a BEC

PRA **69,** 013605 (2004)



Pumped atom laser with feedback

$$i\hbar\frac{\partial}{\partial t}\psi_t(\mathbf{x}) = \left(-\frac{\hbar^2}{2m}\nabla^2 + V_{trap}(\mathbf{x}) + U_{tt}|\psi_t|^2 + U_{tu}|\psi_u|^2 - i\hbar\gamma_t^{(1)} - i\hbar\gamma_{tt}^{(2)}|\psi_t|^2 - i\hbar\gamma_{tu}^{(2)}|\psi_u|^2 + i\kappa\rho\right)\psi_t + \hbar\Omega e^{i\mathbf{k.x}}\psi_u$$

$$i\hbar\frac{\partial}{\partial t}\psi_u(\mathbf{x}) = \left(-\frac{\hbar^2}{2m}\nabla^2 + V_{gravity}(\mathbf{x}) + U_{uu}|\psi_u|^2 + U_{tu}|\psi_t|^2 - i\hbar\gamma_u^{(1)} - i\hbar\gamma_{uu}^{(2)}|\psi_u|^2 - i\hbar\gamma_{tu}^{(2)}|\psi_t|^2\right)\psi_u + \hbar\Omega e^{-i\mathbf{k.x}}\psi_t$$

$$\frac{\partial}{\partial t}\rho(\mathbf{x}) = r - \gamma_{res}\rho - \kappa(\mathbf{x})|\psi_t|^2\rho + \lambda\nabla^2\rho$$

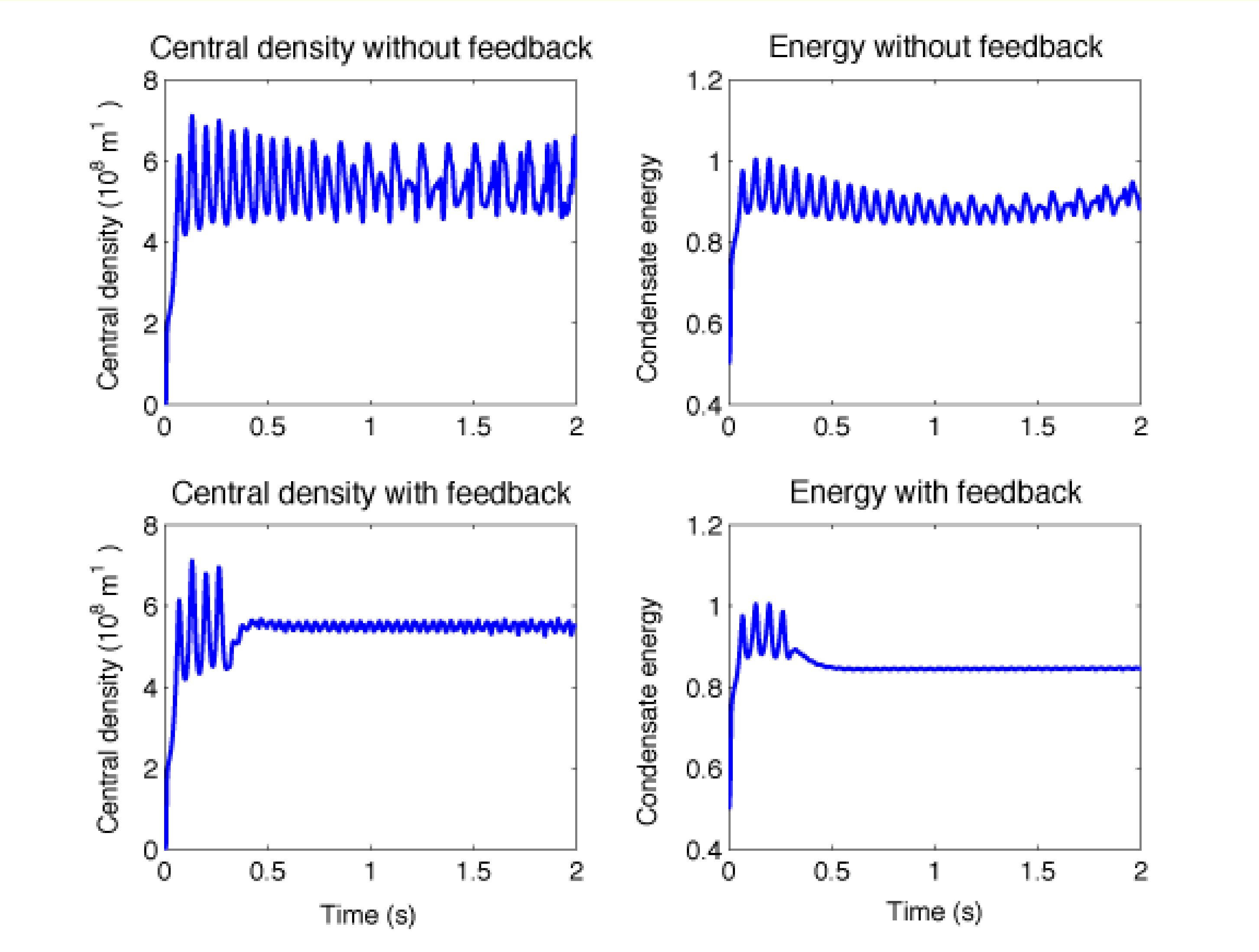
$$\frac{dE_0}{dt} = -c_1\left(\frac{d\langle x\rangle}{dt}\right)^2 - c_2\left(\frac{d\langle x^2\rangle}{dt}\right)^2 - u_1\left(\frac{d\langle|\psi|^2\rangle}{dt}\right)^2$$

