EPR, entanglement and macroscopic quantum paradoxes

Eric G. Cavalcanti and Margaret D. Reid
University of Queensland, Brisbane

AUSTRALIAN CENTRE FOR QUANTUM-ATOM OPTICS

1935: a *cool* year

- EPR paradox

→ local realism **VS.** nonseparability (entanglement)

- Schroedinger's Cat

→ macroscopic realism **VS.** macroscopic superpositions

EPR Paradox

- Positions x_A and x_B and momenta p_A and p_B are perfectly correlated
- With the assumption of local realism;
- one can predict with certainty the result of both x and p by measuring at B => *elements of reality*
- But for any *quantum state*

$$\Delta x \Delta p \geq \hbar/2$$

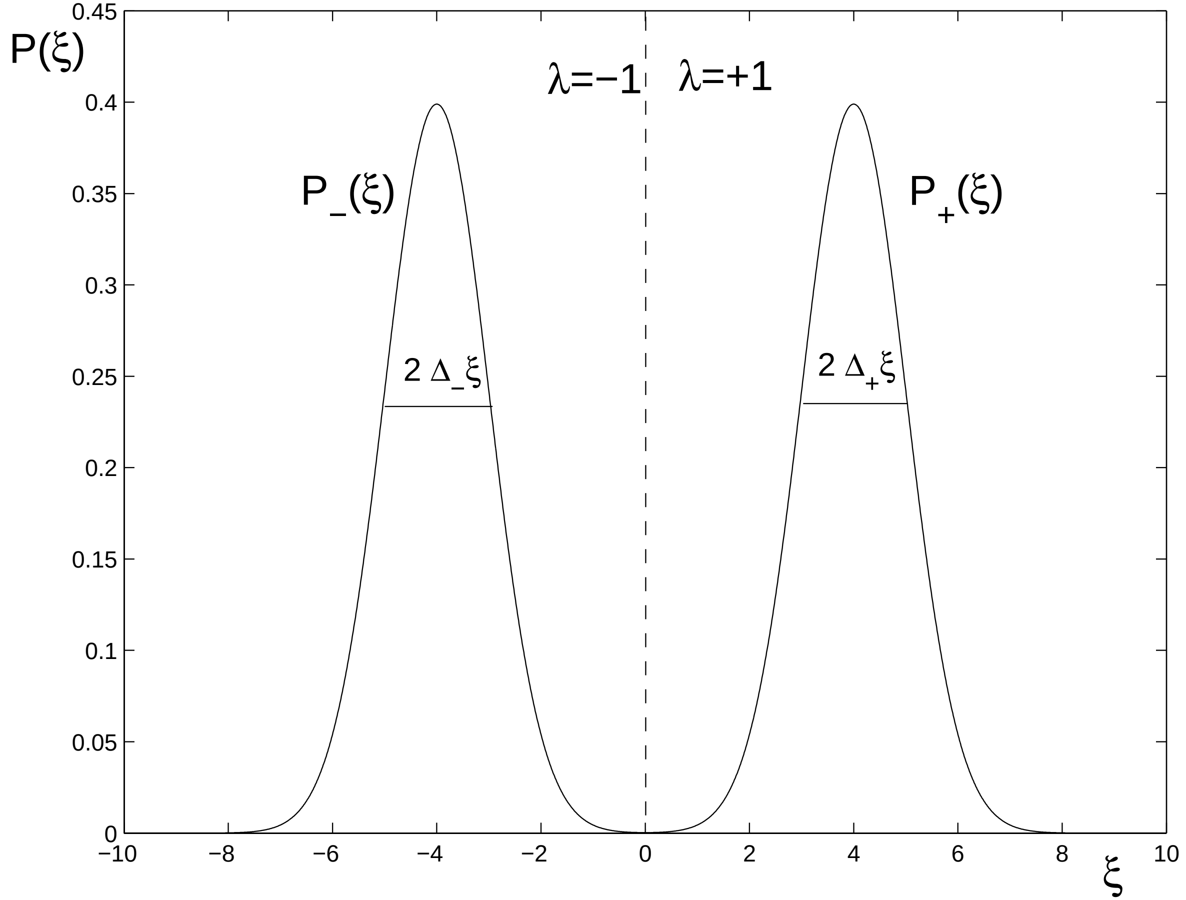
⇒ Either local realism is false or QM is incomplete

