Coherent transport of matter waves

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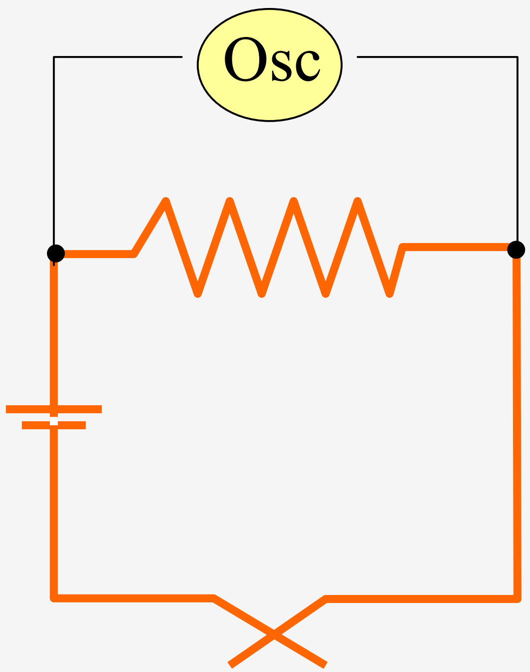
Quantum effects: phase coherence length smaller than system size

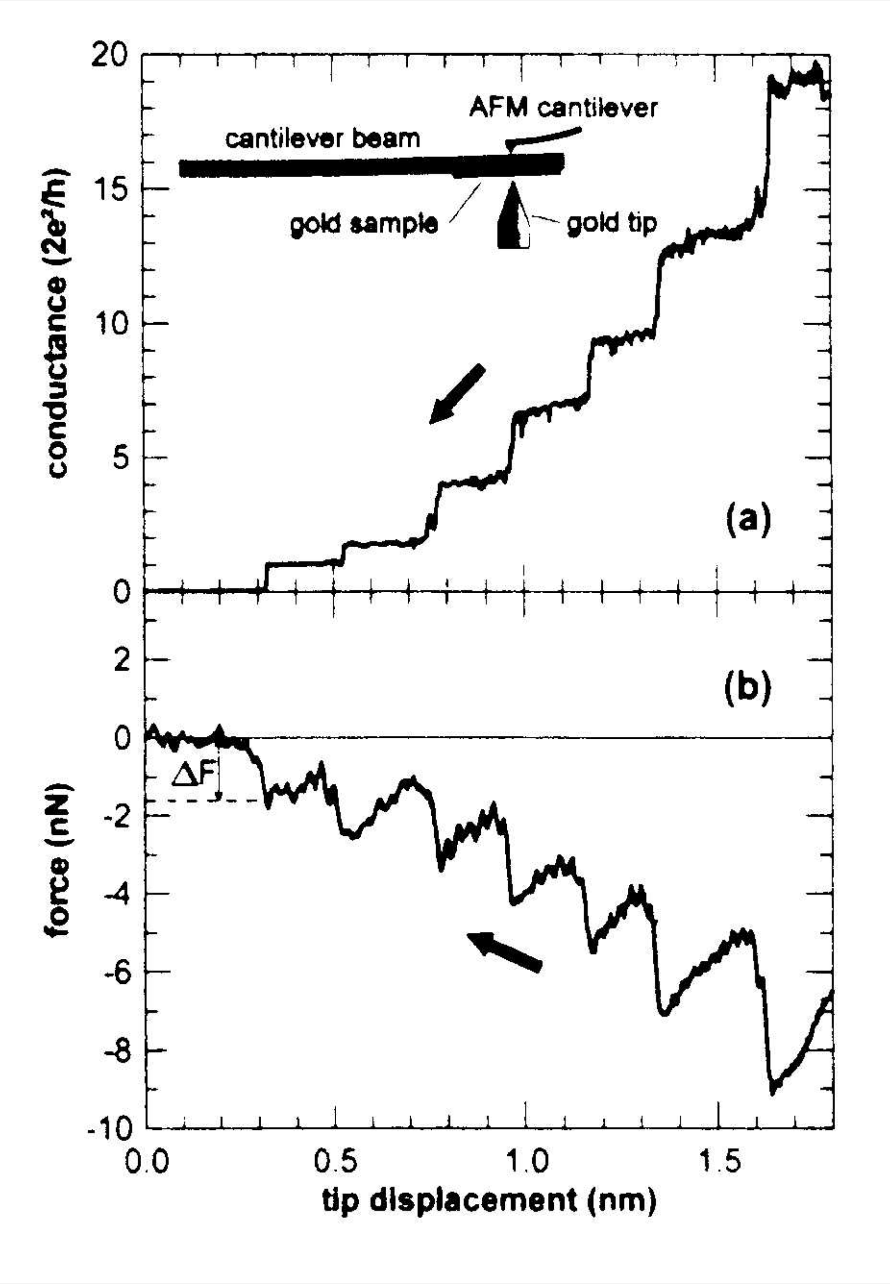
Many times, the quantum corrections appear as oscillations superimposed to a « classical » smooth behavior

Electronic systems → mesoscopic physics

- Magnetic susceptibility in quantum dots
- Persistent currents in metallic rings → no classical effect
- Shell effects in the energy of metallic particles (stability)
- conductance quantization
- force in metallic nanocontacts
- ….