$+$coupling terms

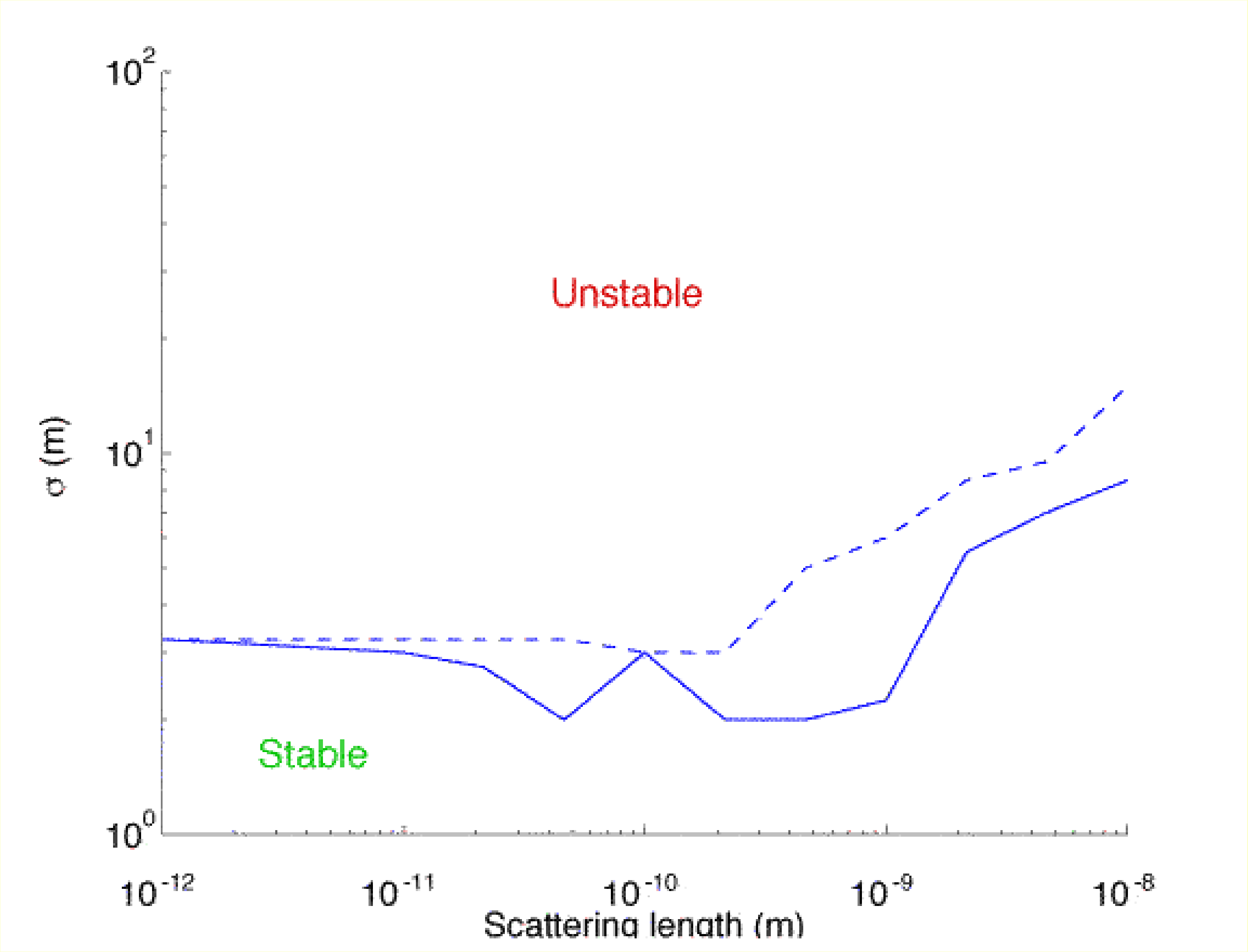
$+$pumping terms $\quad +$damping terms

A ridiculous number of computer hours later…

Pumped atom laser with feedback



Effect of feedback on the phase plot



Conclusions:

1. Atom lasers are single mode for large interactions

2. Single mode atom lasers are better for low interactions

3. Spatial feedback for pumped atom lasers
    - Highly effective over a finite timescale
    - Hardly affects the long term stability

Future issues:

- Reality of pumping processes
- Detection
- Few mode atom laser model combining **both** feedback schemes
- More advanced feedback schemes on modal/phase stability

[**Job**!]   Postdoctoral position available

Overview:

Pumping atom lasers

- Multimode pumped atom laser models

If atom lasers were stable:

