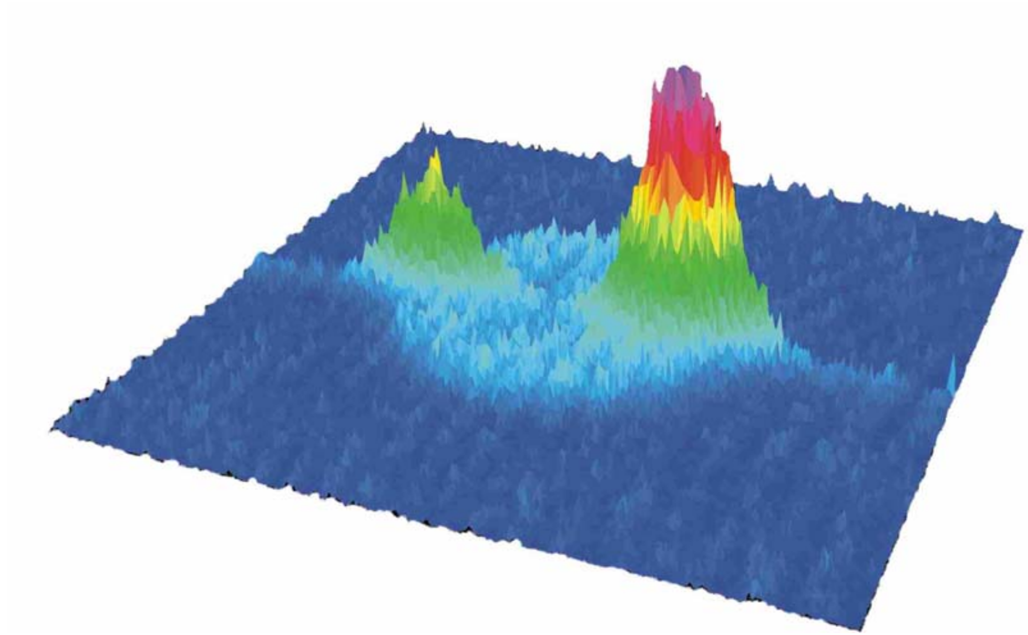
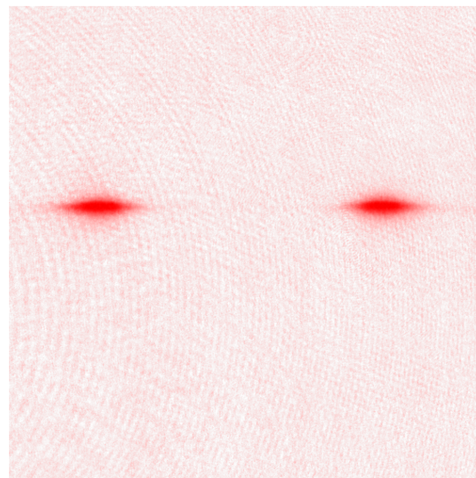
Studying Bose-Einstein Condensates with a linear accelerator



Christian Buggle, Jeremie Leonard, Wolf von Klitzing, Tobias Tiecke, Jook Walraven
Van der Waals – Zeeman Institute
FOM-Institute AMOLF
University of Amsterdam

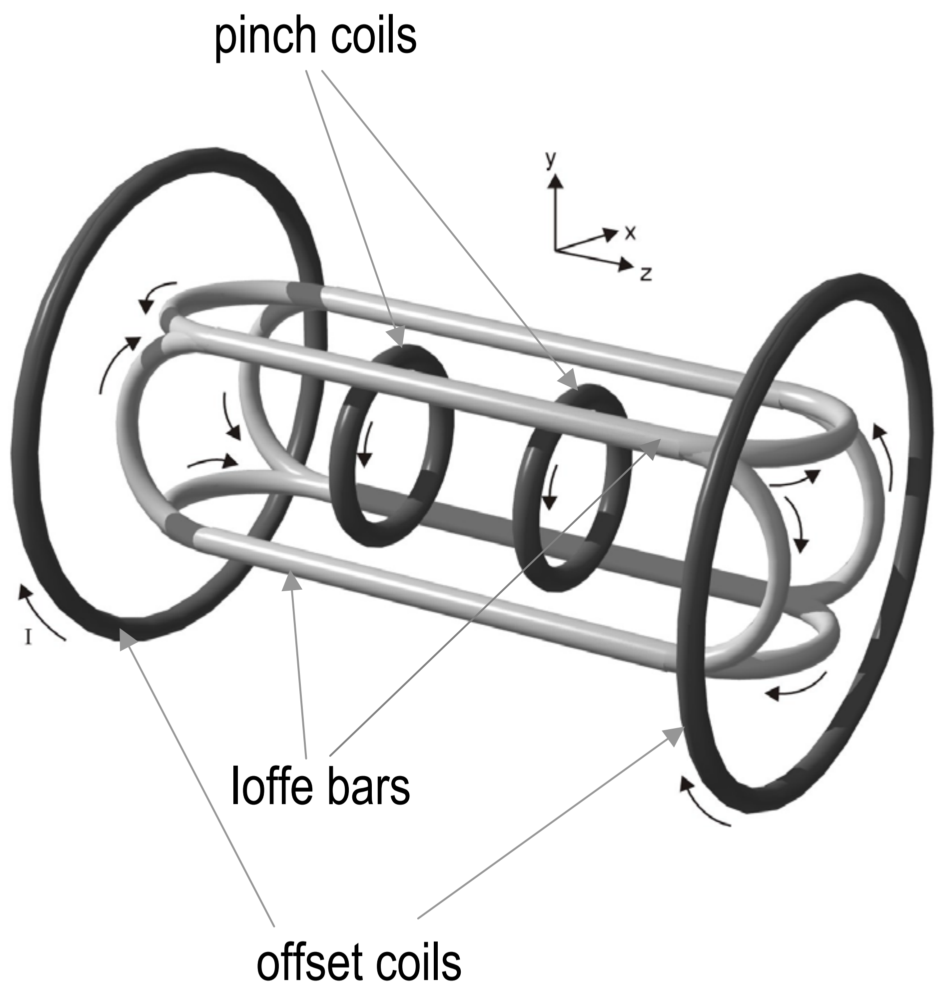
Les Houches: 14 February 2005



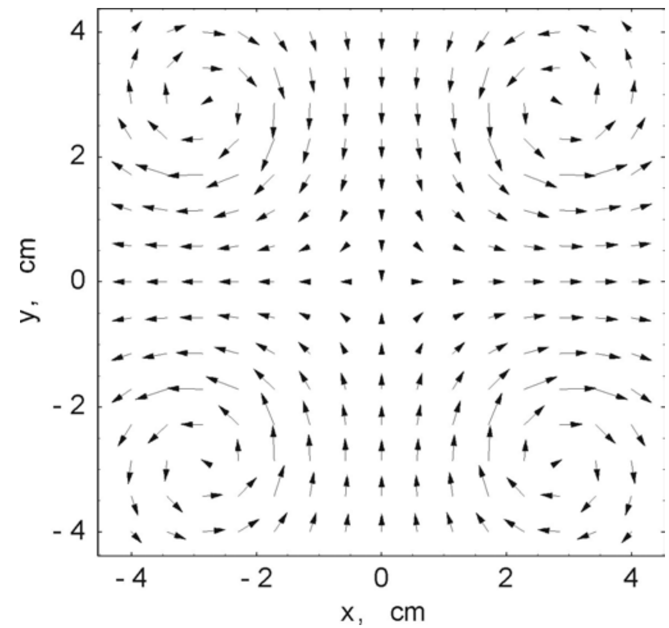
references

1. Production and Acceleration of two clouds
   - Tiecke, Kemmann, Buggle, Shvarchuck, von Klitzing and Walraven, J. OPT. B 5 (2003) 119
   - Thomas, Wilson, Foot PRA 65 (2002) 063406
2. Observing the collision of two condensates
   - Buggle, Leonard, von Klitzing, Walraven, PRL 93 (2004) 173202
   - Thomas, Kjaergaard, Julienne, Wilson, PRL 93 (2004) 173201
3. Getting the elastic scattering amplitudes at any low energy
   - Buggle, Leonard, von Klitzing, Walraven, PRL 93 (2004) 173202
4. Interference observed in Feshbach-dissociation of ultra-cold molecules
   - Volz, Durr, Syassen, Rempe, van Kempen, Kokkelmans, cond-mat/0410083

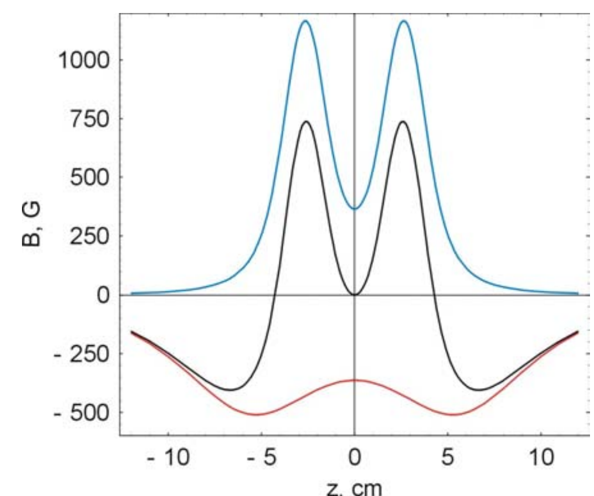
Ioffe Quadrupole Trap



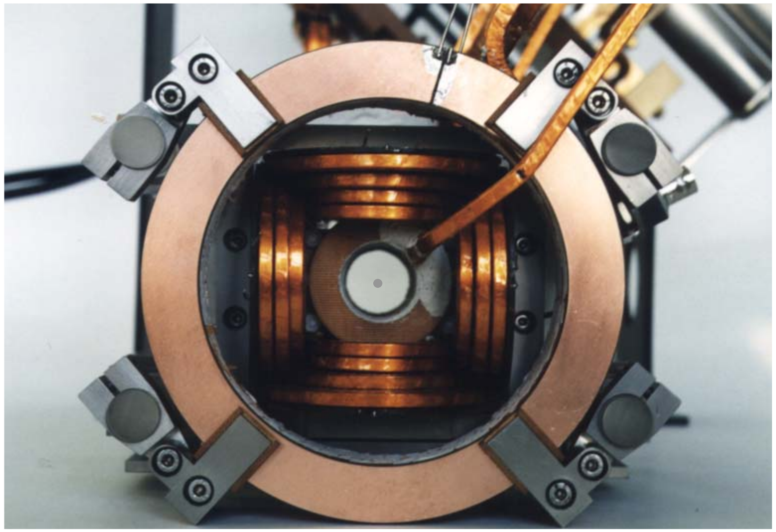
Radial

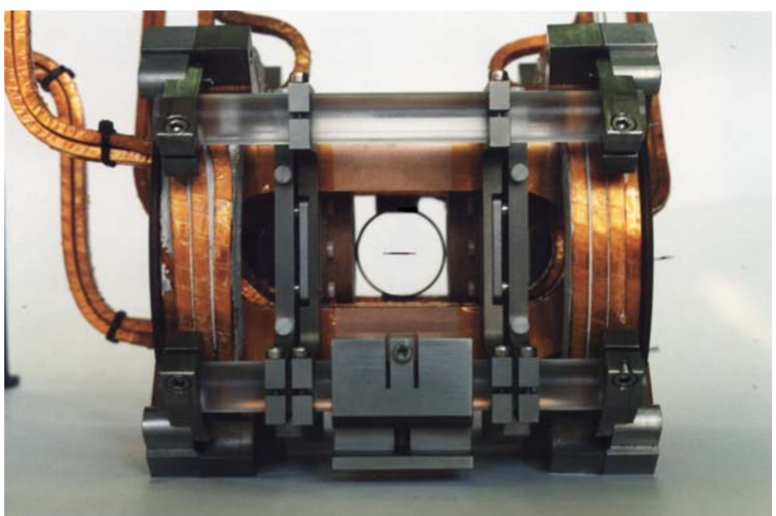


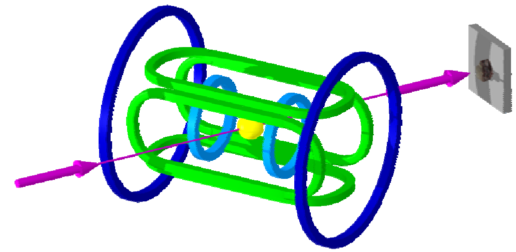
Axial



Ioffe-Pritchard Quadrupole Trap

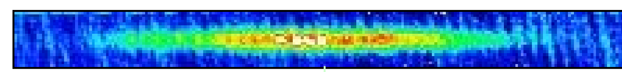






$$\frac{\omega_\perp}{\omega_\parallel} \approx 23$$

$$\omega_\perp = 2\pi \times 477 \text{ s}^{-1}$$



$$\omega_\parallel = 2\pi \times 21 \text{ s}^{-1}$$

$$N = 3.5 \times 10^6$$
$$n_0 = 4 \times 10^{14} \text{ cm}^{-3}$$
$$T_0 = 1.17 \ \mu\text{K}$$

Trap parameters:

- axial level splitting 1 nK
- radial level splitting 23 nK

Ioffe-Pritchard Trap

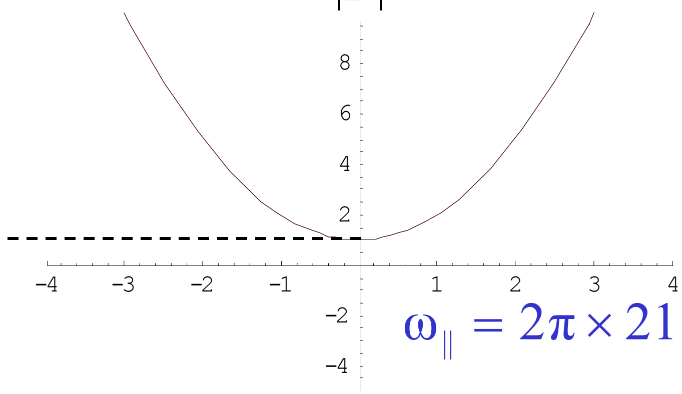
Trapping potential: $E = -\vec{\mu}.\vec{B} \propto |B|$