Metallic Nanocontact

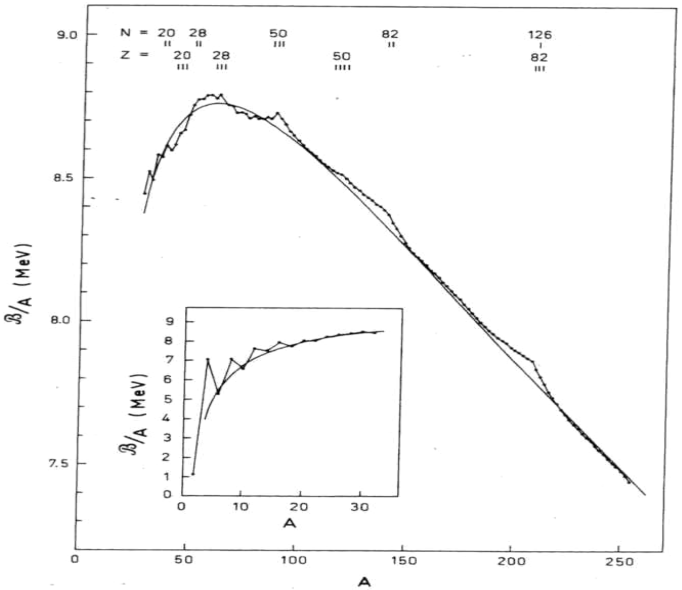




G. Rubio, N. Agrait, S. Veira, Phys. Rev. Lett. 76 (1996) 2302

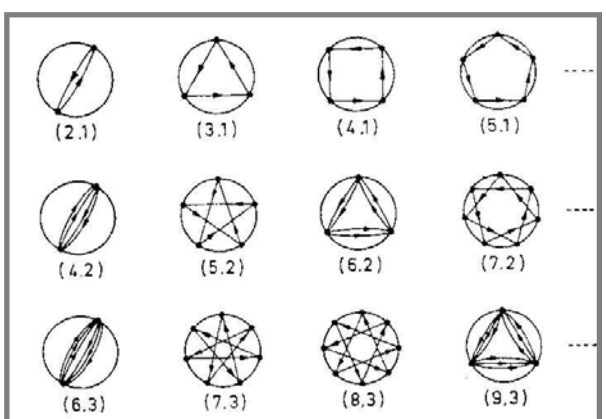
Nuclear masses: Average and Fluctuations

$$M = Z^*M_P + N^*M_N - \mathcal{B}(Z,N)/c^2 \quad \Longrightarrow \quad \widetilde{\mathcal{B}} = \overline{\mathcal{B}} - \mathcal{B}$$

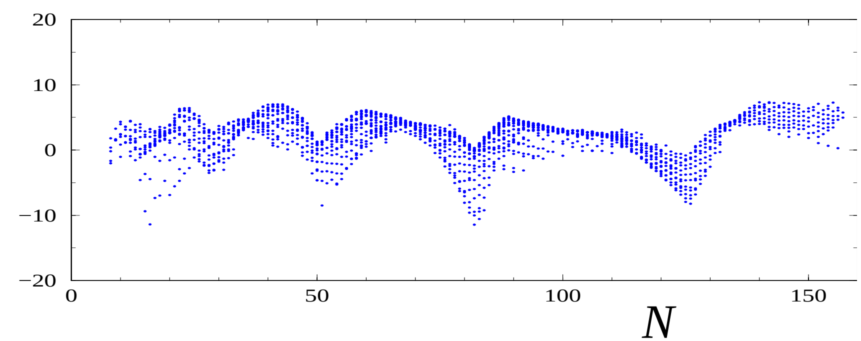


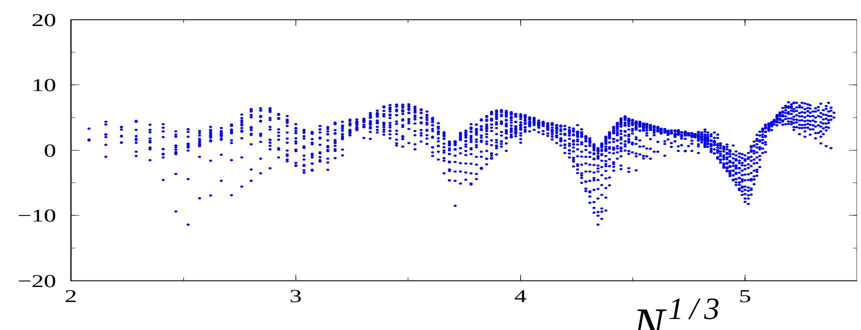
$$\overline{\mathcal{B}} = a_v A - a_s A^{2/3} - a_c \frac{Z^2}{A^{1/3}} - a_A \frac{(N-Z)^2}{A} - a_p \frac{t_1}{A^{1/2}}$$

$$a_v = 15.67, a_s = 17.26, a_c = 0.714, a_A = 23.29, a_p = 11.2, t_1 = +1,0,-1$$