Macroscopic Superpositions

- Copenhagen Interpretation
  - quantum world / macroscopic world ; ill-defined boundary
- Decoherence *does not* solve the problem
- Alternative theories to QM (dynamical collapse theories) aim to determine a limit
  - Ex: Ghirardi, Rimini, Weber and Pearle (GRWP)

    length scale (a)   *rate scale (λ)*
- Development of experimental techniques start to make it a feasible programme to push the boundary
- How to experimentally identify a “Schroedinger Cat”?
  - Indirectly (dynamical signatures)
  - Directly (measurement statistics)

Macroscopic variable ξ



In general

$$\rho = \sum_R P_R |\psi_R\rangle\langle\psi_R|. \qquad (1)$$

Where (assuming that we have two particles at *A* and *B*)

$$|\psi_R\rangle = \sum_{i,r} c^R_{i,r} |o_i\rangle_A |o_r\rangle_B. \qquad (2)$$

To prove the existence of a macroscopic superposition, ***we want to prove the existence*** in expansion (1) of

$$|\psi_R\rangle = c^R_+ |\psi_+\rangle + c^R_- |\psi_-\rangle, \qquad (3)$$

where $|\psi_+\rangle = \sum_{o_i \in \lambda_+, r} c^R_{i,r} |o_i\rangle_A |o_r\rangle_B$, and similarly for $|\psi_-\rangle$

It is easy to see that ***if there is no superposition of the type (3),*** the density matrix can be written as

$$\rho_{mix} = P^0_+ \rho_+ + P^0_- \rho_- \qquad (4)$$

Proof of failure of (4) is then proof of the existence of a superposition of type (3).

Inequalities for a single system

In any system which can be described by mixture (4), the variances of two observables ξ *and* η *satisfy*

$$\Delta\xi\Delta\eta \geq \Delta_{ave}\xi\Delta\eta \geq \sum_{\lambda} P_{\lambda}^{0}\chi_{\lambda}, \qquad (5)$$

$\Delta_{\lambda}\xi\Delta_{\lambda}\eta \geq \chi_{\lambda}$ -> Heinsenberg Uncertainty Relation

Defining an ***average variance***

$$\Delta_{ave}^{2}\xi = \sum_{\lambda} P_{\lambda}\Delta_{\lambda}^{2}\xi$$

If mixture (4) is valid, then

$\Delta^{2}\xi \geq \Delta_{ave}^{2}\xi$, and similarly for η

$$[\hat{A}, \hat{B}] = i\hat{C}$$

$$\Delta^{2}A\Delta^{2}B \geq \frac{1}{4}|\langle C\rangle|^{2}$$

=> By the Cauchy-Schwarz inequality, result (5) follows

Inequalities for composite systems

Given a system composed of two subsystems *A* and *B* and a third observable O^B to be measured at subsystem *B;*

We define an ***average inference variance*** of ξ given a particular result $O^B{}_i$

$$\Delta^2_{inf}\xi = \sum_i P(O^B_i)V[\xi|O^B_i]$$

therefore $\Delta^2\xi \geq \Delta^2_{inf}\xi$, and similarly for η

The uncertainty relations now read

$$V[\xi|O^B_i]V[\eta|O^B_i] \geq (w^\lambda_i)^2$$

$$\Delta_{inf,\lambda}\xi\Delta_{inf,\lambda}\eta \geq \bar{w}^\lambda \qquad \bar{w}^\lambda = \sum_i P_\lambda(O^B_i)w^\lambda_i$$

If mixture (4) is valid, then by the Cauchy-Schwarz inequality,

$$\Delta\xi\Delta_{inf}\eta \geq \Delta_{ave}\xi\Delta_{inf}\eta \geq \sum_\lambda P^0_\lambda\bar{w}^\lambda, \qquad (8)$$

Example: two-mode squeezed state

$$|\psi\rangle = \sum_{n=0}^{\infty} c_n |n\rangle_A |n\rangle_B \qquad c_n = tanh^n r / coshr$$

Defining the quadrature operators $\hat{x} = (\hat{a}^\dagger + \hat{a}),\ \hat{p} = i(\hat{a}^\dagger - \hat{a})$

$$\hat{x}^B = (\hat{b}^\dagger + \hat{b}),\ \hat{p}^B = i(\hat{b}^\dagger - \hat{b})$$

They obey the HUP

$$\boxed{\Delta^2 x \Delta^2 p \geq 1}$$

The probability distribution for *x* is a gaussian

$$P(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-x^2/2\sigma} \text{ where } \sigma = cosh2r$$

Wait a minute. Gaussian? What the…!?