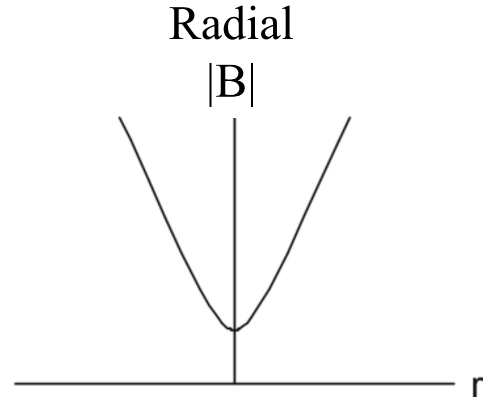
Axial (z)

Radial

$B_0 > 0$

B_0

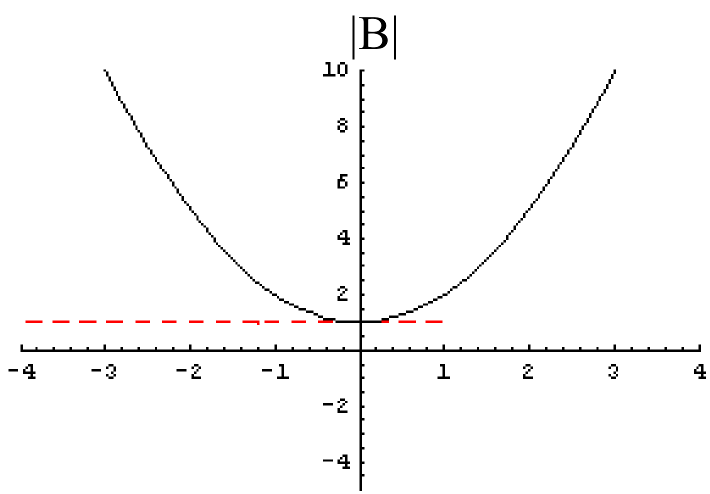


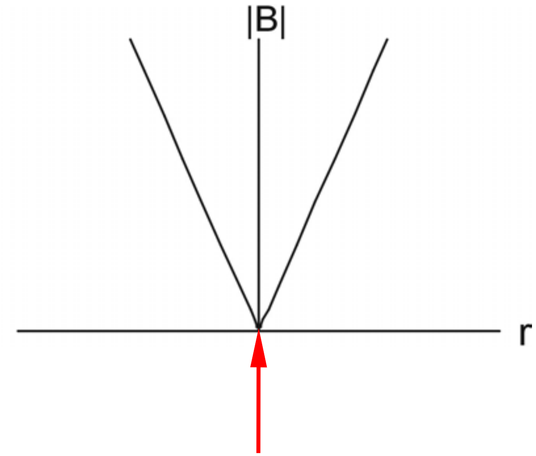


$\omega_{\parallel} = 2\pi \times 21\ s^{-1}$

$\omega_{\perp} = 2\pi \times 477\ s^{-1}$

$B_0 < 0$



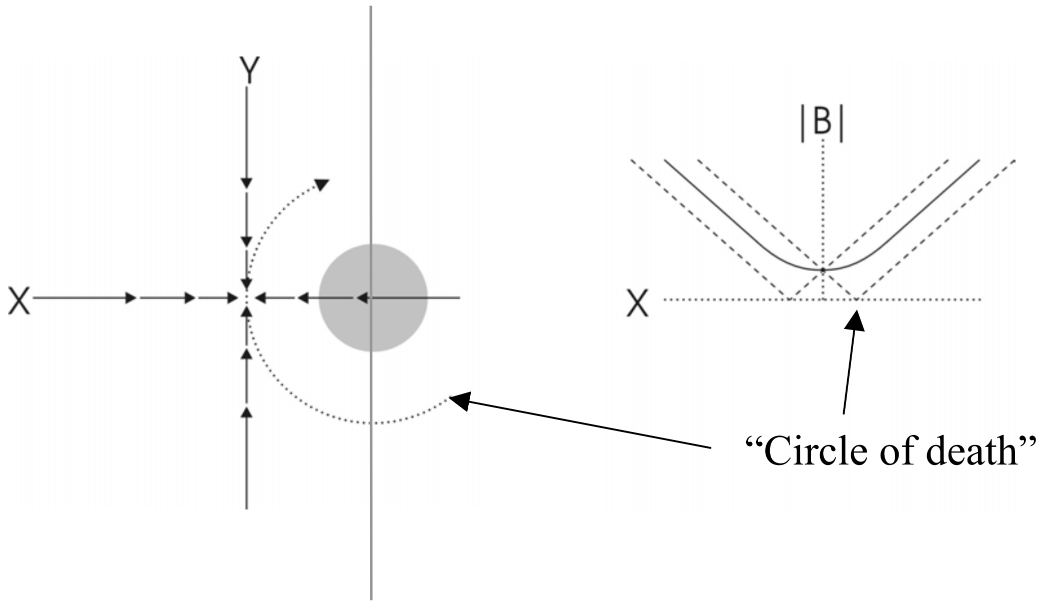


B_0

Two wells, but... Majorana spin flips

Rotating the Trap Axis (TOP-field)

Add a rotating magnetic field perpendicular to Z:



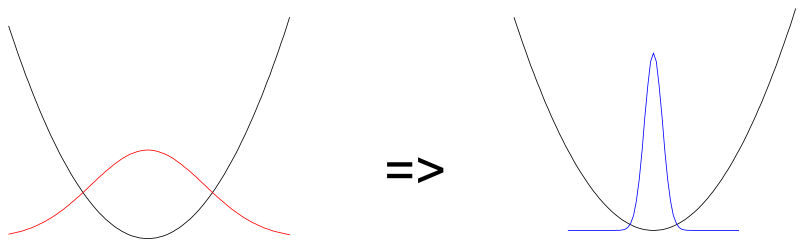
Petrich, Anderson, Ensher, Cornell, PRL 74 (1995) 3352

Thomas, Wilson, Foot PRA 65 (2002) 063406

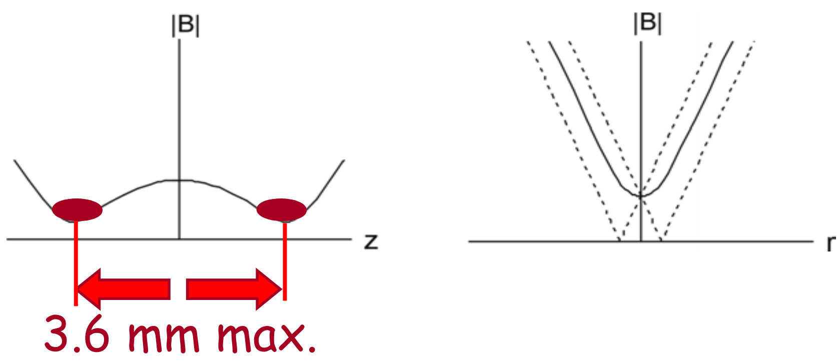
Tiecke, Kemmann, Buggle, Shvarchuck, von Klitzing and Walraven, J. OPT. B 5 (2003) 119

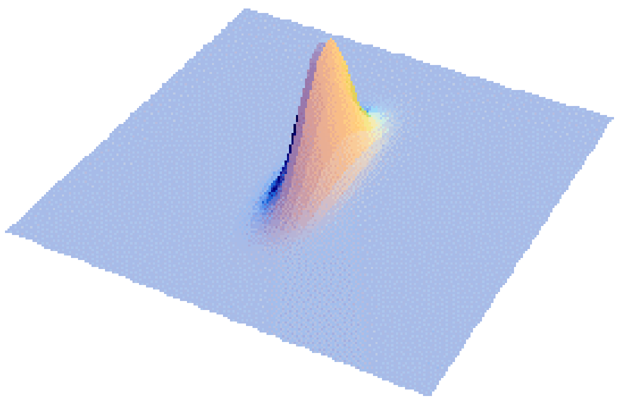
Producing two clouds

1) Pre-cooling in standard Ioffe-Pritchard trap



2) Apply rotating field and
ramp B_0 to negative values

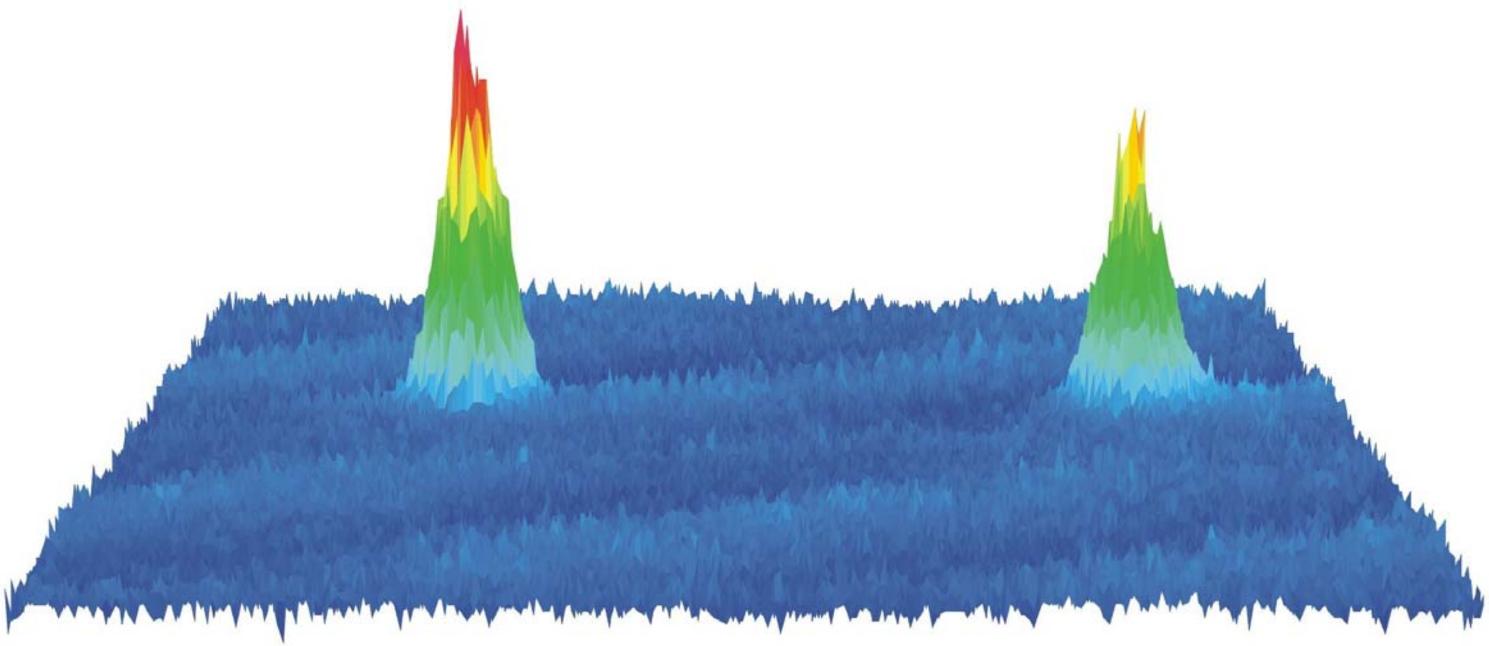




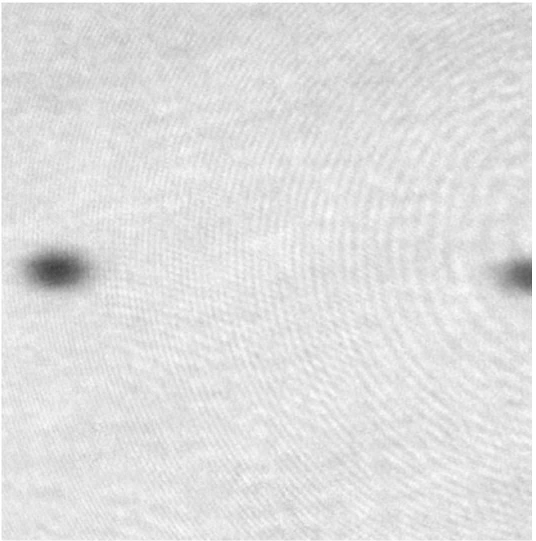
3) Final cooling to BEC

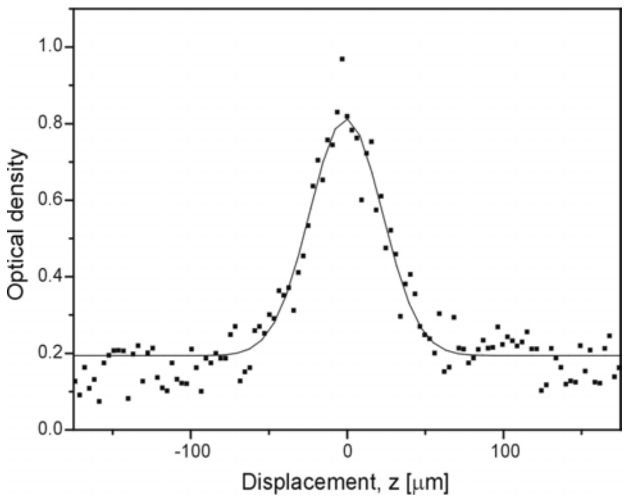
Tiecke, Kemmann, Buggle, Shvarchuck, von Klitzing and Walraven, J. OPT. B 5 (2003) 119

Double BEC



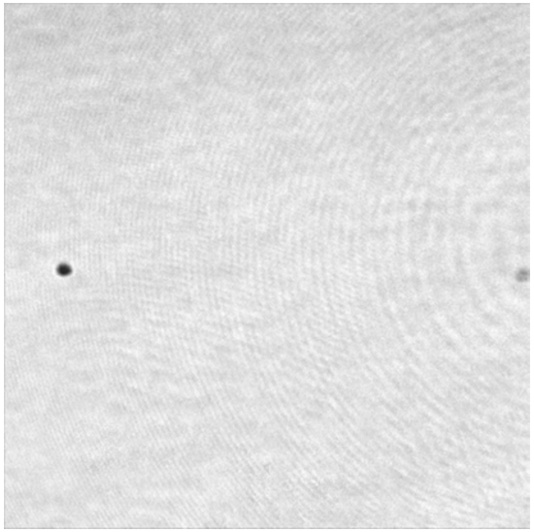
Production of a Double cloud.