$\widetilde{\mathcal{B}}$

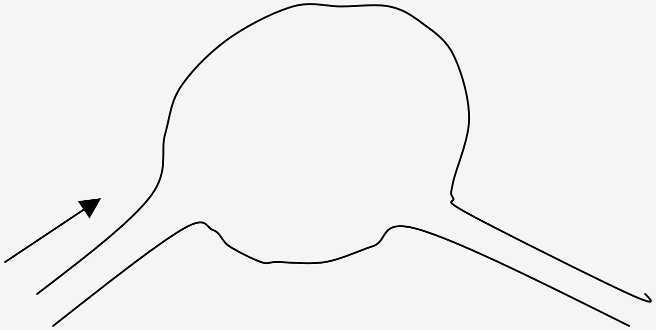




In other cases, the quantum terms appear as non-oscillatory corrections to a « classical » behavior

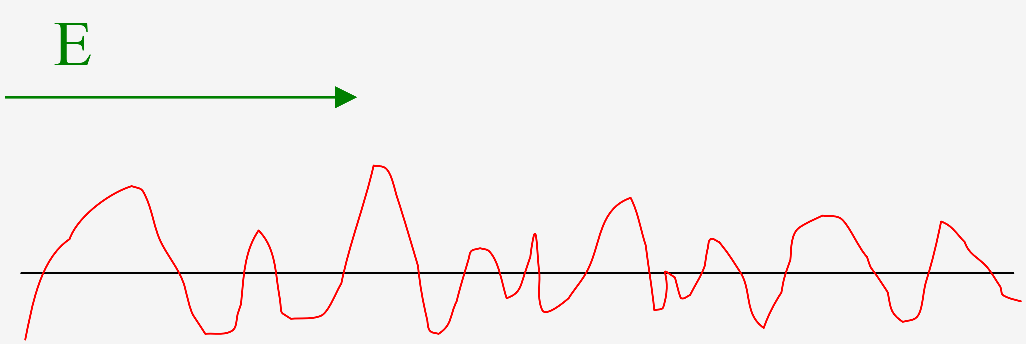
- Weak localization

$$T = \frac{1}{2} - \delta, \quad \delta > 0$$



In extreme cases, the quantum behavior could be totally different from the classical one

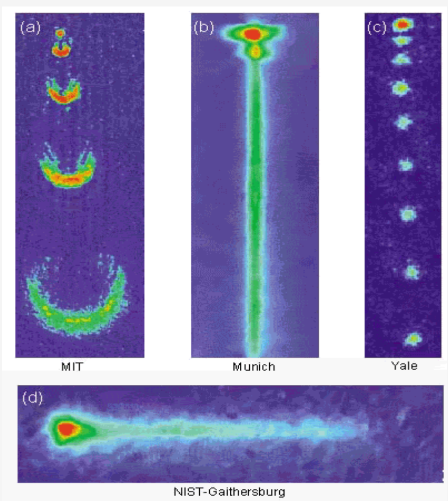
- Anderson localization

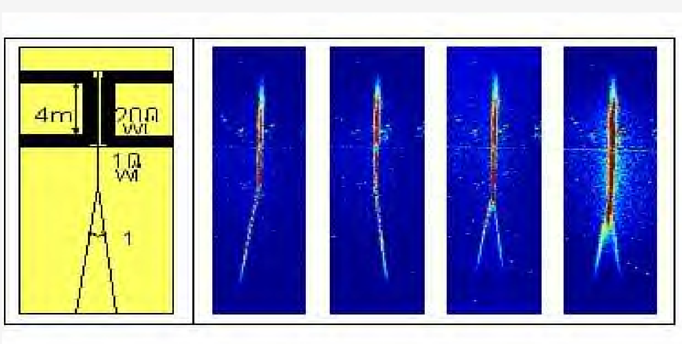


$$T = 1 - '1' = 0$$

(1D)

Qualitatively, all the effects described above may be understood within a single—particle picture → « universality » in wave systems





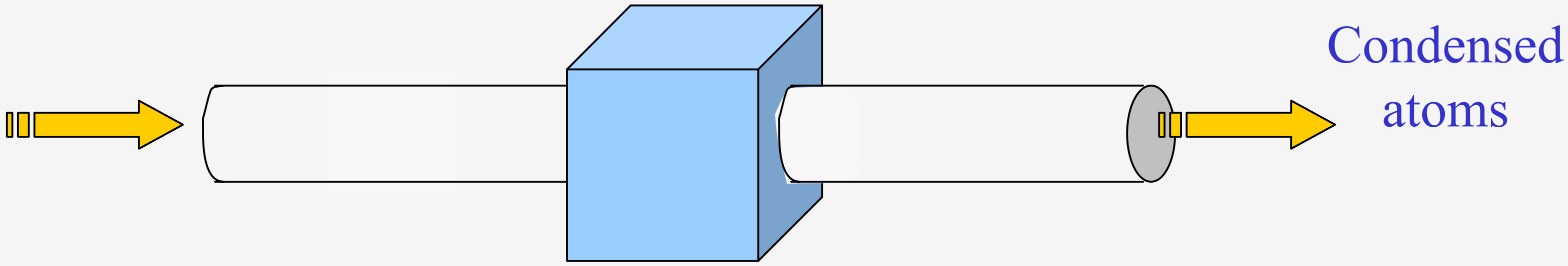
Atom lasers (Atom-Chip experiments,…):

Fundamental properties

- (de)coherence effects
- interference
- localization
- dissipation, forces,…

Effects of interactions

Propagation of a Bose condensate through a magnetic guide:
Scattering of Bose beams



- a bend in the guide
- a red or blue detuned laser beam
- a change in the shape of the guide
- …..