- An ideal laser in an ideal world
- … with interactions
- … … and feedback

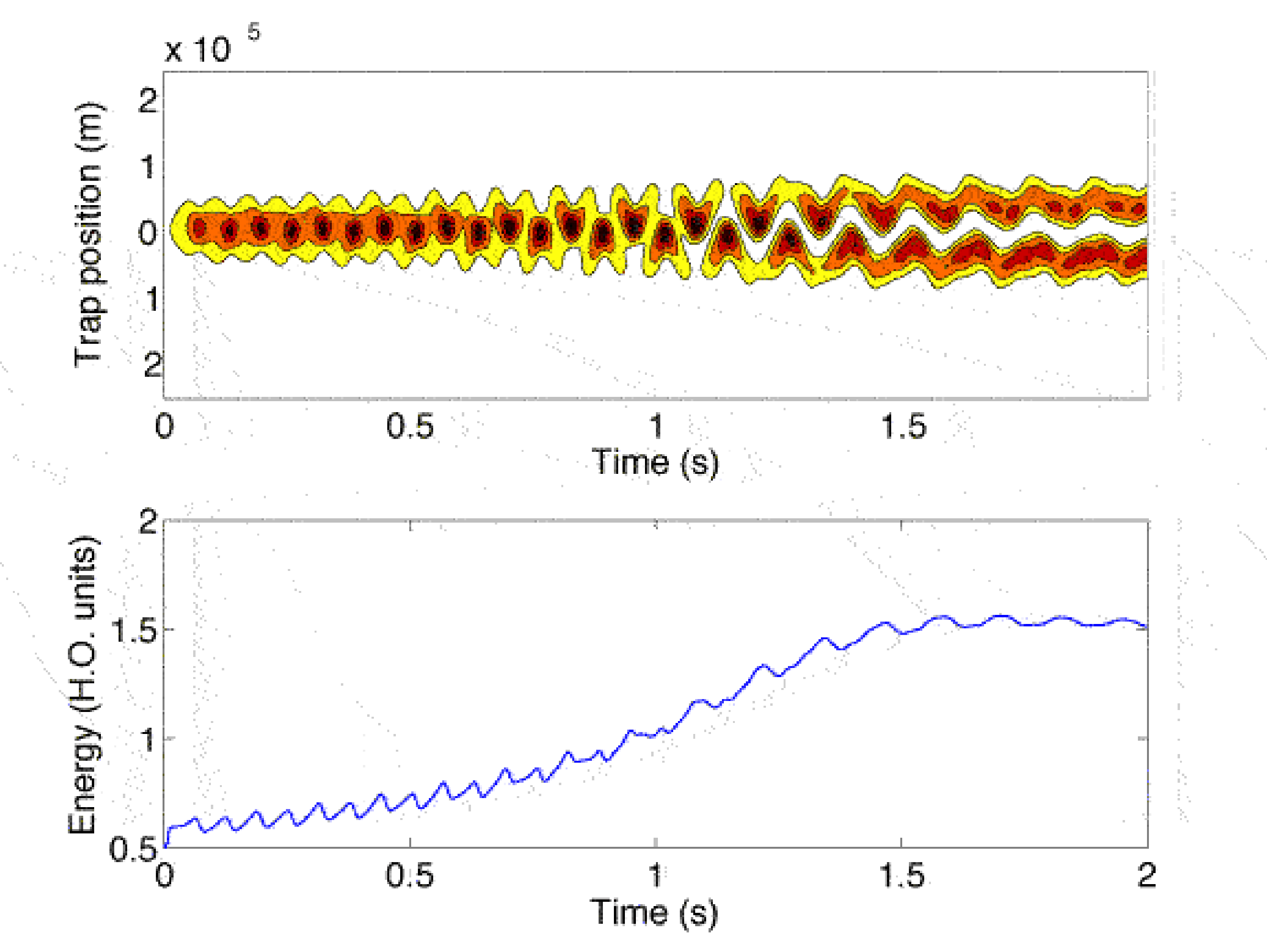
Feedback

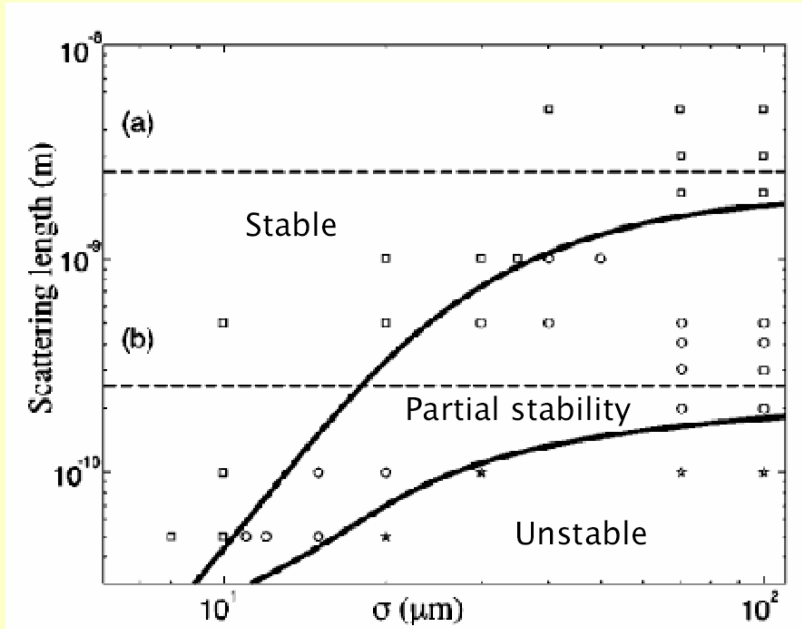
Pumping atom lasers with active feedback

The future

- Reality of pumping processes
- Detection
- Few mode atom laser model combining both schemes
- More advanced feedback schemes on modal/phase stability