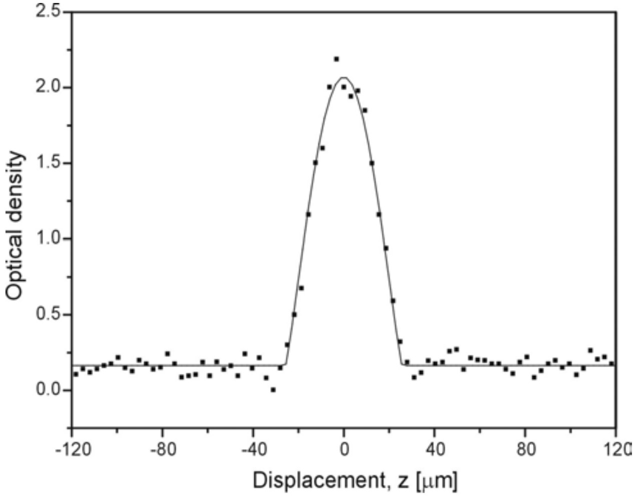




Thermal:
Gaussian shape

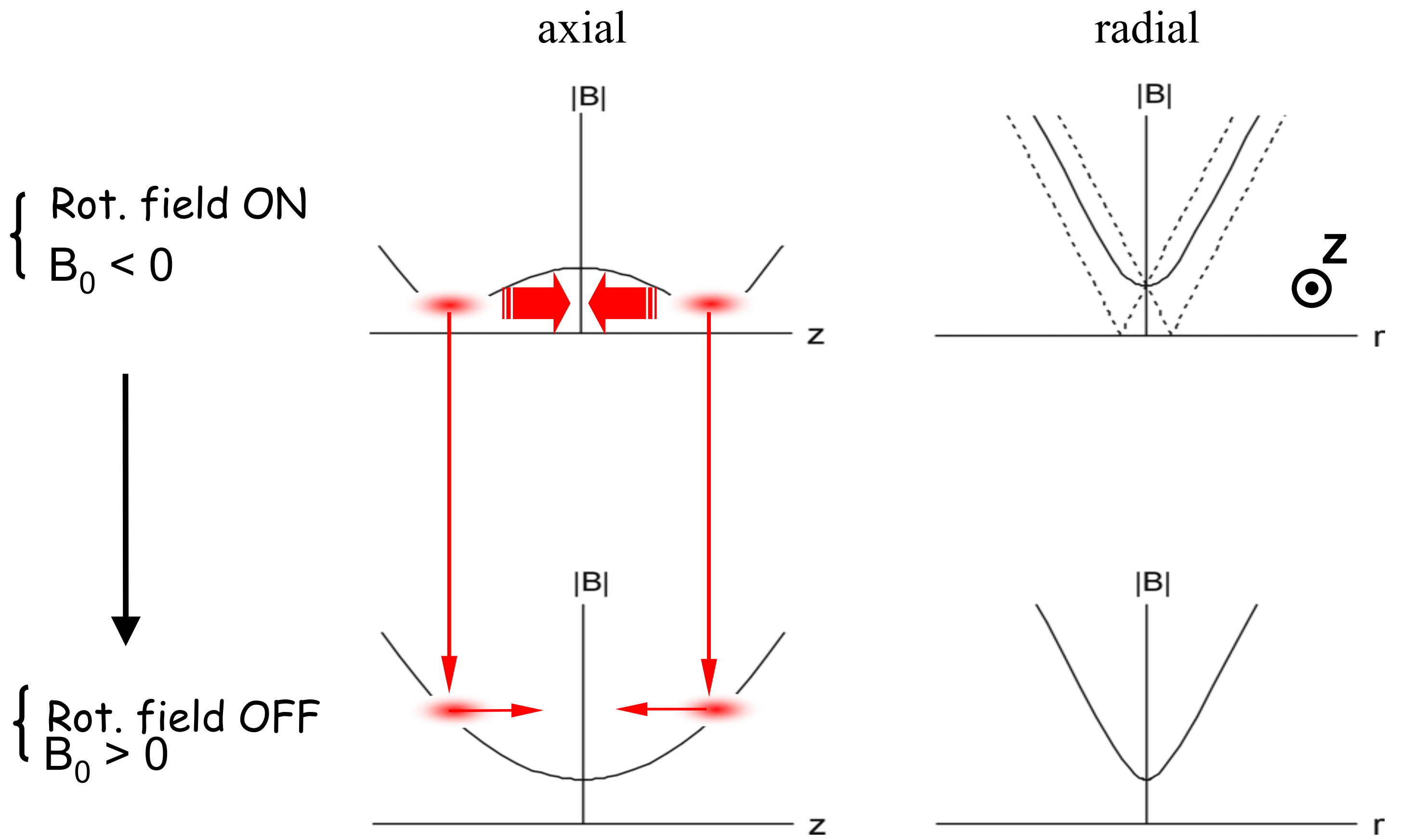
~ 1 mm





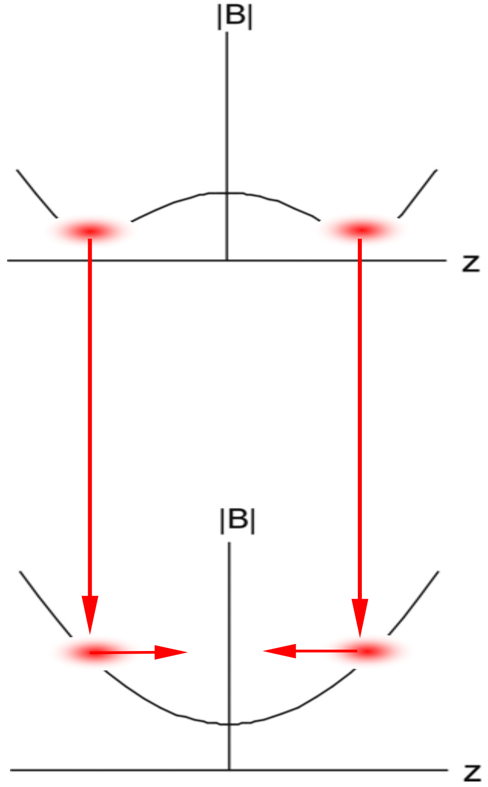
Condensed:
Parabolic shape
(Thomas-Fermi)

Acceleration Principle

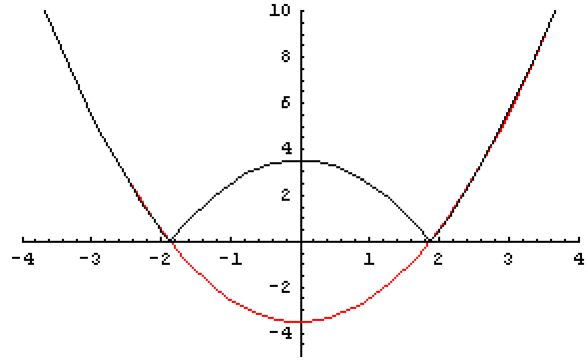


Acceleration Principle

axial



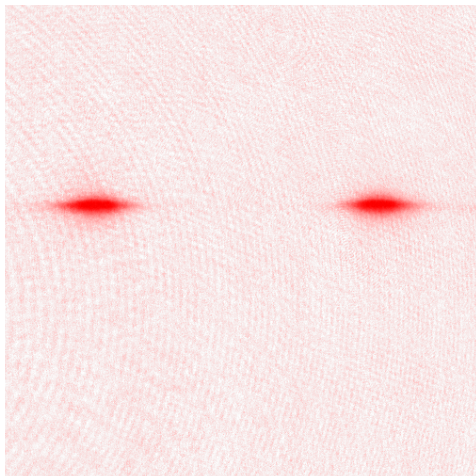
Principle
B_0-jump



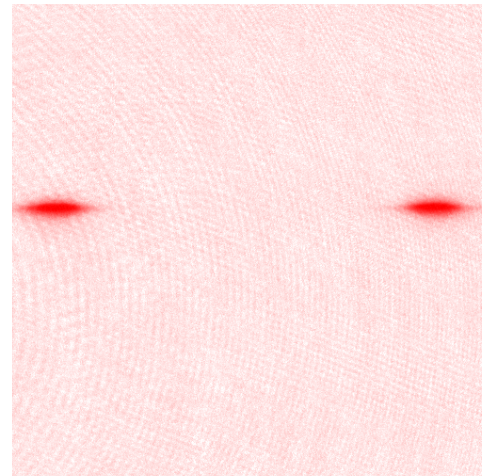
Practice
B_0-ramp

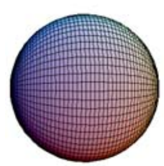
collision of two ultracold clouds

E_c/k_B=138 μK



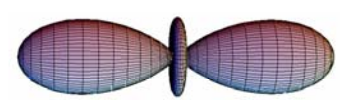
E_c/k_B=1230 μK





$Y_0^0(\theta)$

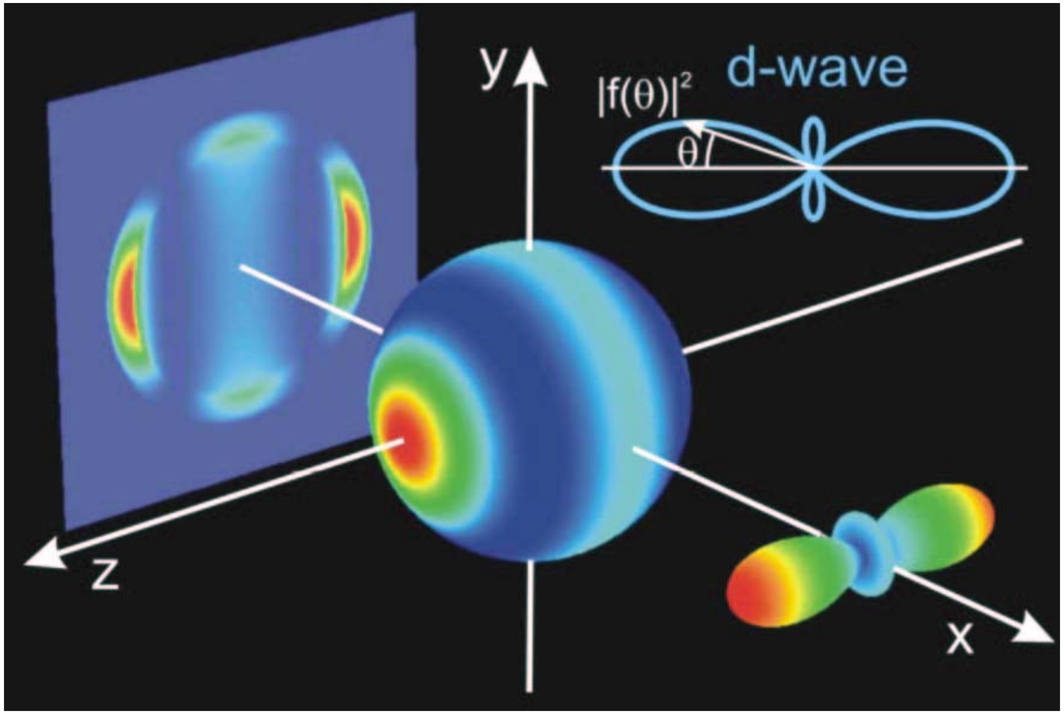
s-wave



$Y_2^0(\theta)$

d-wave

3D spatial distribution to be retrieved



Thomas, Kjaergaard, Julienne, Wilson, PRL 93 (2004) 173201

=>TOMOGRAPHY transformation

tomography

Reconstruction of 3D images from a set of 2D x-ray pictures (scanner)

Nobel prize in medicine in 1979 (A. M. Cormack and G. N. Hounsgield)

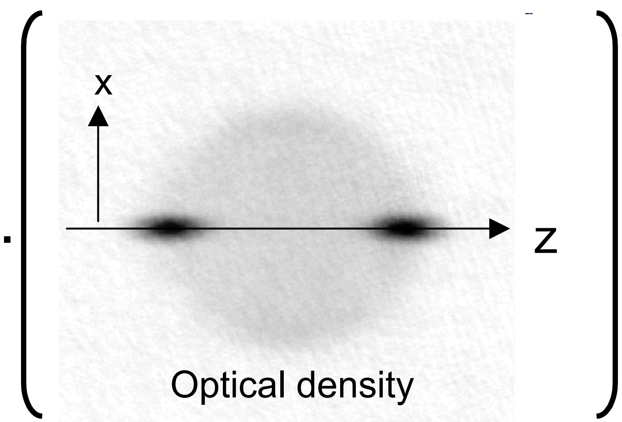
AXIAL SYMMETRY: only ONE picture is needed

Bessel Function

$$n\left(\rho,z\right) = \frac{1}{4\pi}\int_{-\infty}^{\infty} \tilde{n}_2\left(\kappa_x, z\right) J_0\left(\kappa_x \rho\right) \left|\kappa_x\right| d\kappa_x$$

3D density

1D F.F.T.