Equations of motion:

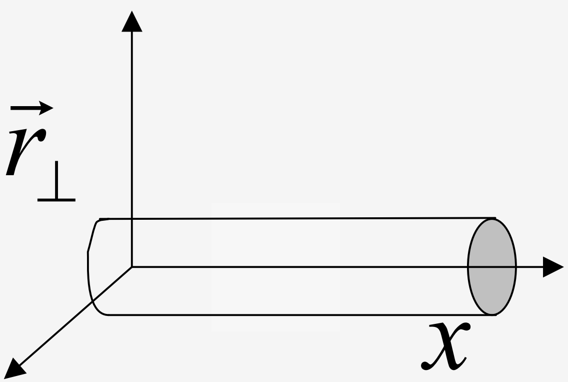
a_{sc} : s - wave scattering length (>0)

$$\delta S = 0$$

$$\begin{cases} S = \frac{i}{2}\int d^3r\, dt\,(\Psi^*\partial_t\Psi - \Psi\partial_t\Psi^*) - \int dt\ \mathrm{E}[\Psi] \\ \mathrm{E}[\Psi] = \int d^3r\left[\frac{1}{2}\left|\vec{\nabla}\Psi\right|^2 + 2\pi a_{sc}|\Psi|^4 + V|\Psi|^2\right] \end{cases}$$

Adiabatic approximation:

$$\Psi(r,t) = \psi(x,t)\,\phi(r_\perp, n)$$



- $n(x,t) = \int d^2r_\perp |\Psi|^2 = |\psi(x,t)|^2$ — Longitudinal density
- $V_\perp(\vec{r}_\perp) = \frac{1}{2}\omega_\perp^2\, r_\perp^2$ — Transverse confinement
- $V_{=}(x)$ — Longitudinal potential
- $\hbar = m = 1$ — Units

Adiabatic equations of motion:

$$\begin{cases} -\frac{1}{2}\nabla_{\perp}^{2}\phi + \left(V_{\perp} + 4\pi a_{sc} n|\phi|^{2}\right)\phi = \varepsilon(n)\phi \\ -\frac{1}{2}\nabla_{x}^{2}\psi + \left(V_{=} + \varepsilon(n)\right)\psi = i\partial_{t}\psi \end{cases}$$

- $\varepsilon(n) =$ Lagrange multiplier for $n(x,t) = \int d^{2}r_{\perp}|\Psi|^{2} = |\psi(x,t)|^{2}$

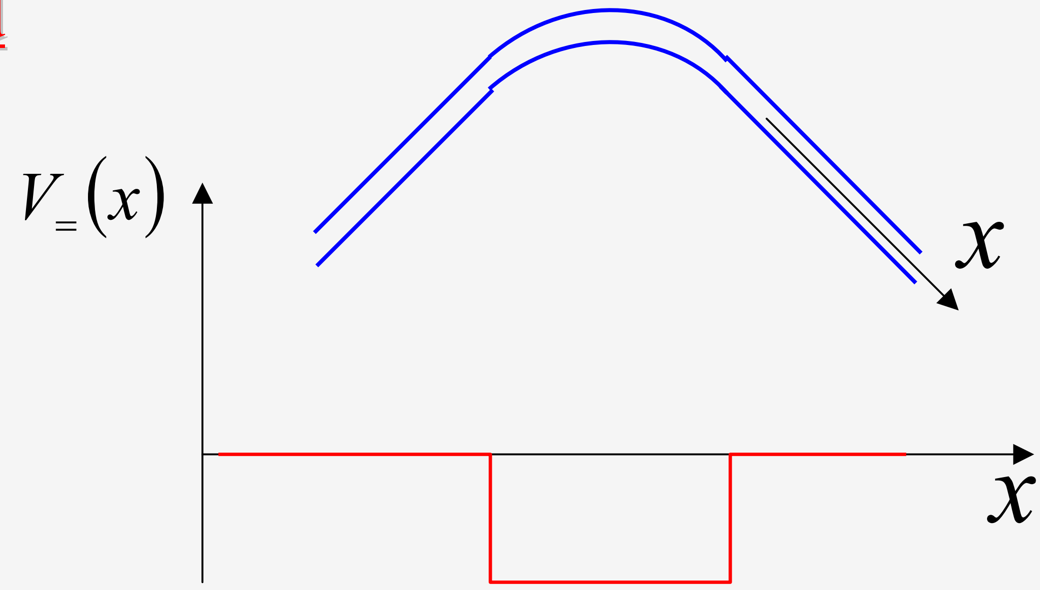
$$\varepsilon(n) = \begin{cases} \varepsilon_{0} + 2a_{sc}n/a_{\perp}^{2} & a_{sc}n << 1 \\ \varepsilon_{0} + 2\omega_{\perp}\sqrt{a_{sc}n} & a_{sc}n >> 1 \end{cases}$$

- $a_{\perp}^{-2} = 2\pi\int|\phi_{0}|^{4}d^{2}r_{\perp}$
- $n_{3D}a_{sc}^{3} << 1$