This system presents the *EPR correlations*

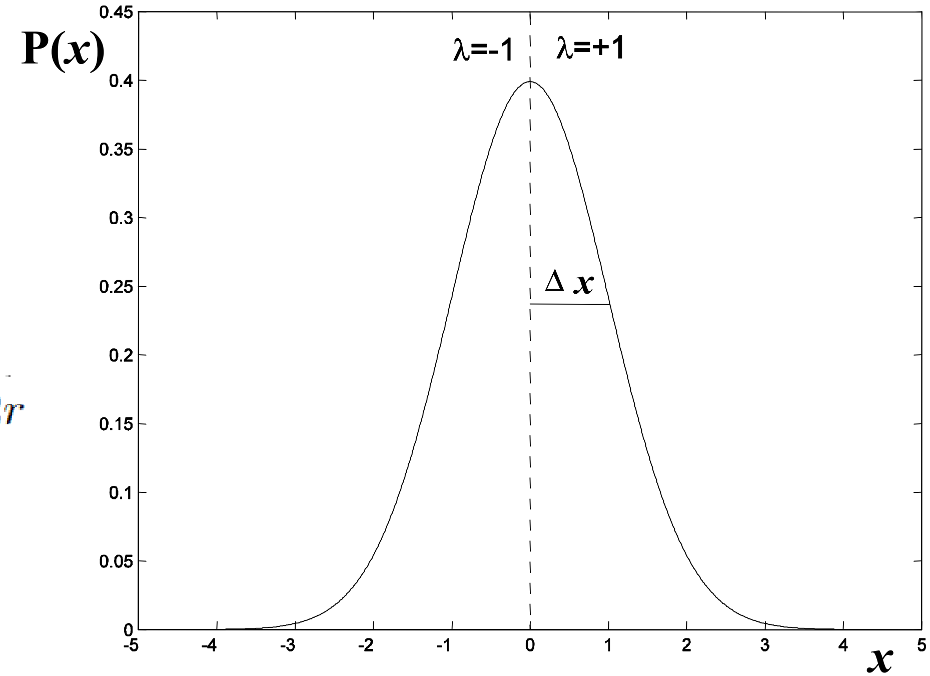
$$x = x^B$$
$$p = -p^B$$

with inference variances

$$V[x|x_i^B] = V[p|p_i^B] = 1/cosh2r$$

With the binning shown in the graph, the variances for each λ are

$$\Delta_\lambda^2 x = \sigma(1 - 2/\pi) = .36\sigma$$



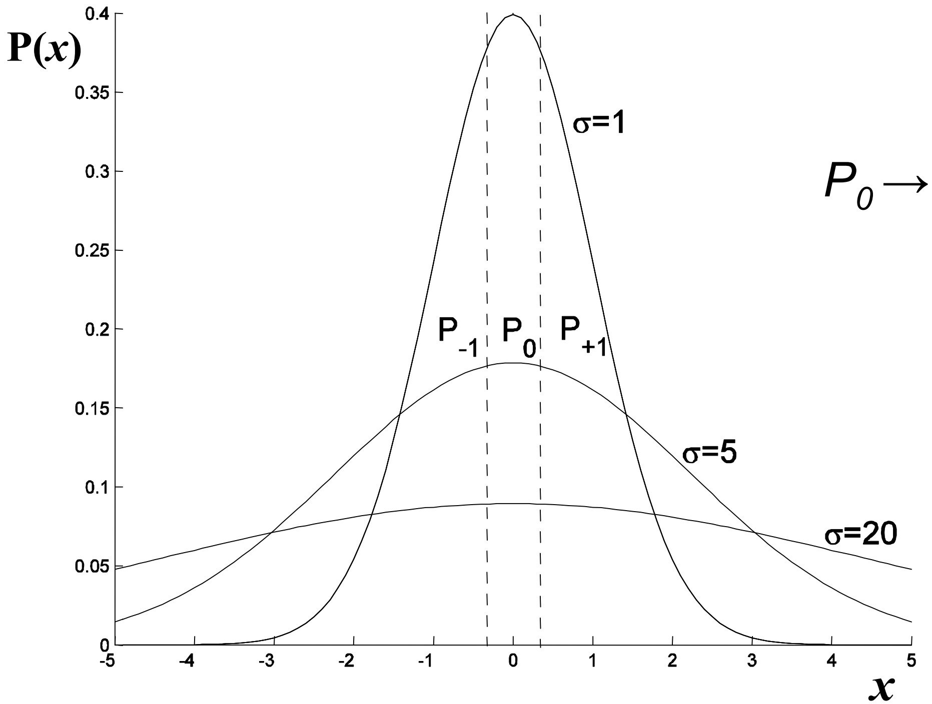
To prove the existence of superposition between + and -, *we want to violate the inequality*