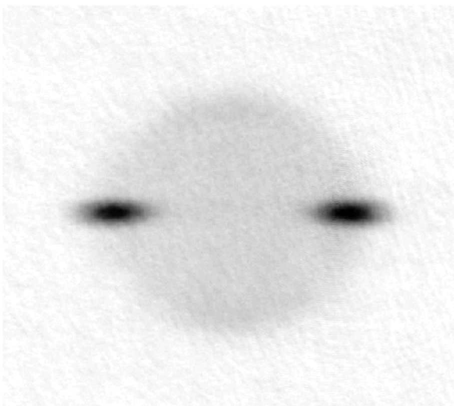
Optical density

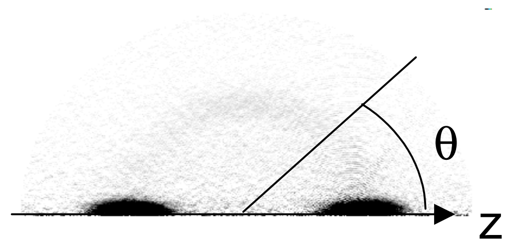
see *e.g.* : M. Born and E. Wolf, Principles of Optics, 7th (expanded) Edition, Cambridge University Press, Cambridge 1999.

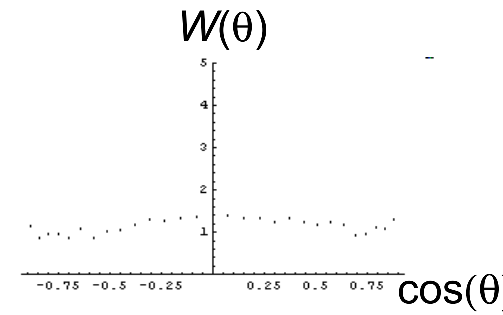
collision energy = 138 μK

(almost) pure s-wave

Experiment:







$\mathrm{OD}(x,y) \rightarrow n(r,\theta)$

$n(r,\theta) \rightarrow W(\theta)$

Optical density → **Tomography** → **Differential cross section**

Theory : $\sigma(\theta) = 2\pi \left| f(\theta) + f(\pi - \theta) \right|^2$ (identical bosons => symetrization)

$$\sigma(\theta) = 2\pi \left| \frac{2}{k} \sum_{l=even} (2l+1) P_l(\cos\theta)\, e^{i\eta_l} \sin\eta_l \right|^2$$

η_l : phase shift for the partial wave l
P_l : Legendre polynomials

$$\approx \frac{8\pi}{k^2} \left| \underbrace{e^{i\eta_0} \sin\eta_0}_{\text{s-wave}} + \underbrace{\frac{5}{2}(3\cos^2\theta - 1) e^{i\eta_2} \sin\eta_2}_{\text{d-wave}} \right|^2$$

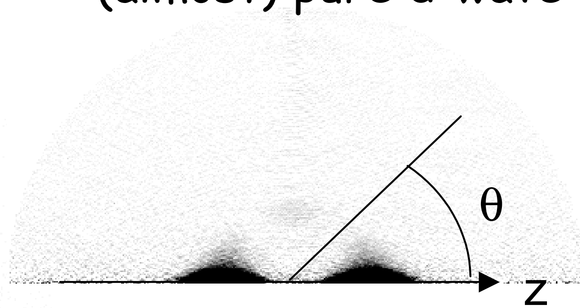
(Low Energy)

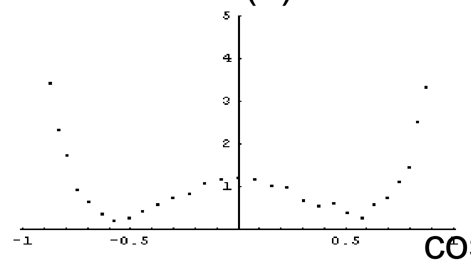
| s-wave + d-wave | ^{2} ⇒ matter-wave interference

collision energy = 1.23 mK

(almost) pure d-wave

Experiment:





cos(θ)

$$\mathrm{OD}(x,y) \rightarrow n(r,\theta)$$

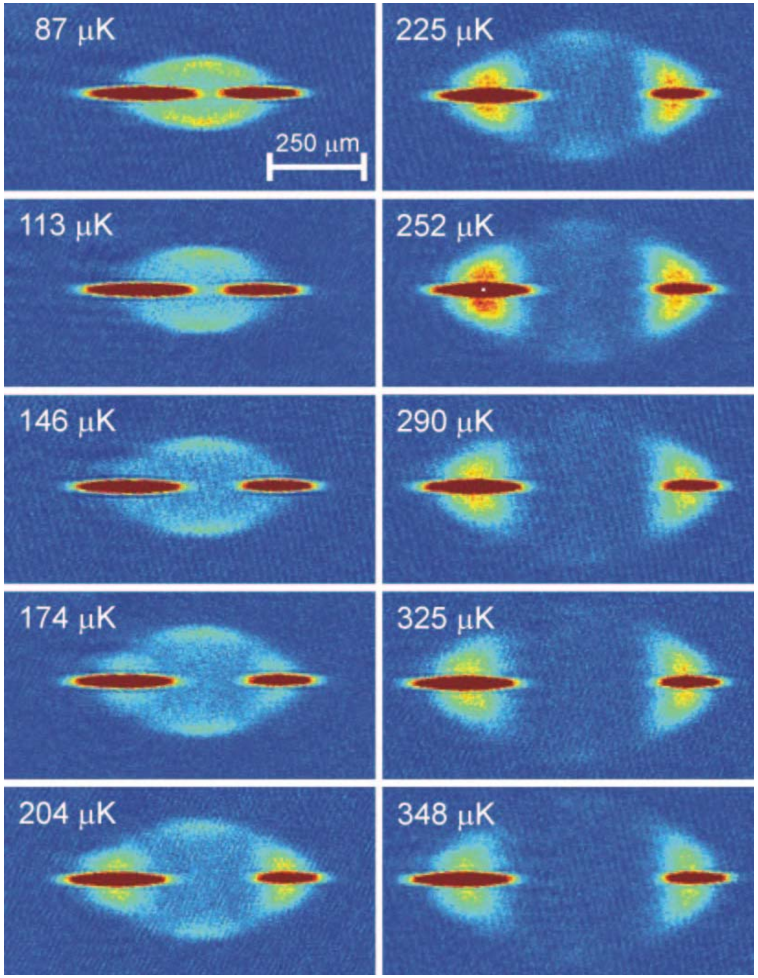
$$n(r,\theta) \rightarrow W(\theta)$$

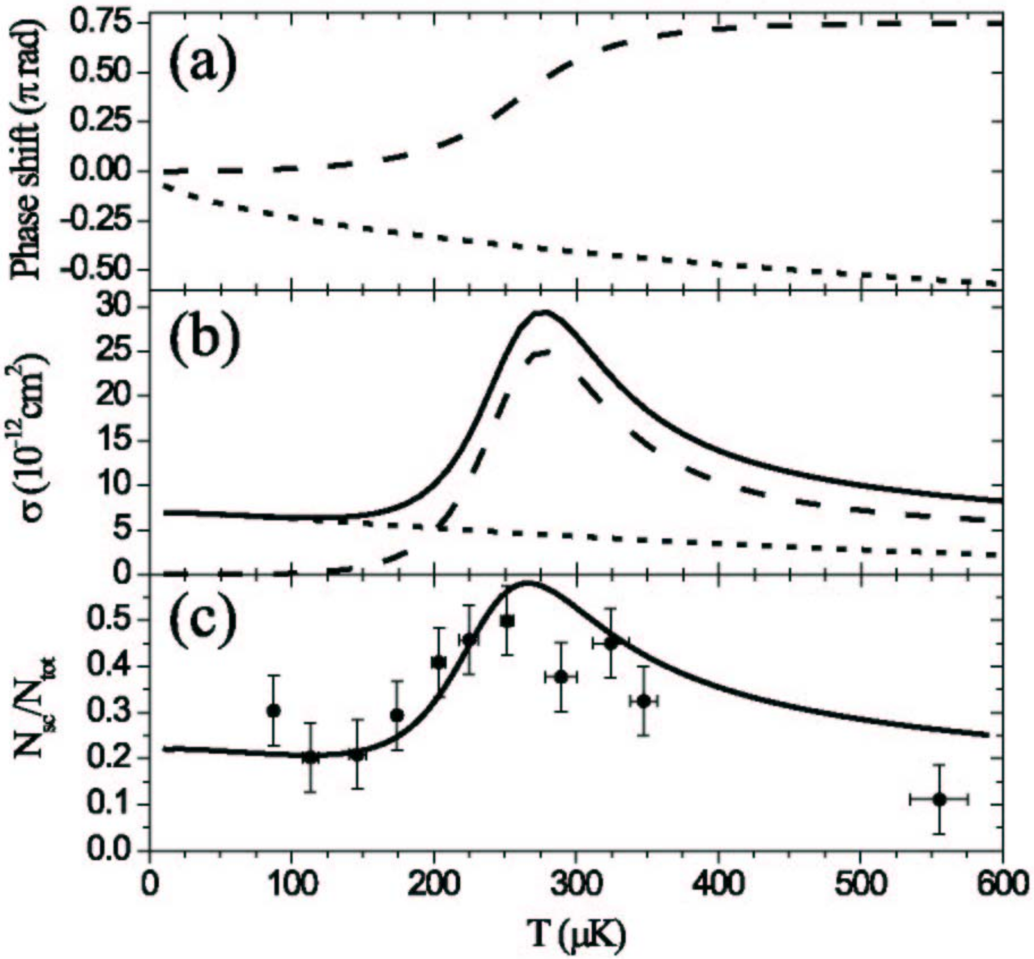
Optical density → **Tomography** → **Differential cross section**

$$W(\theta) \propto \sigma(\theta) \approx \frac{8\pi}{k^2}\left| e^{i\eta_0}\sin\eta_0 + \frac{5}{2}e^{i\eta_2}(3\cos^2\theta - 1)\sin\eta_2 \right|^2$$

$$\sigma = \int_0^{\pi/2} \sigma(\theta)\sin\theta \, d\theta$$

Otago results



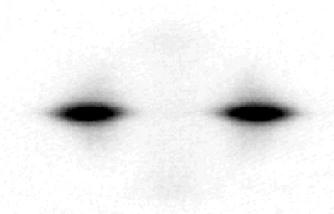


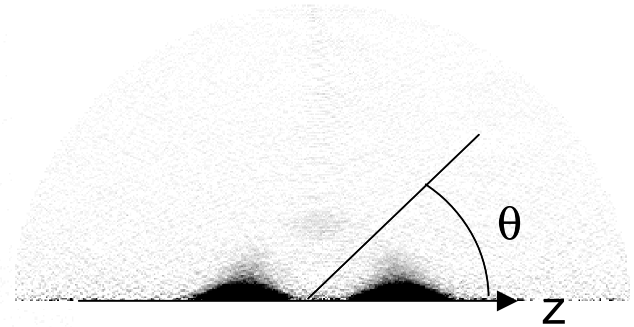
Thomas, Kjaergaard, Julienne, Wilson, PRL 93 (2004) 173201

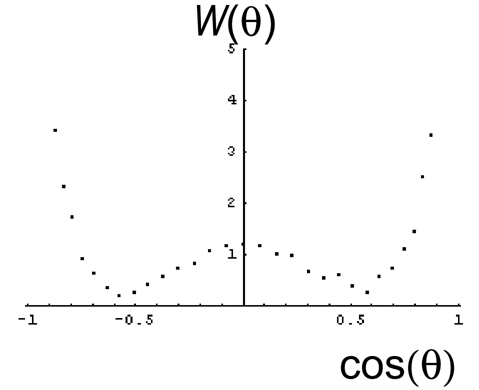
collision energy = 1.23 mK

(almost) pure d-wave

Experiment:







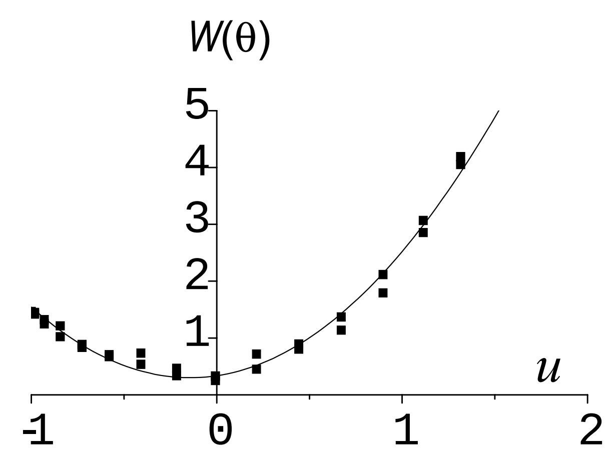
$$\mathrm{OD}(x,y) \rightarrow n(r,\theta) \rightarrow W(\theta)$$

Optical density → **Tomography** → **Differential cross section**

$$W(\theta) \propto \sigma(\theta) \approx \frac{8\pi}{k^2} \left| e^{i\eta_0} \sin\eta_0 + \frac{5}{2} e^{i\eta_2} (3\cos^2\theta - 1) \sin\eta_2 \right|^2$$

$$= C(1 + Au + Bu^2)$$

$$u \equiv (3\cos^2\theta - 1)$$



parabolic fit => (A, B) => (η_0, η_2)

Scattered waves and Phase shifts

$$H\psi(\vec{r}) = \frac{\hbar^2 k^2}{m}\psi(\vec{r})$$ with $$\psi(\vec{r}) = \frac{\chi_l(r)}{kr}\cdot Y_l^m(\theta,\varphi)$$

Radial Schrödinger Equation : $$\ddot{\chi}_l(k,r) + \left[k^2 - V(r) - \frac{l(l+1)}{r^2}\right]\chi_l(k,r) = 0$$

Asymptotic solution ($r \rightarrow \infty$): $$V(r) \approx -\frac{C^6}{r^6} \xrightarrow[r\rightarrow\infty]{} 0$$ $$\frac{l(l+1)}{r^2} \xrightarrow[r\rightarrow\infty]{} 0$$

$$\Rightarrow \chi_l \approx \sin(kr + \eta_l - l\frac{\pi}{2})$$

Phase Shift

- Scattering amplitude, differential cross section

$$\sigma(\theta) \approx \frac{8\pi}{k^2}\left| e^{i\eta_0}\sin\eta_0 + \frac{5}{2}e^{i\eta_2}(3\cos^2\theta - 1)\sin\eta_2 \right|^2$$

- Obtained from the experiments

Obtaining
the differential cross section
at ANY (low) energy ...