Pumped atom laser



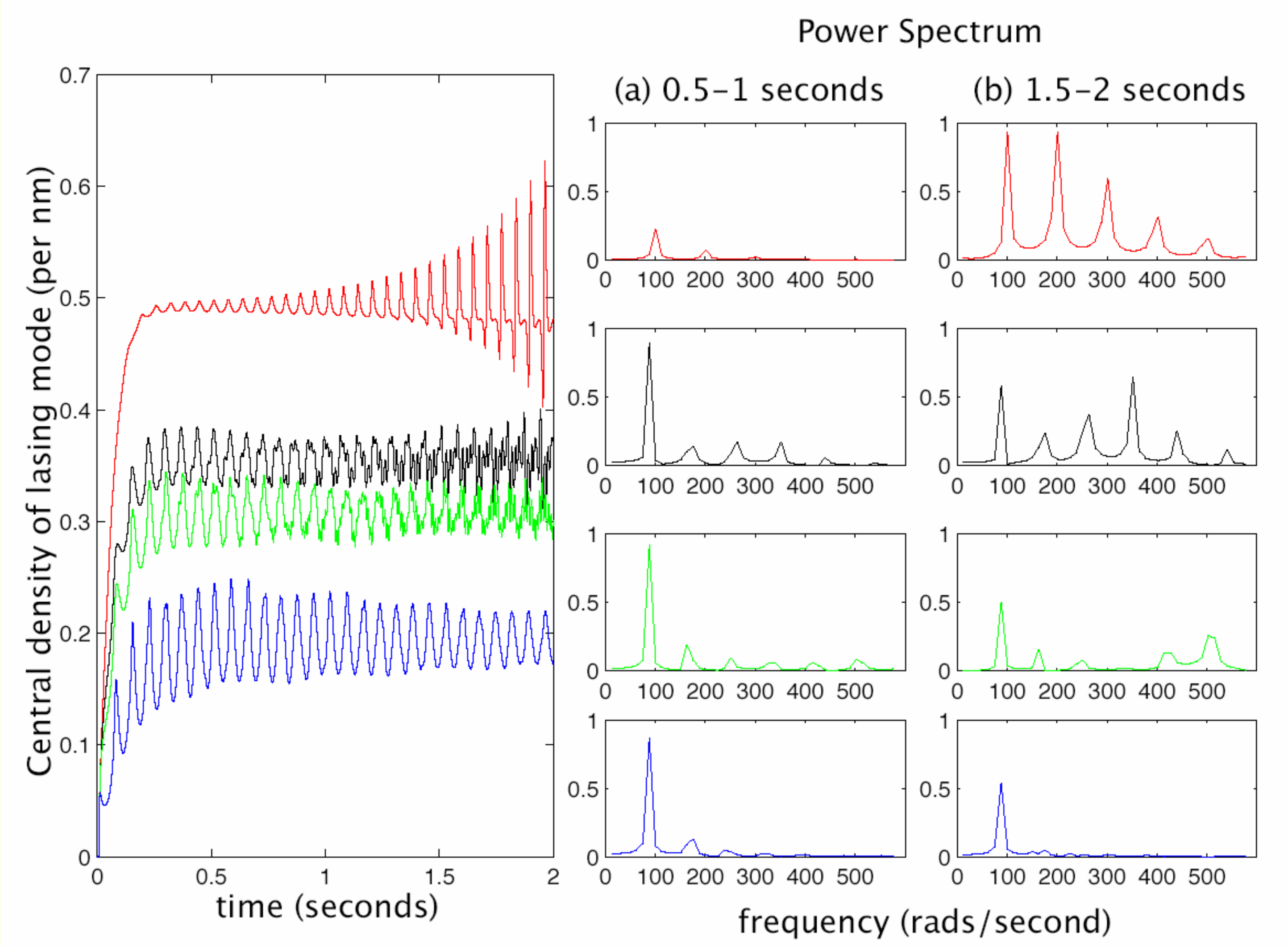


Scattering length (m)
σ (μm)
(a)
(b)
Stable
Partial stability
Unstable

No interactions, weak pumping

Moderate interactions, weak pumping

Moderate interactions, strong pumping