P.L. and N. Pavloff, Phys. Rev. A 64 (2001) 033602

Bound states → Bend

$$V_{=}(x) = -\frac{\kappa(x)}{8}$$



→ $$\psi(x,t) = A(x)e^{-i\mu t}$$

→ $$\mu = \varepsilon_0 - \frac{1}{2}\left[\int_{-\infty}^{+\infty} V_{=}(x)\,dx + \frac{a_{sc}}{a_\perp^2} N\right]$$

Bend

Repulsive atomic inter.

$$N_{max} = \frac{a_\perp^2}{a_{sc}}\left|\int_{-\infty}^{+\infty} V_{=}(x)\,dx\right|$$

- $R_C \approx 5a_\perp$
- $a_\perp \approx 10\ \mu m$
- ^{23}Na atoms ($a_{sc} = 2.75$ nm)

$\Rightarrow N_{max} \approx 150$

Stationary transmission modes

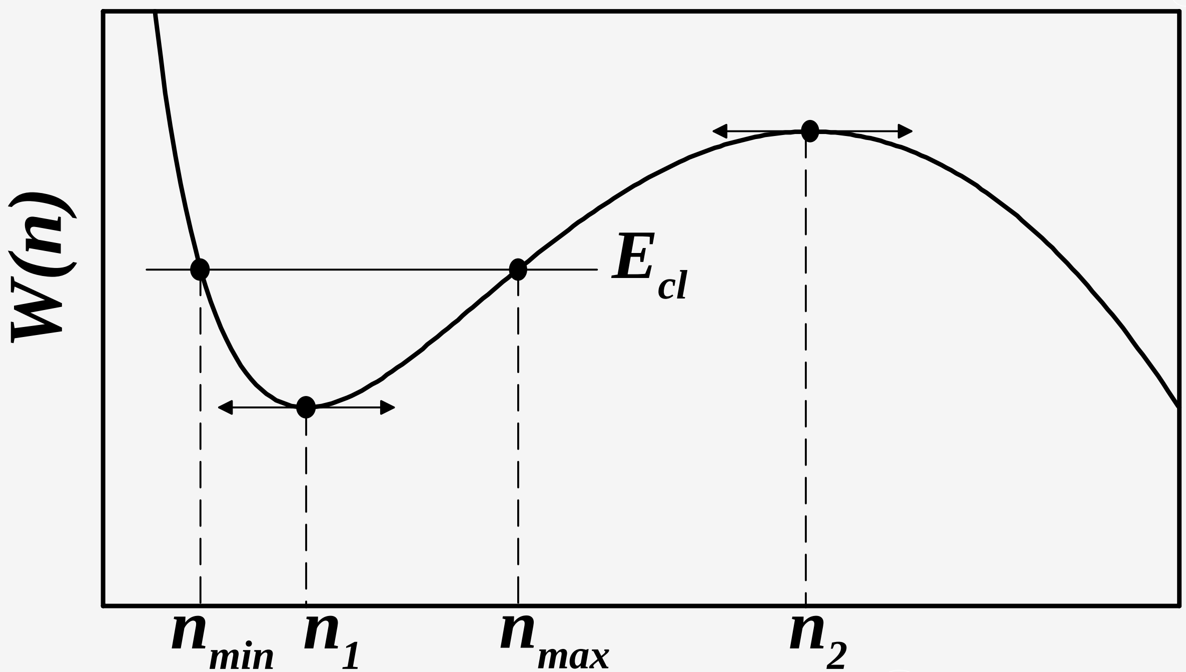
$$\longrightarrow \quad \psi(x,t) = A(x)\, e^{-i\mu t}\, e^{iS(x)}$$

- $n(x) = A^2$
- $v(x) = S'(x)$
- $\mu > \varepsilon_0$

$$\begin{cases} n(x)\, v(x) = J_\infty & \text{(flux conservation)} \\ -\frac{1}{2}\nabla_x^2 A + \left[V_=(x) + \varepsilon(n) + \dfrac{J_\infty^2}{2n^2} \right] A = \mu A \end{cases}$$

Free modes $\longrightarrow$ $V_{=}(x) = 0$ (straight tube)

$$\frac{1}{2}A'^2 + W(n) = E_{cl} \quad \text{with} \quad \begin{cases} W(n) = \sigma(n) + \mu\, n + J_\infty^2 / 2n \\ \sigma(n) = \int_0^n \varepsilon(\rho)\, d\rho \end{cases}$$