Integrating the Schrödinger equation

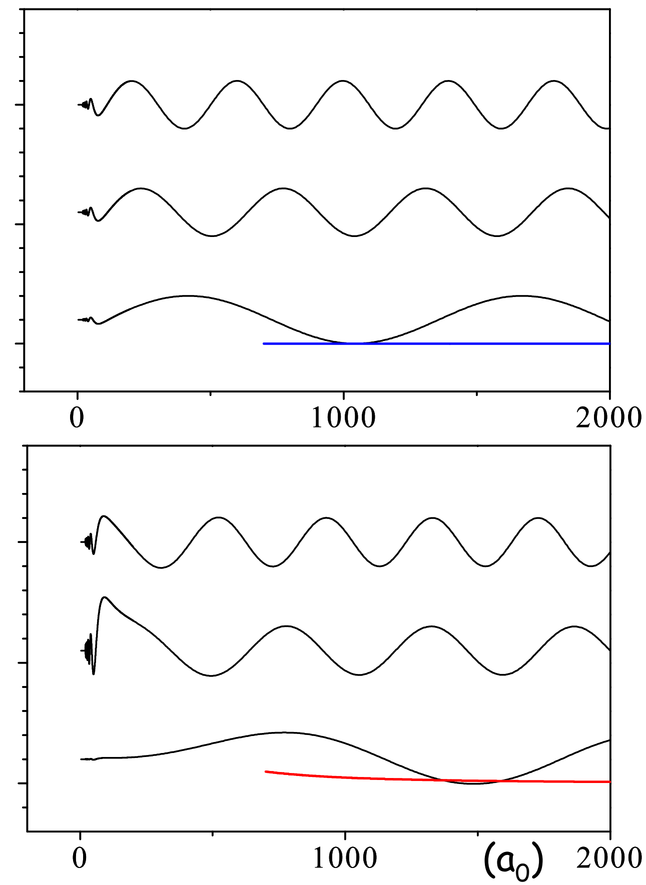
inward ←

experimental phase shifts: boundary condition for $r \to \infty$

Long-range part only:

$$V(r) = -\frac{C_6}{r^6}$$

$$\ddot{\chi}_l(k,r) + \left[k^2 - \frac{l(l+1)}{r^2} + \frac{C_6}{r^6}\right]\chi_l(k,r) = 0$$



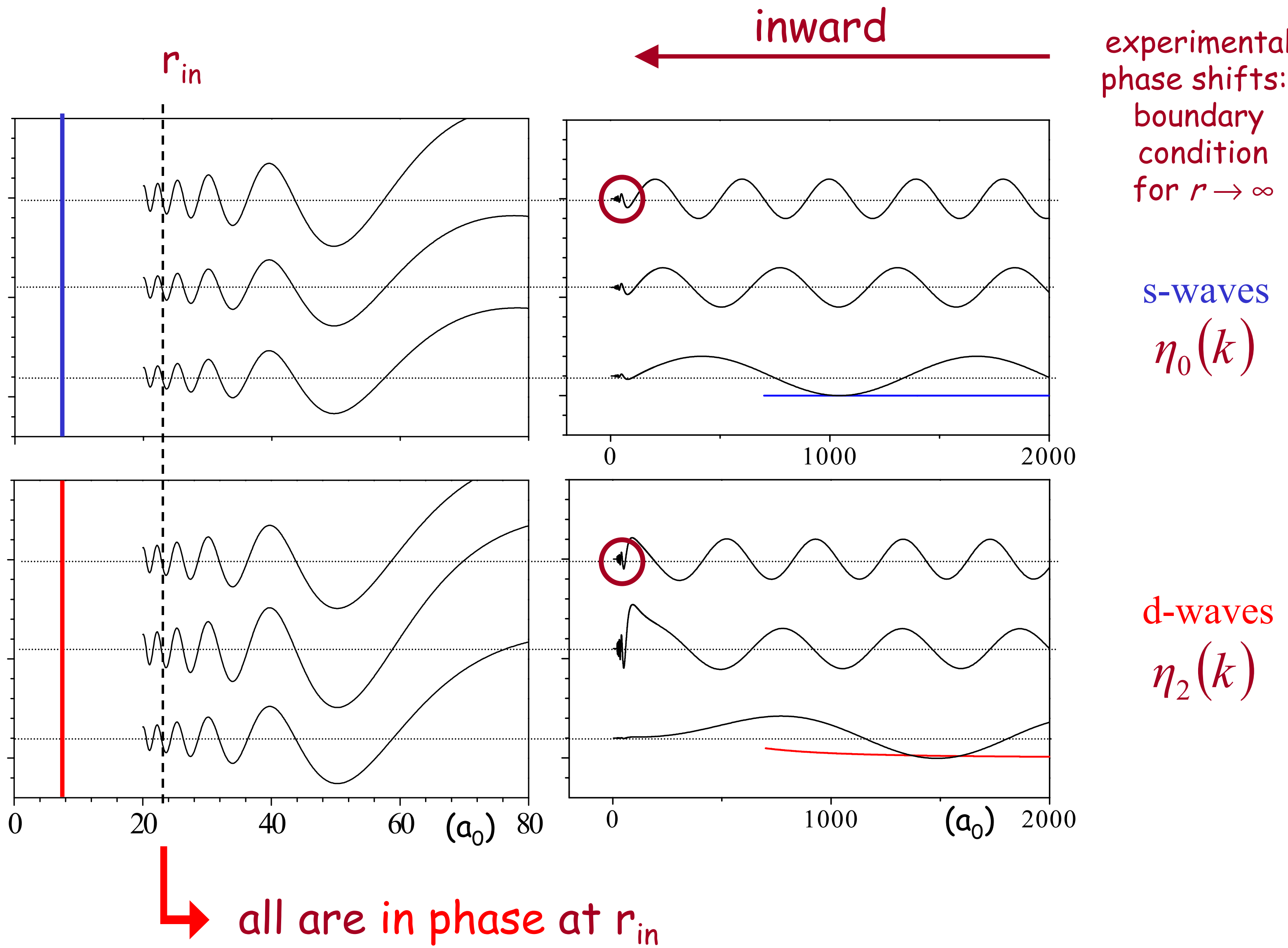
s-waves

$\eta_0(k)$

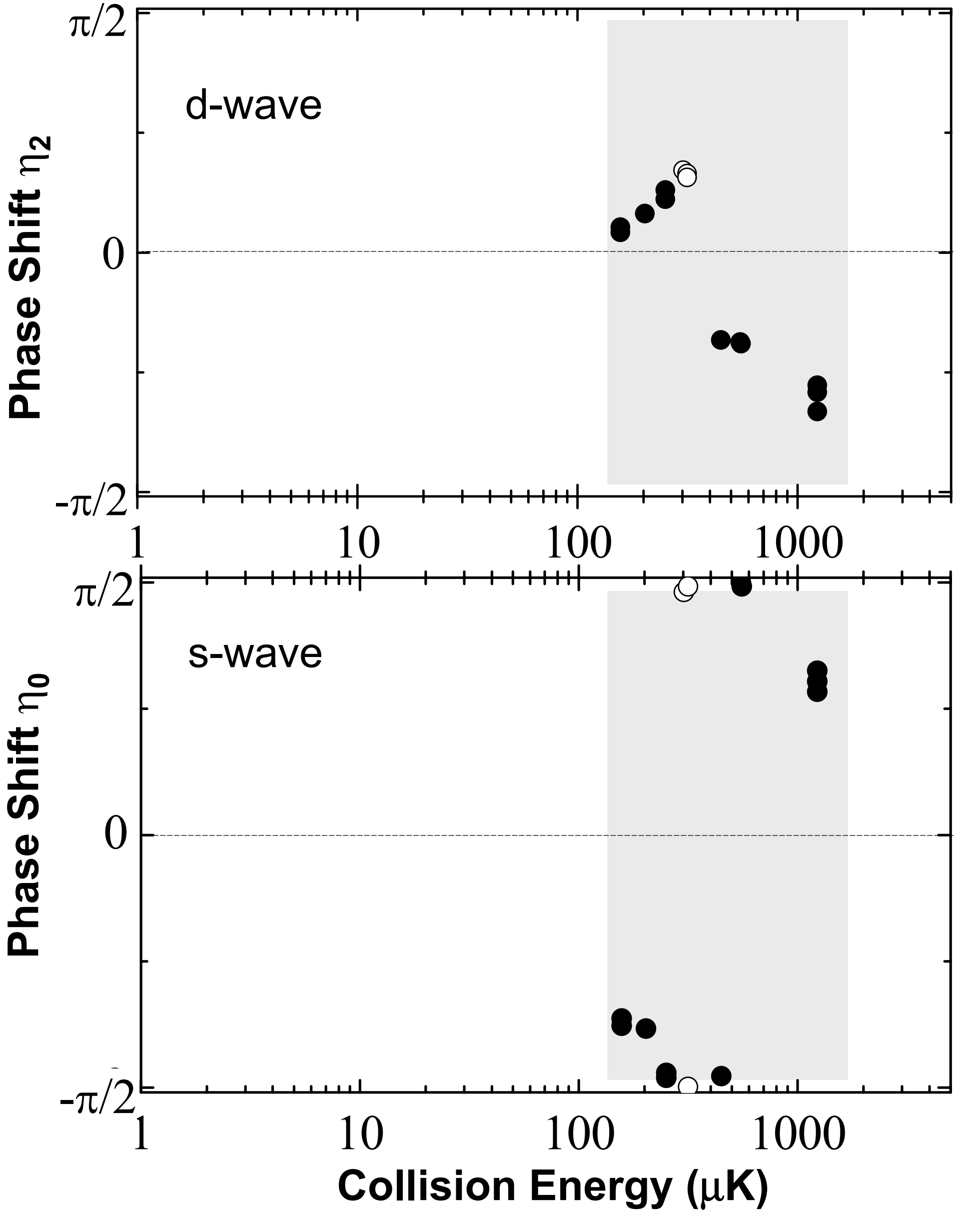
d-waves

$\eta_2(k)$

Integrating the Schrödinger equation

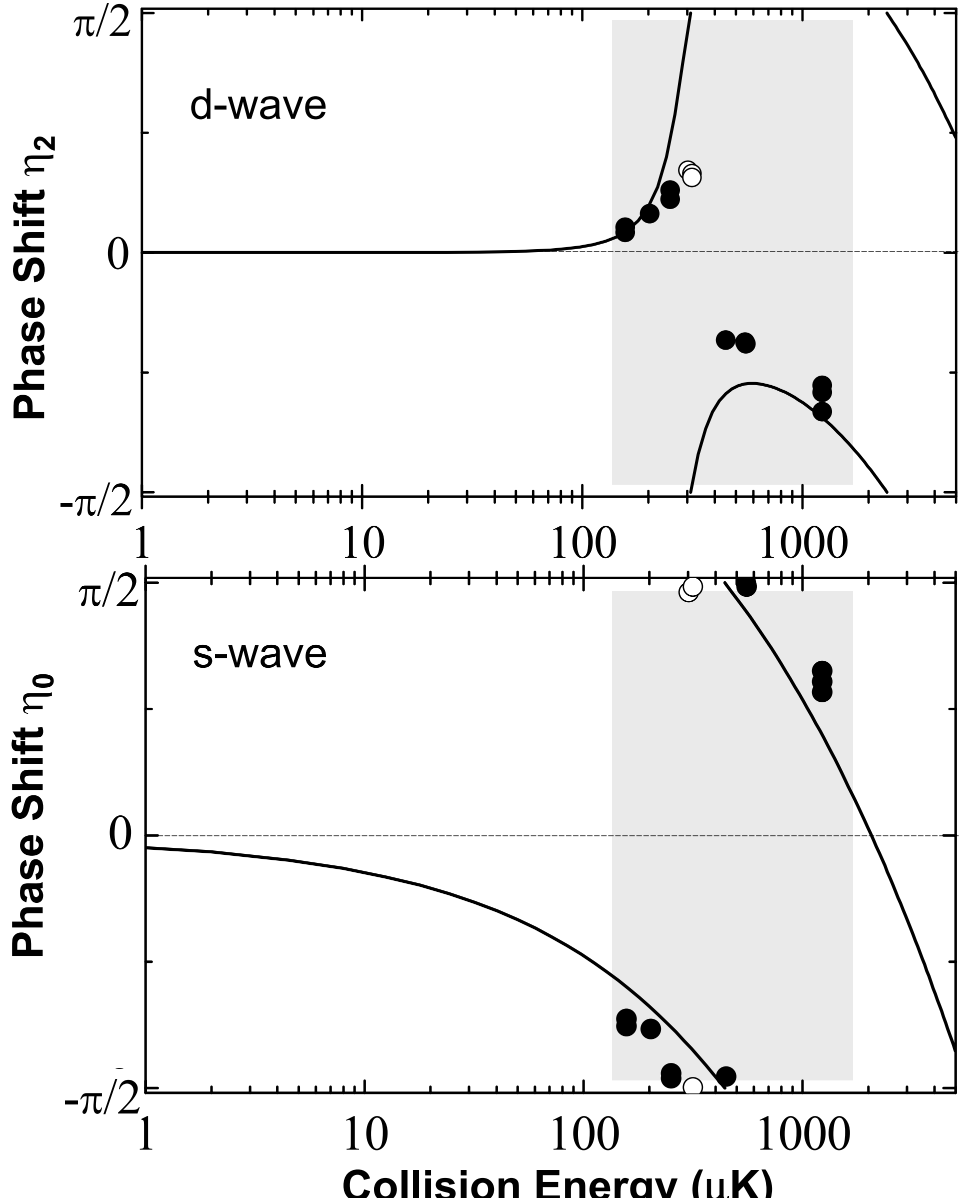


Results



knowing only C_6/r^6
we extract
ONE accumulated phase
at r_{in} = 20 a_0
It allows us to calculate
the phase shifts at
ANY (low) energy