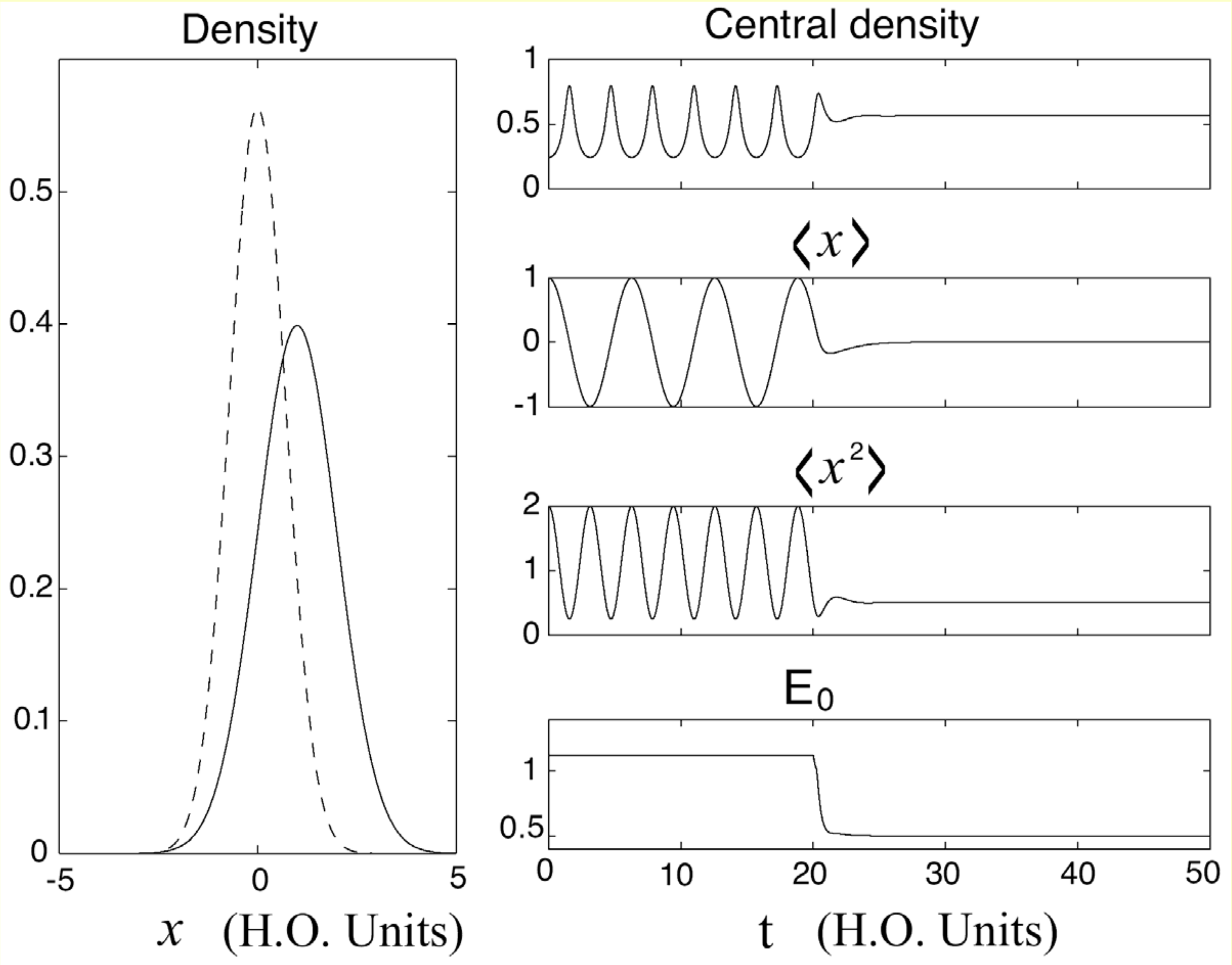


Power Spectrum
(a) 0.5-1 seconds
(b) 1.5-2 seconds
Central density of lasing mode (per nm)
time (seconds)
frequency (rads/second)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.5
1
1.5
2
100
200
300
400
500