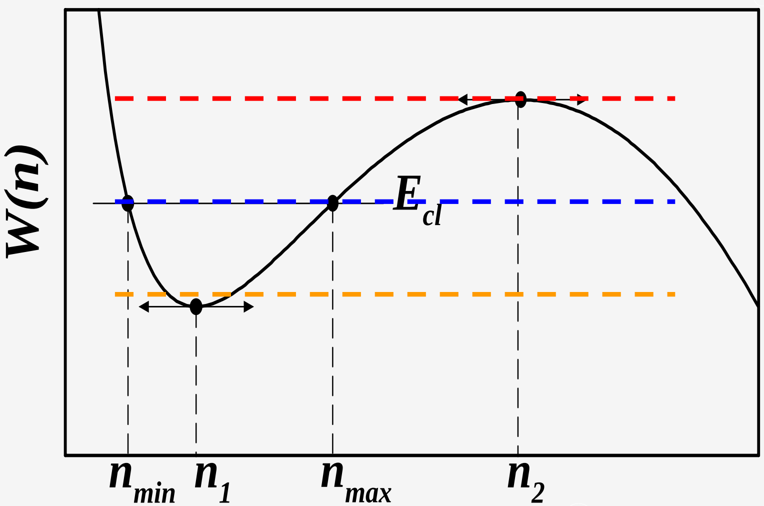
fixed

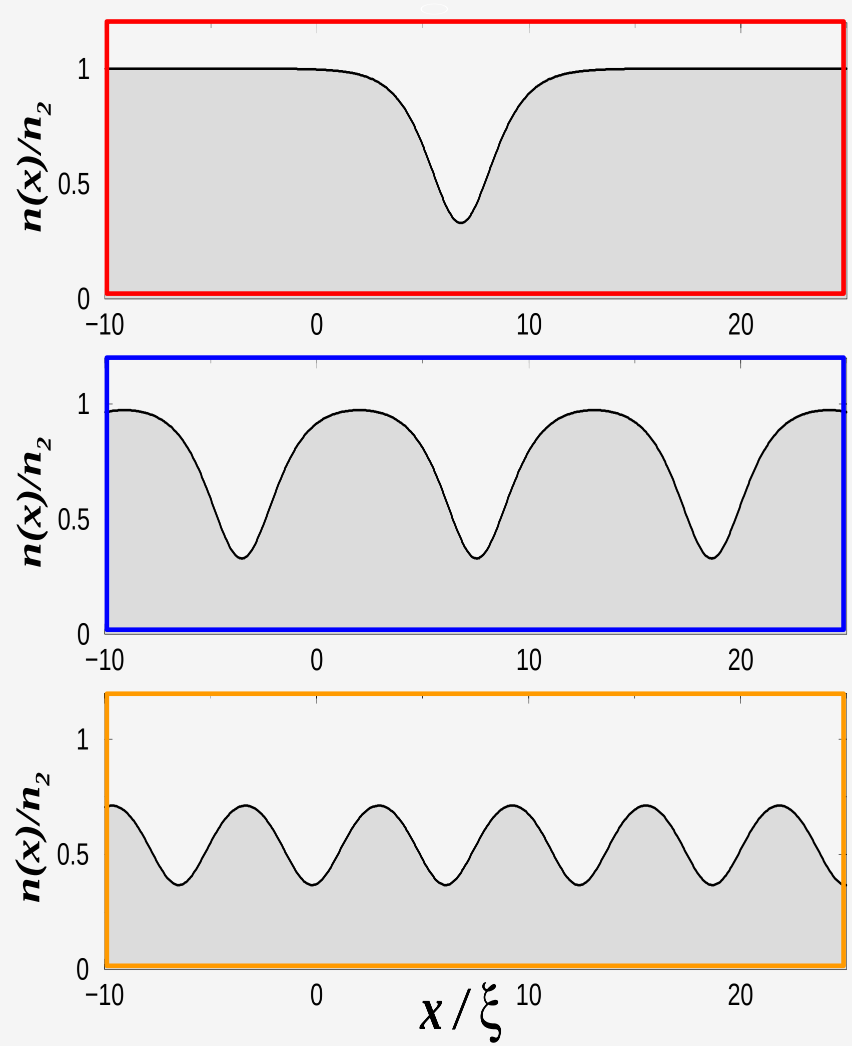
μ and J_∞

$A \mapsto$ position

$x \mapsto$ time

Free modes


W(n)
E_cl
n_min
n_1
n_max
n_2


n(x)/n_2
1
0.5
0
−10
0
10
20
n(x)/n_2
1
0.5
0
−10
0
10
20
n(x)/n_2
1
0.5
0
−10
0
10
20
x / ξ

Scattering potential: → Matching problem

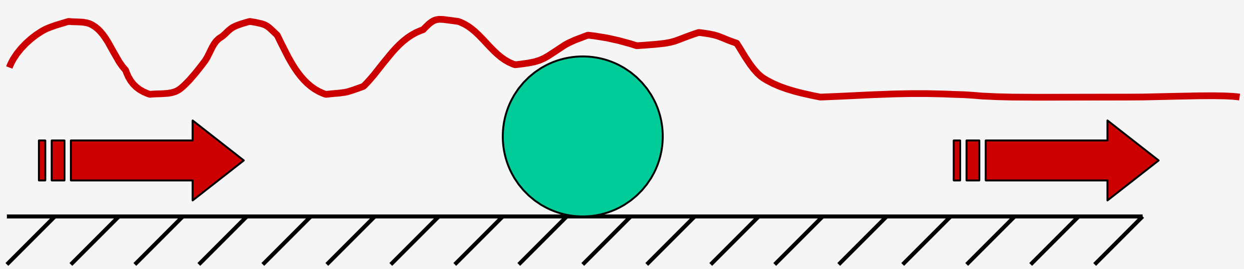
Boundary conditions: compare

$$\textit{group velocity } v_g \leftrightarrow \textit{phase velocity } v_p$$

speed of the energy transferred to the fluid

speed of the obstacle with respect to the beam (stationarity)

$$\textit{Radiation conditions} + v_g \geq v_p \qquad \left(\omega^2(k) = k^2 \left(n\frac{d\varepsilon}{dn} + \frac{k^2}{4} \right) \right)$$