Results



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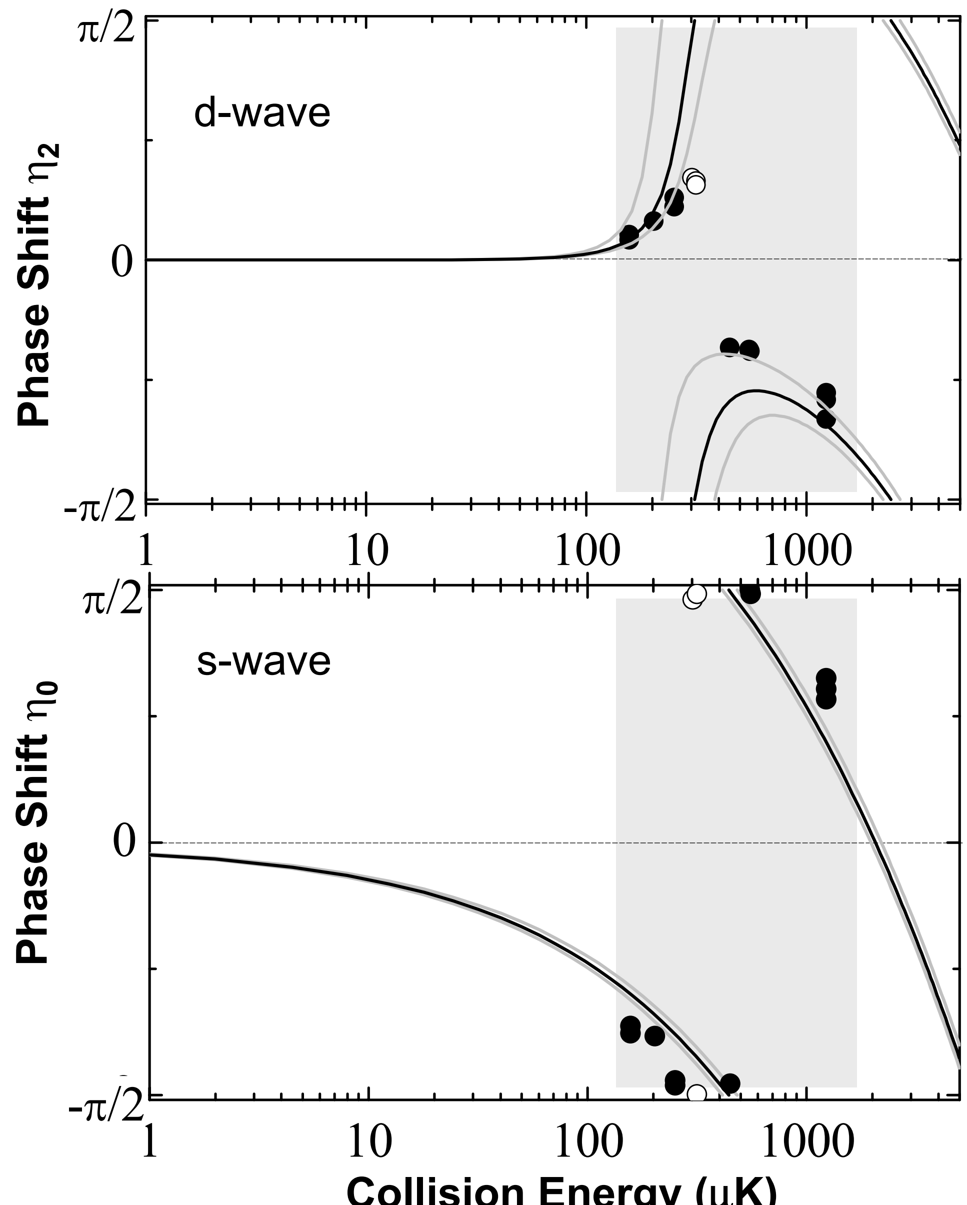
Buggle, Leonard, von Klitzing, Walraven, PRL 93 (2004) 173202

Results



knowing only C_6/r^6
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ONE accumulated phase
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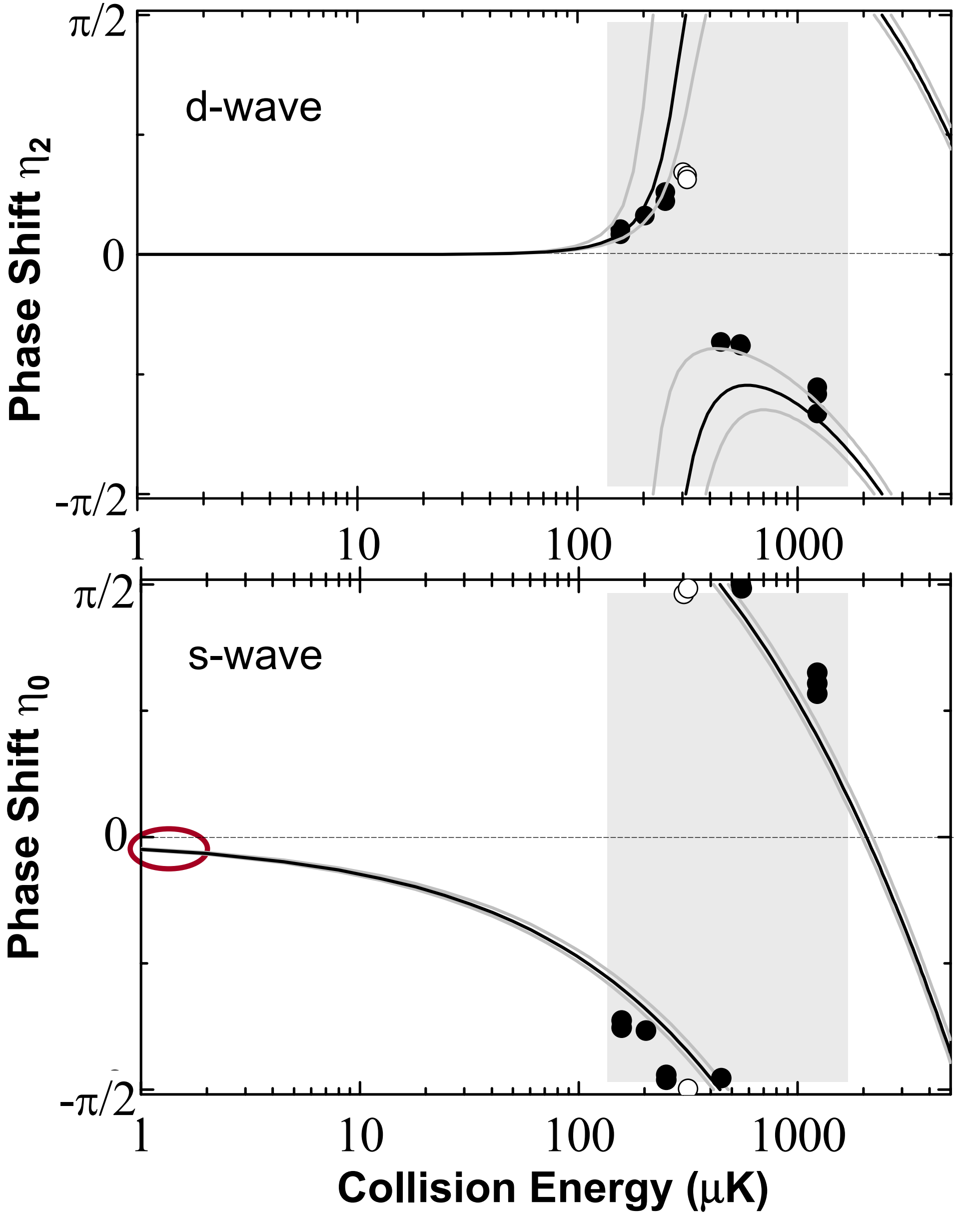
Buggle, Leonard, von Klitzing, Walraven, PRL 93 (2004) 173202

Results



$$\lim_{k \to 0} \eta_0(k) = -ka$$

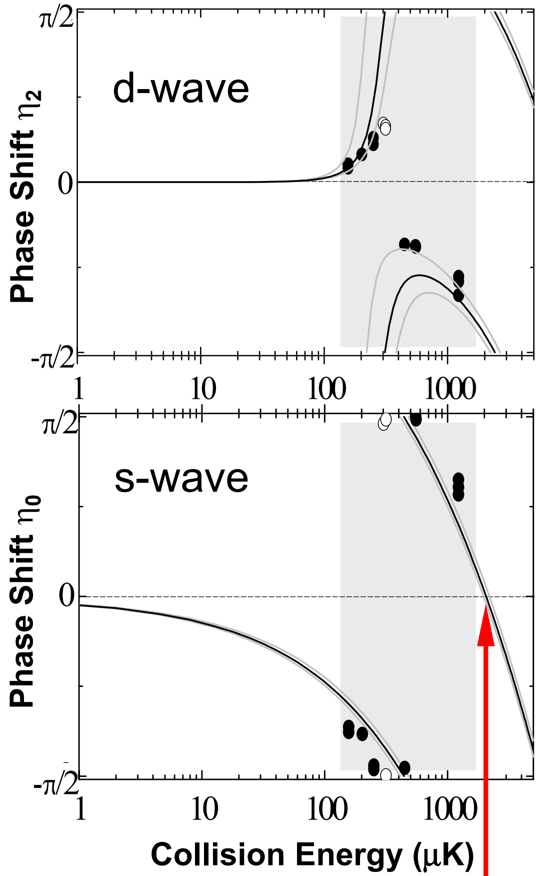
$a_{triplet} = +\ 102(6)a_0$

$a_{triplet} = 98.99(2)a_0$

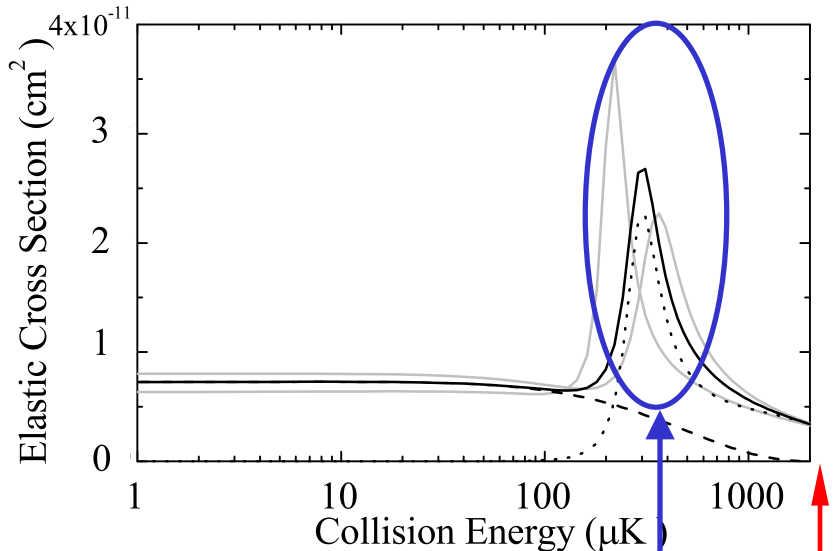
Van Kempen, Kokkelmans, Heinzen, Verhaar PRL **88**, 93201 (2002)

Buggle, Leonard, von Klitzing, Walraven, PRL 93 (2004) 173202

Phase Shifts and Elastic Cross-Section



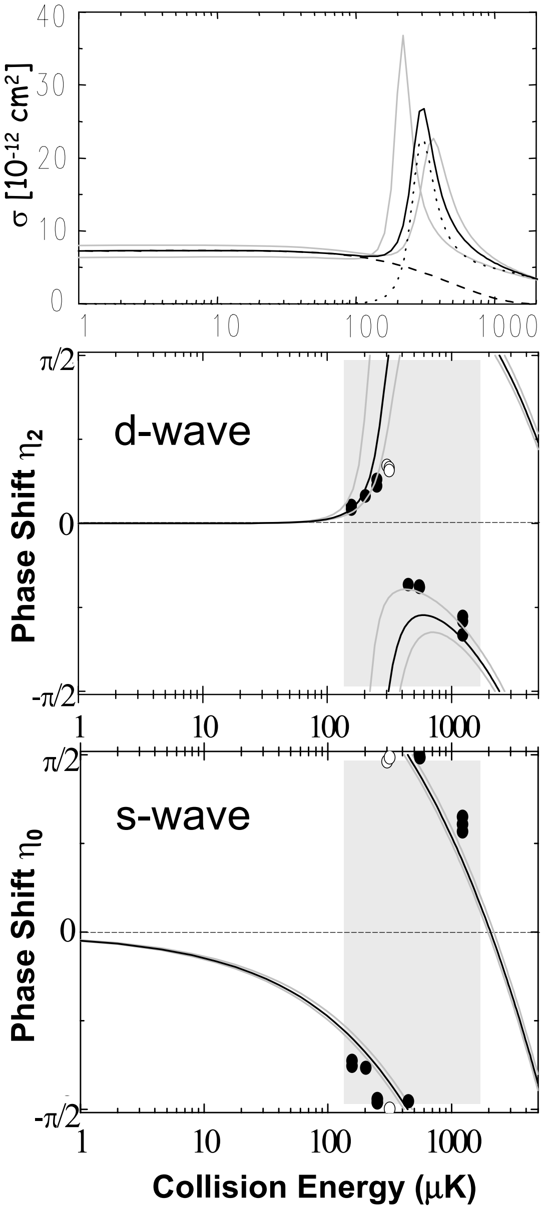
$$\sigma = \int_0^{\pi/2} \sigma(\theta) \sin\theta \, d\theta = \frac{8\pi}{k^2} \sum_{l=even} (2l+1) \sin^2 \eta_l$$