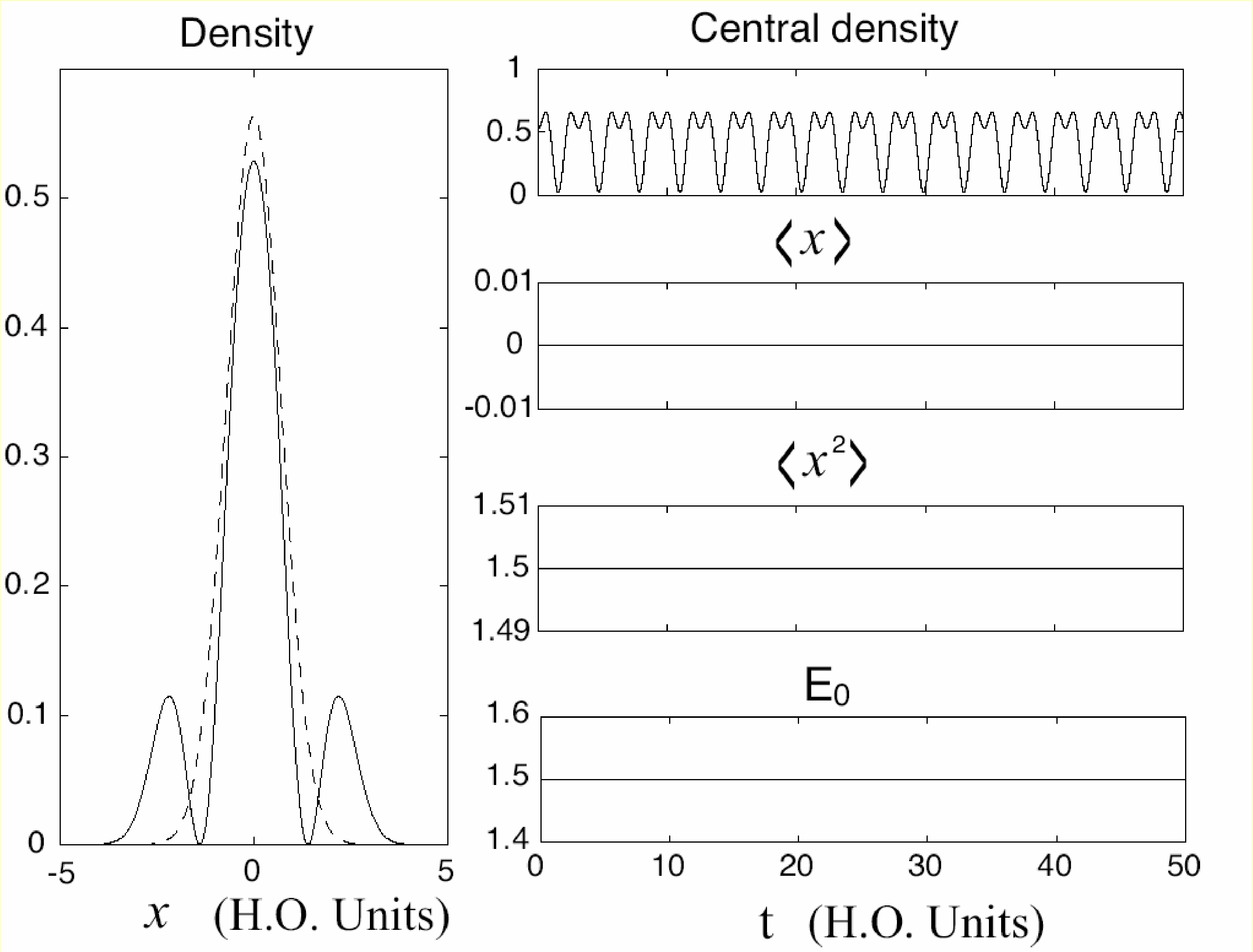
A simple diagnostic: density at a point

QuickTime™ and a
Animation decompressor
are needed to see this picture.