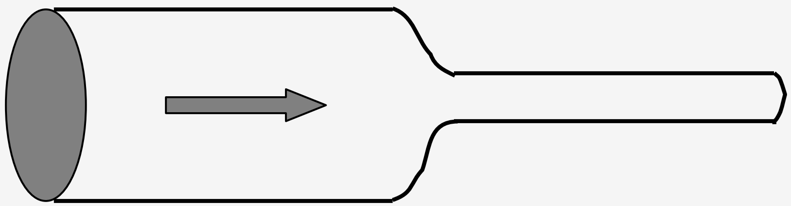


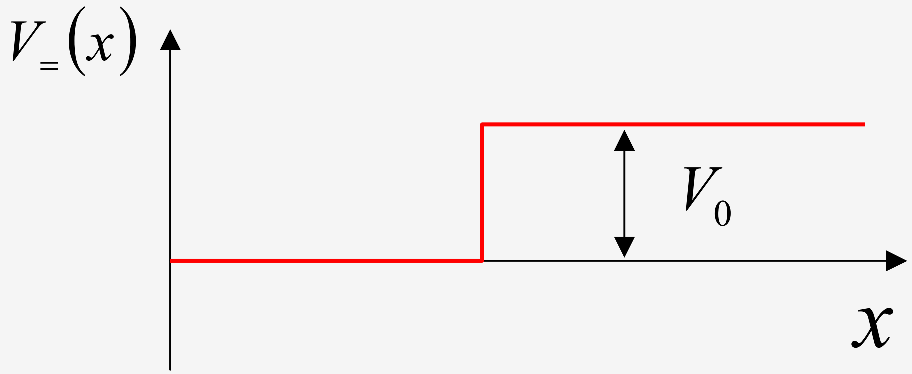
Minimum

or

Saddle

Particular geometry or scattering potential:





$$V_0 = \omega_\perp^> - \omega_\perp^< = (\alpha - 1)\,\omega_\perp^< \;\; ; \;\; \omega_\perp^> = \alpha\,\omega_\perp^<$$

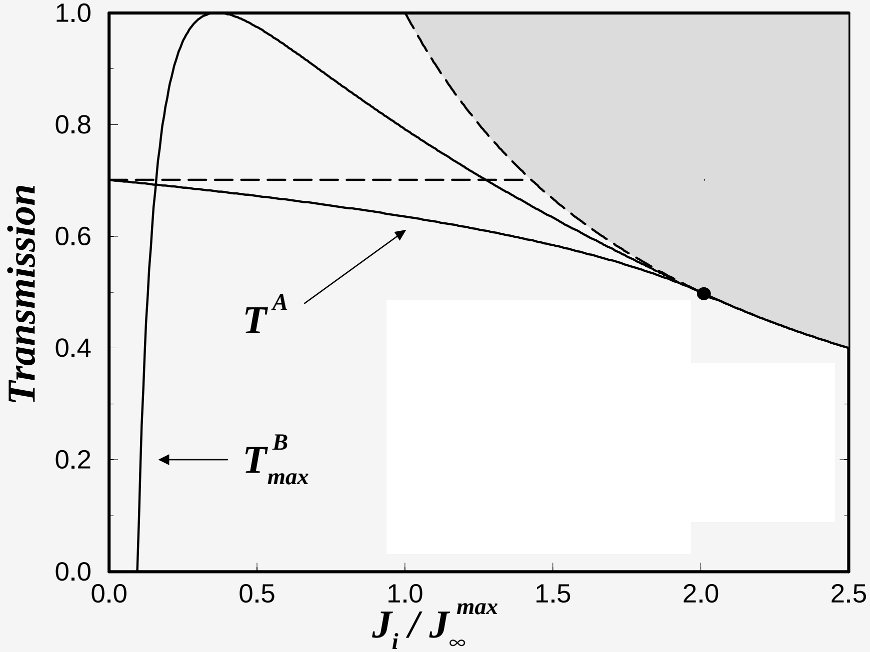
- Solve the matching problem
- Define a transmission coefficient (?)

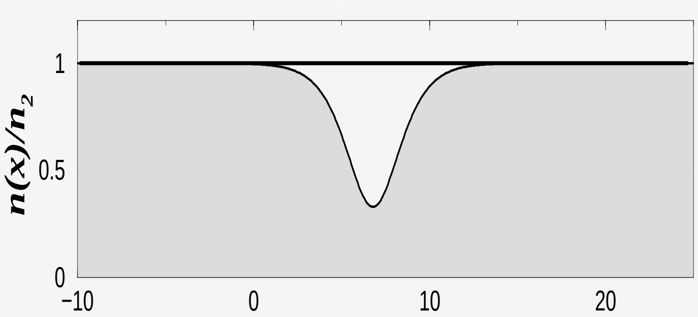
Concrete example

- ^{23}Na atoms $(a_{sc} = 2.75 \text{ nm})$
- $\omega_\perp^< = 2\pi \times 2 \text{ kHz}$
- $\omega_\perp^> = 3\,\omega_\perp^<$