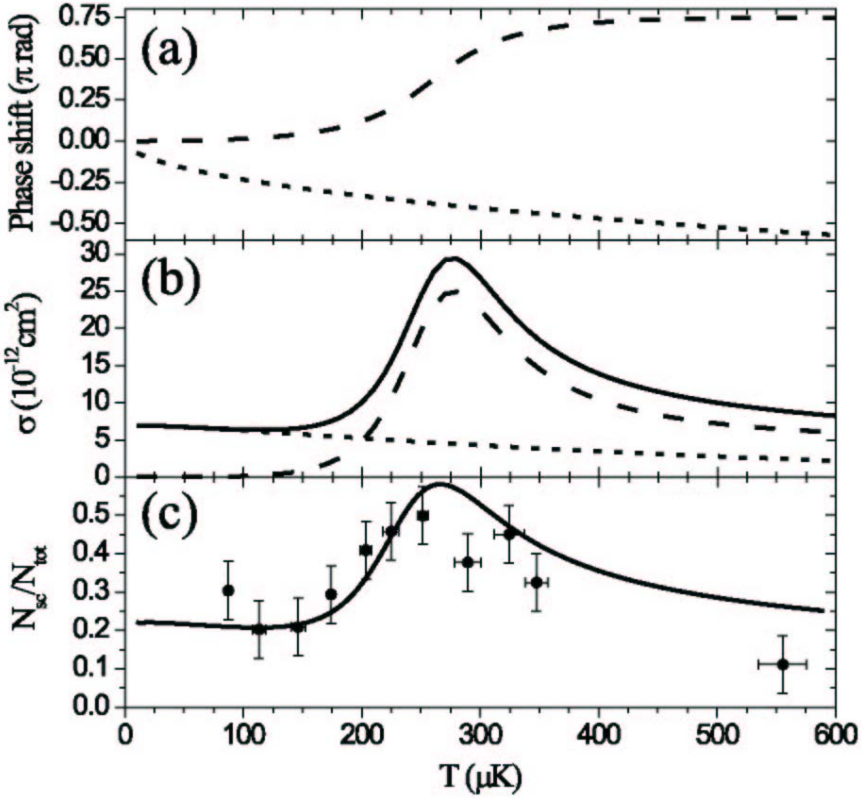


first Ramsauer minimum at 2.1(2) mK

d-wave resonance obtained at 300(70) μK

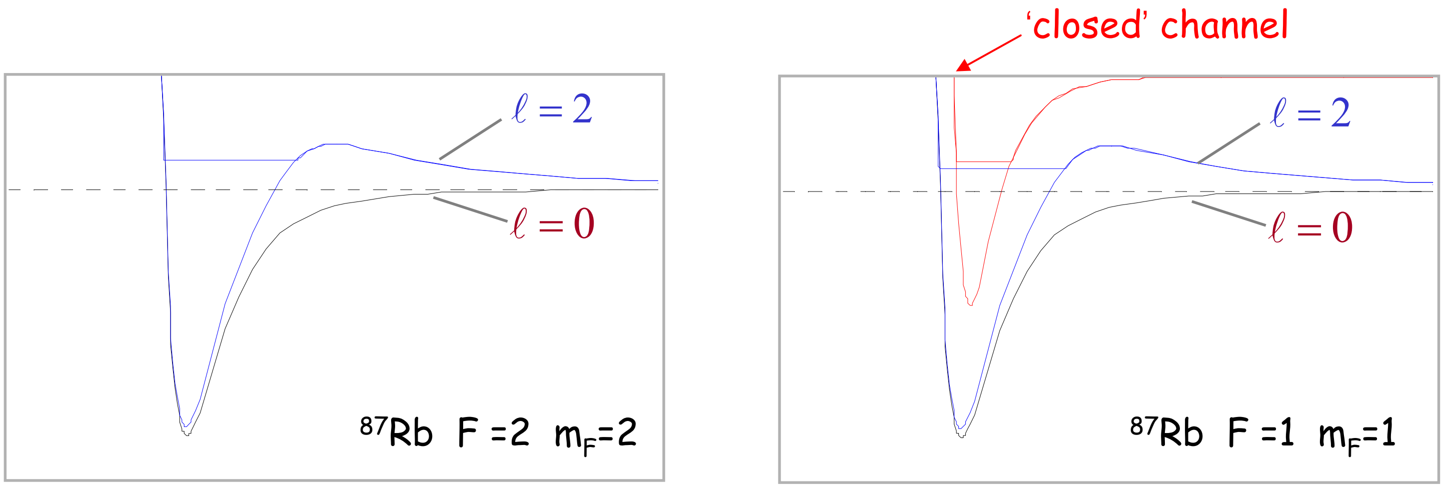
Comparison with the Otago results





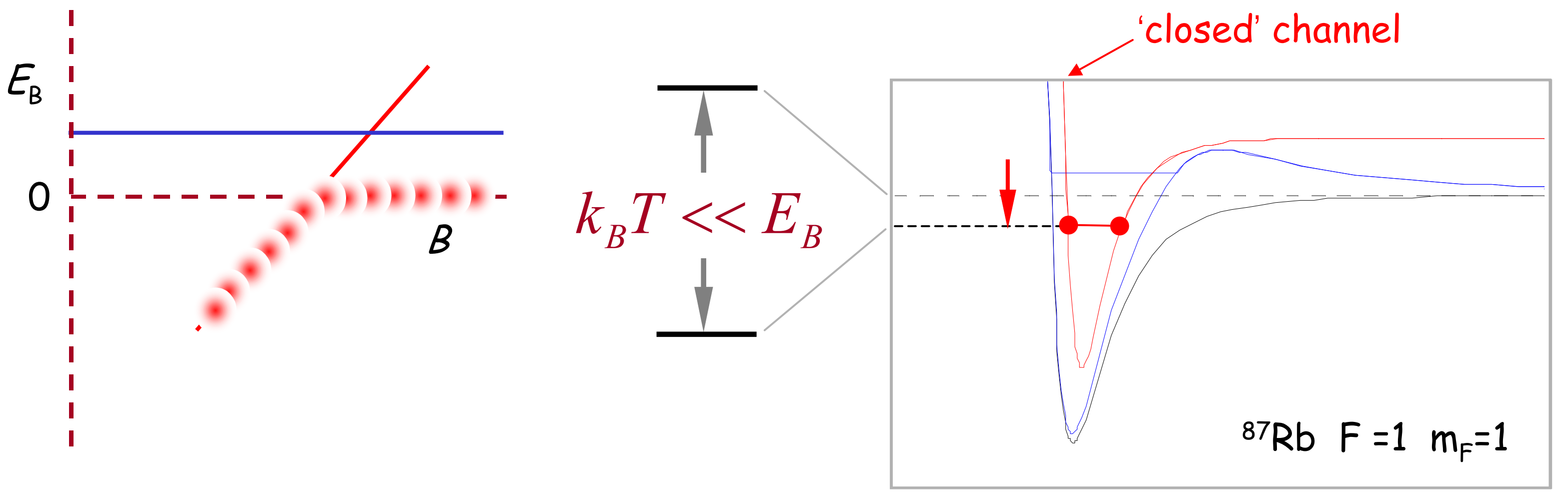
Thomas, Kjaergaard, Julienne, Wilson,
PRL 93 (2004) 173201

Comparison with the Garching results



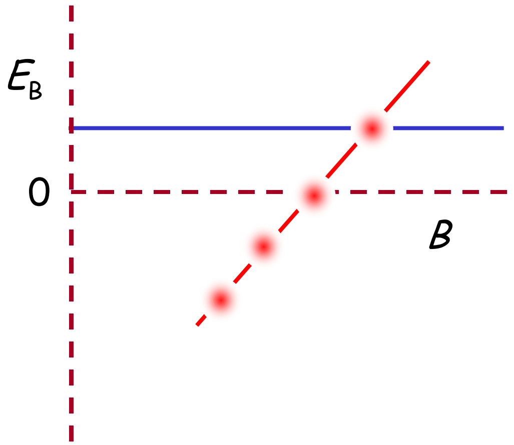
Volz, Durr, Syassen, Rempe, van Kempen, Kokkelmans, cond-mat/0410083

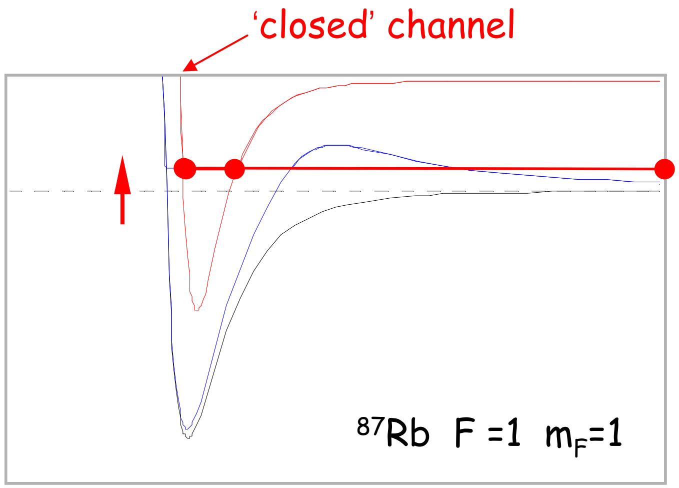
Molecular *association* by Feshbach tuning



Volz, Durr, Syassen, Rempe, van Kempen,
Kokkelmans, cond-mat/0410083

Molecular *dissociation* by Feshbach tuning



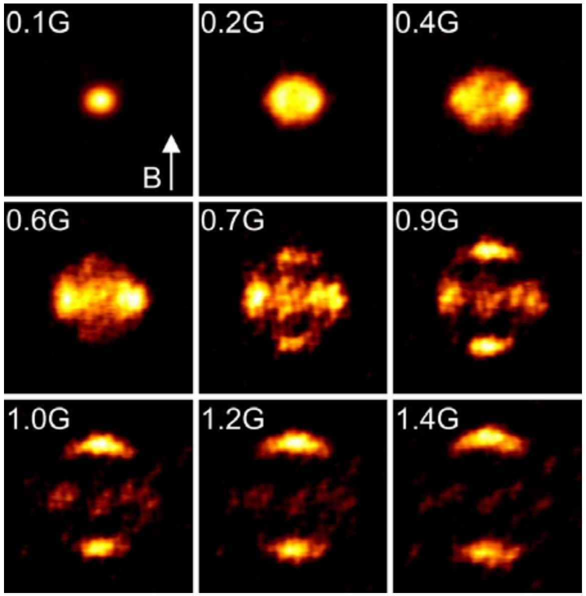


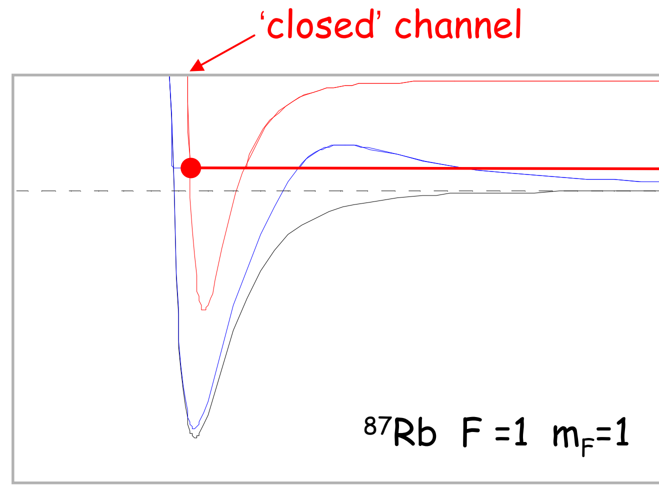
Volz, Durr, Syassen, Rempe, van Kempen, Kokkelmans, cond-mat/0410083

$$\Psi(\vec{r}) = g(\vec{r},t)\{e^{i\delta_0}\sqrt{\beta_0}Y_0^0 - e^{i\delta_2}\sqrt{\beta_2}Y_2^0(\theta)\}$$