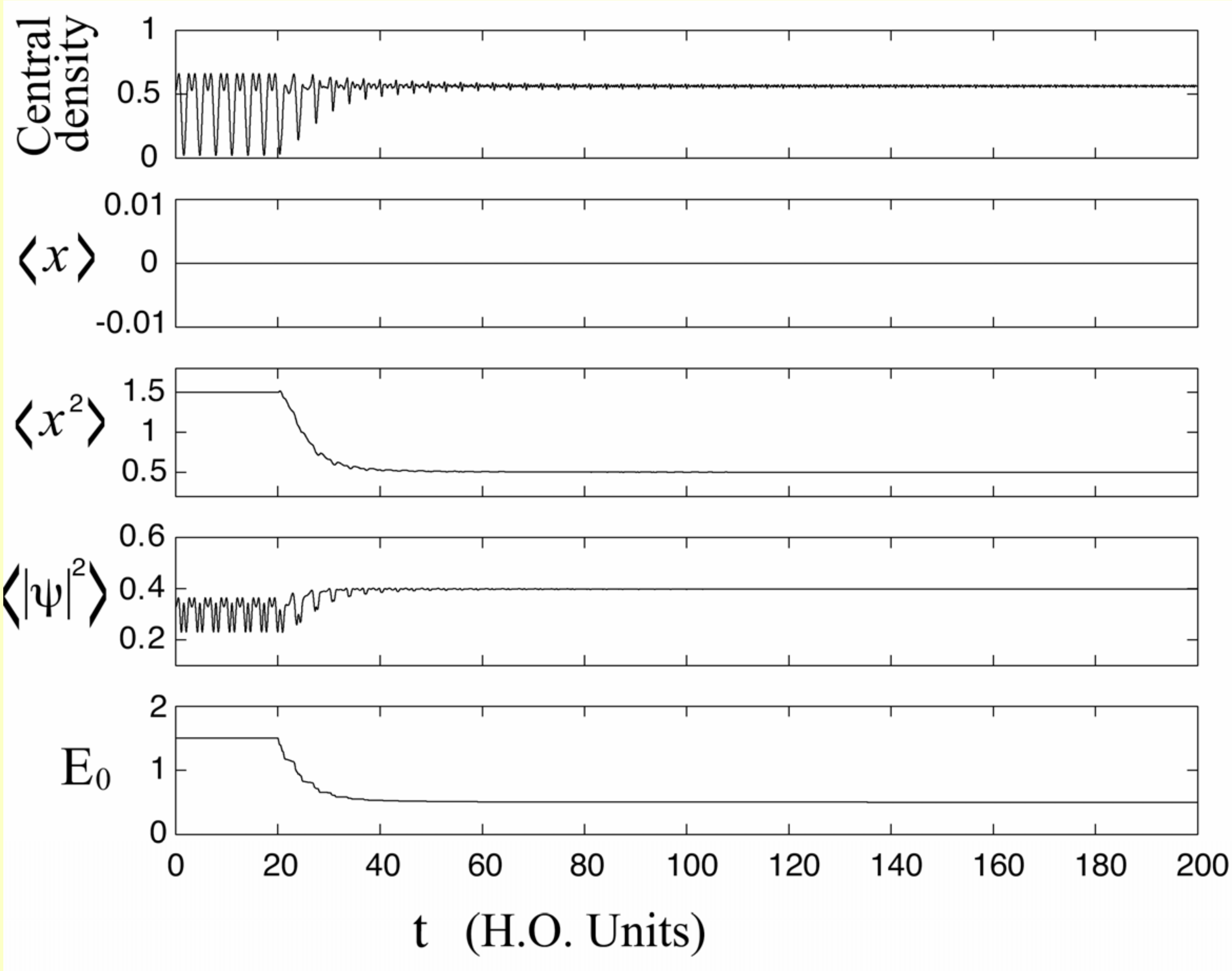
Example: linear system, linear feedback



Example: linear system, linear feedback



Example: linear system, general feedback



Pumped atom laser with feedback