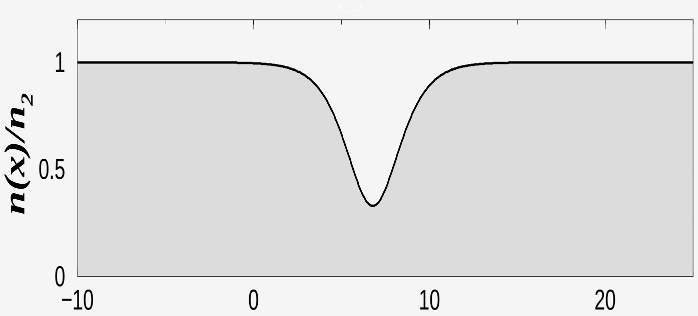
  $\Big\}\; V_0 = 192 \text{ nK}$
- $\mu = 210 \text{ nK}$
- $J_\infty^{\max} = 1.6 \times 10^4$ atoms/s

Bose-Einstein condensate





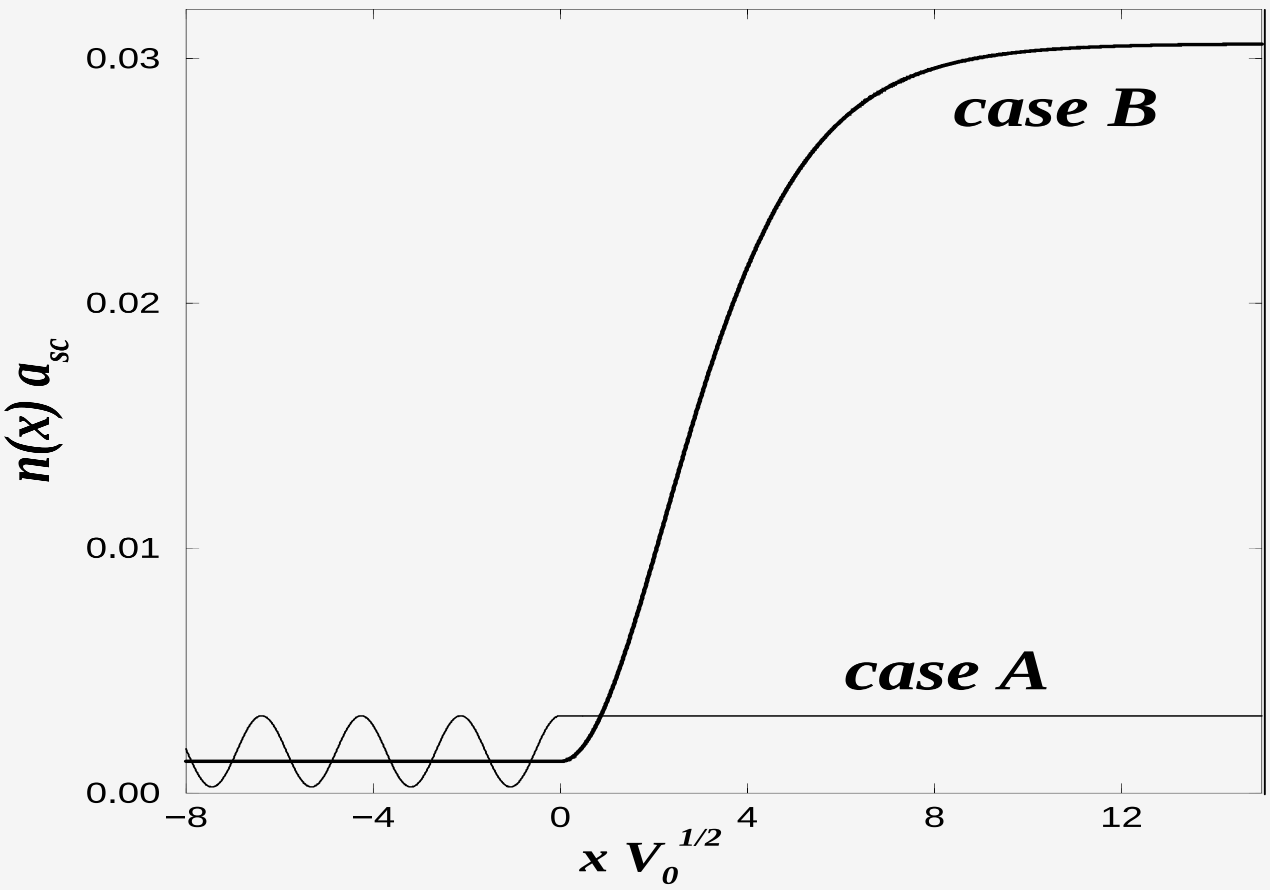
Case A



Case B

P. L., N. Pavloff and S. Sinha, Phys. Rev. A 68 (2003) 063608

Density profiles at incident current with T=1



Concluding remarks

- Importance and interest of quantum effects
- Peculiar scattering features of nonlinear waves
- The transmission coefficient depends on the current
- At given chemical potential, there exists a maximum transmitted current above which no stationary flow exists
- At a given current, several distinct stationary solutions with different T are possible
- For any chemical potential larger than V_0, there is a particular J_i which induces total transmission

- Non-stationary flows
- Dynamical selection of the different solutions
- Localization
- 1D approx: single scattering channel → Full 3D problem
- Scattering theory is missing ….