β_0,β_2 = branching ratios $\qquad \beta_0+\beta_2 = 1$

Molecular *dissociation* by Feshbach tuning





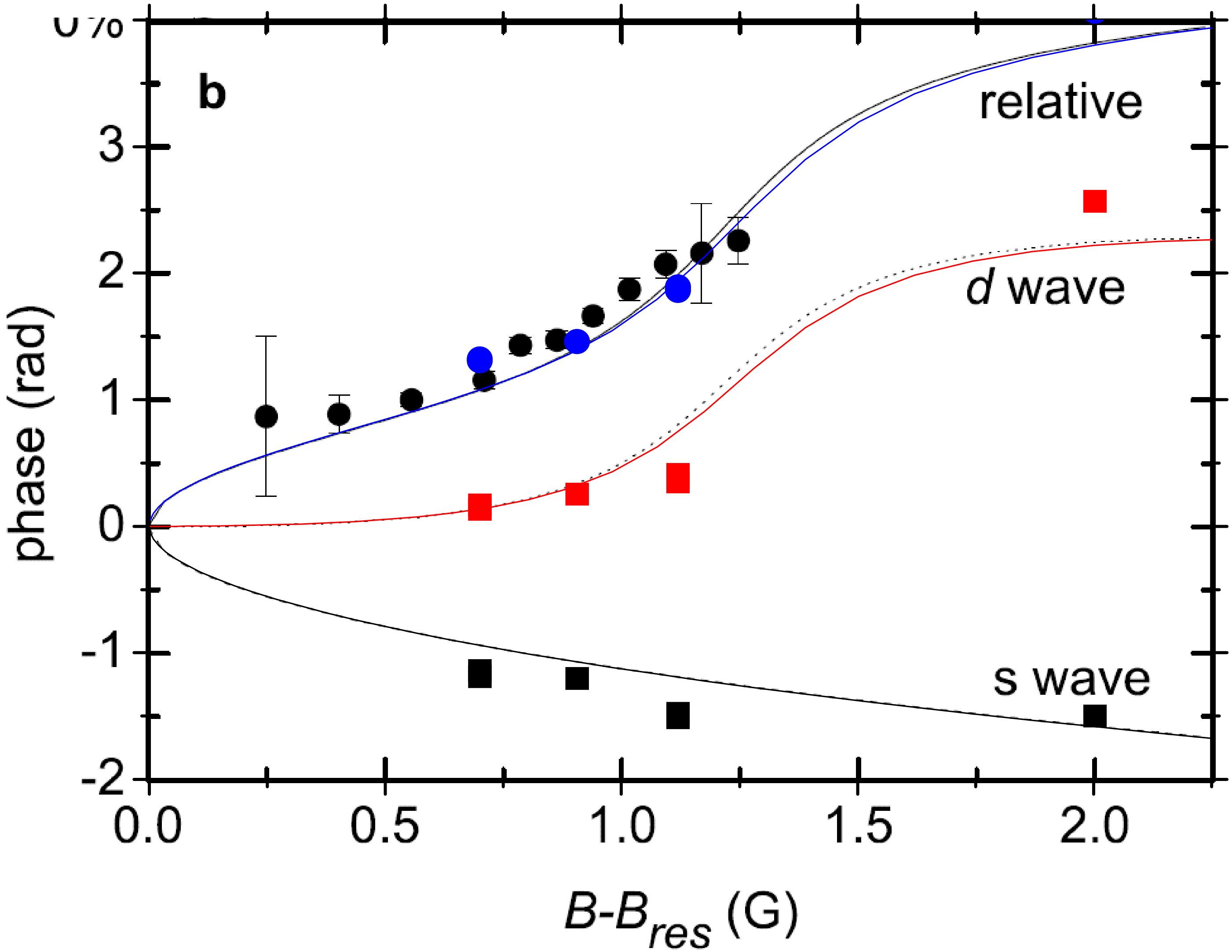
Volz, Durr, Syassen, Rempe, van Kempen, Kokkelmans, cond-mat/0410083

$$\Psi(\vec{r}) = g(\vec{r},t)\{e^{i\delta_0}\sqrt{\beta_0}Y_0^0 - e^{i\delta_2}\sqrt{\beta_2}Y_2^0(\theta)\}$$

β_0,β_2 = branching ratios

$\beta_0+\beta_2 = 1$

dissociation versus collision



Branching ratio's

Molecular dissociation:

$$\Psi(\vec{r}) = g(\vec{r},t)\{e^{i\delta_0}\sqrt{\beta_0}\,Y_0^0 - e^{i\delta_2}\sqrt{\beta_2}\,Y_2^0(\theta)\}$$

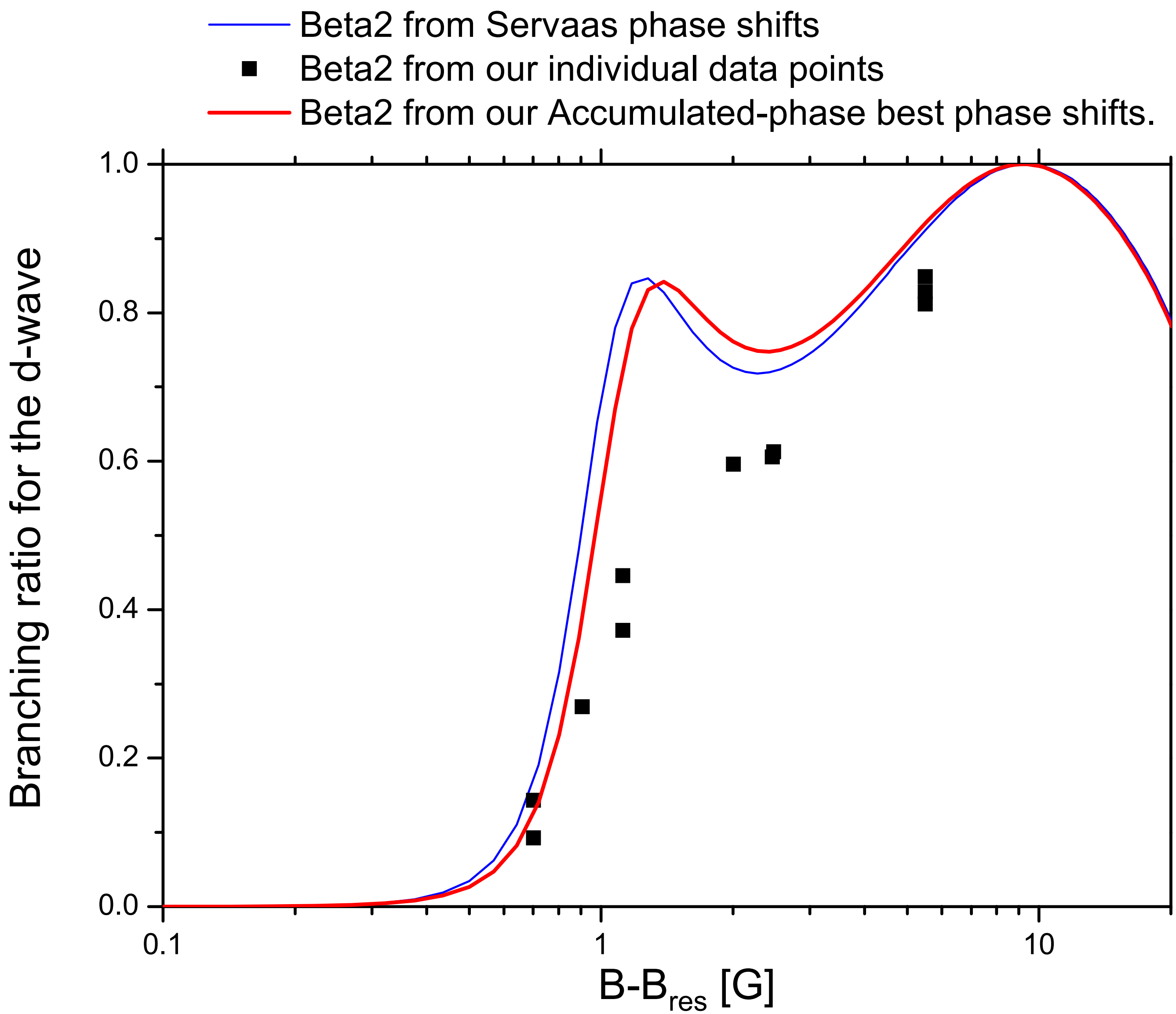
β_0,β_2 = branching ratios $\beta_0+\beta_2 = 1$

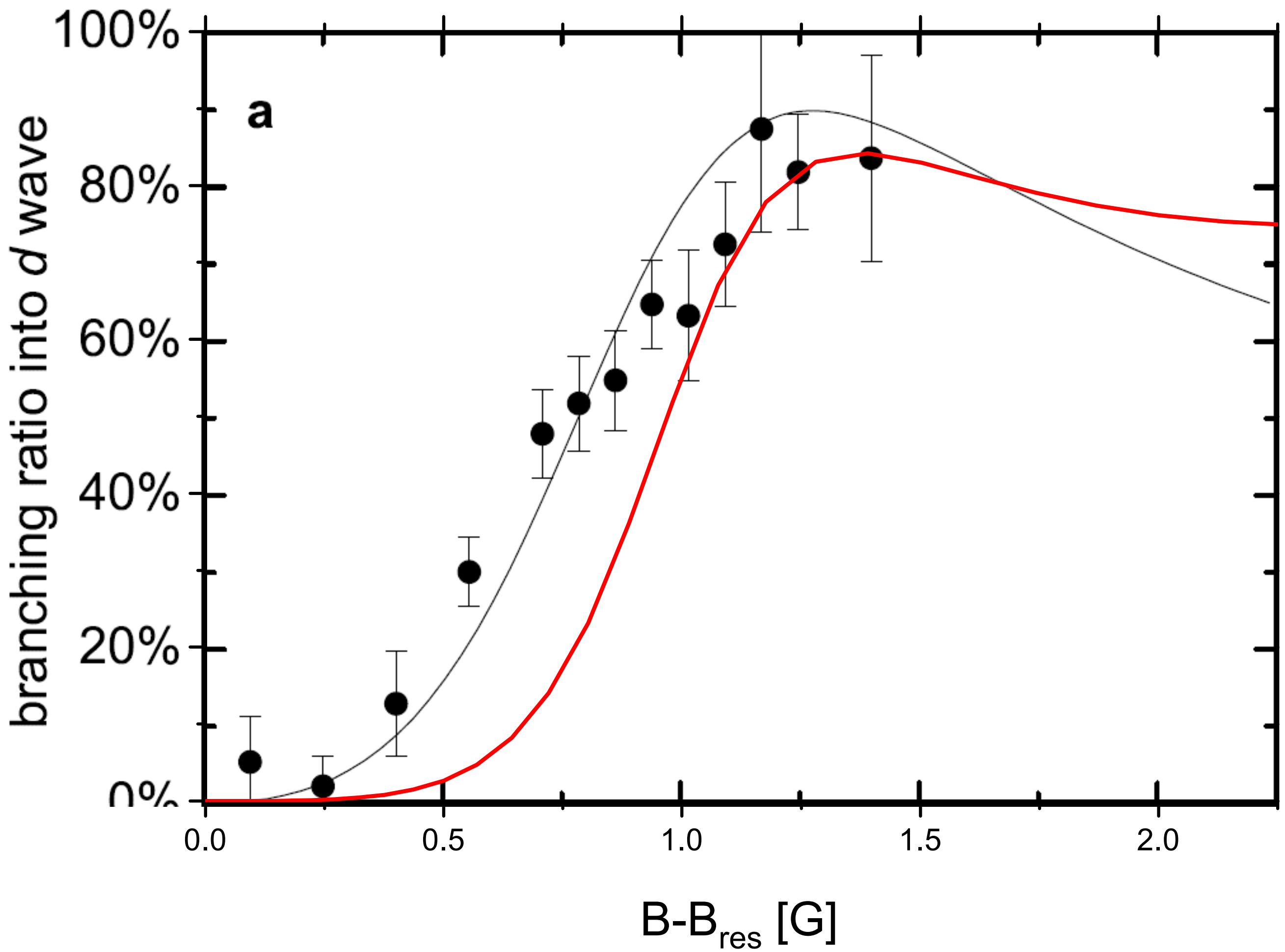
Collision between free atoms (plane incident wave):

$$\Psi(\vec{r}) = g'(\vec{r},t)\{e^{i\delta_0}\sin\delta_0\,Y_0^0 - e^{i\delta_2}\sqrt{5}\sin\delta_2\,Y_2^0(\theta)\}$$

$\beta_0 = 1-\beta_2$ and $\beta_2 = \dfrac{5\sin^2\delta_2}{\sin^2\delta_0 + 5\sin^2\delta_2}$

Comparison with the Garching results

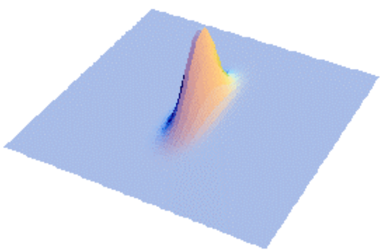


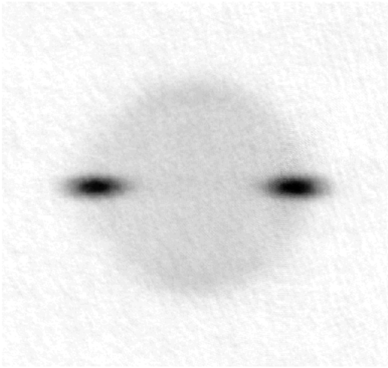


=> The Feshbach resonance shifts the d-wave shape resonance towards lower energies

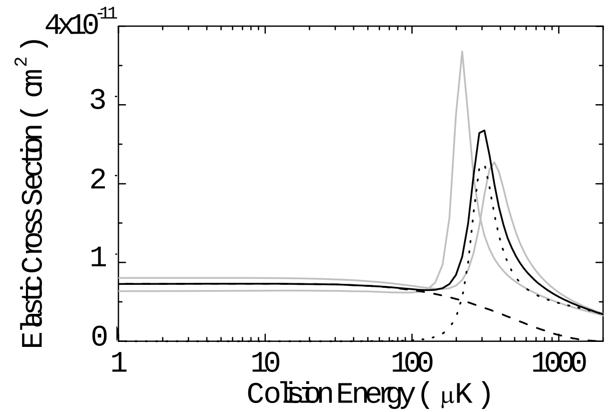
Summary

- ‘New’ method to determine scattering amplitudes





+ C_6/r^6



- Requires very little a priori knowledge of the potential

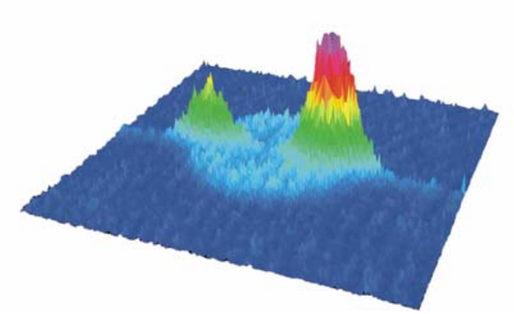
  Only C6 (single-atom property)

- Provides fairly accurate results

- Provides good agreement with other determinations

Group Quantum Gases

Wolf von Klitzing, Paul Cleary, JTMW, Jeremie Leonard, Christian Buggle



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