$$\Delta_{ave} x \Delta_{inf} p \geq 1$$

The two-mode squeezed state does the job:

$$\Delta_{inf}^2 p \Delta_{ave}^2 x = .36$$

…and that's independent of the variance!



$P_0 \rightarrow 0$ as $\sigma \rightarrow \infty$

For large squeezing, there is a *negligible probability* of a result in the middle region, as required.

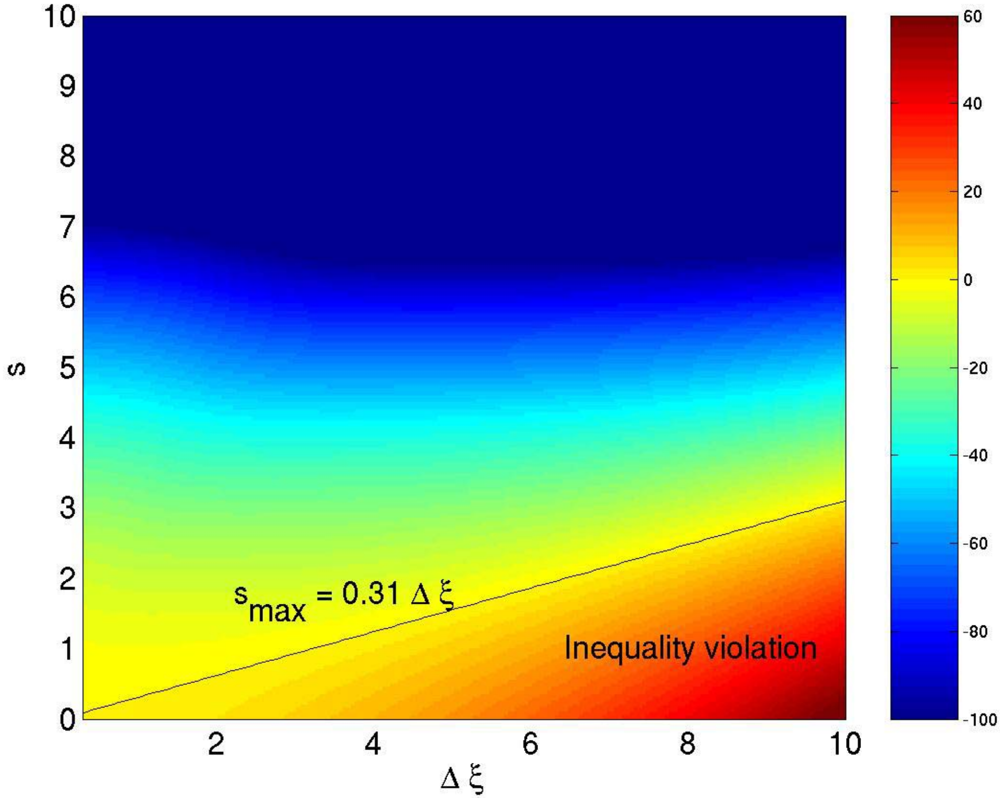
As $r \longrightarrow \infty$, outcomes $+1$ and -1 become macroscopically distinct

Yet as $r \longrightarrow \infty$, $\Delta^2_{ave} x \Delta^2_{inf} p = .36\sigma / \cosh 2r \longrightarrow .36 < 1$

proves macroscopic superposition $|x_-\rangle + |x_+\rangle$

I’m still not convinced…

New inequality taking into account the middle region. Violation proves a superposition of size larger than a given *s*.



Increasing the squeezing we can in principle prove the existence of a superposition of the order of the standard deviation.

What else?

- Discrete (spin) systems
  - EPR-Bohm macroscopic paradox
- Strong proof of violation of macroscopic realism
  - Criteria which don’t need the uncertainty principle

Conclusion

- We derived inequalities to experimentally identify superpositions of macroscopically distinguishable states;
- Can be applied to mixed states;
- Two-mode squeezed states violate the appropriate inequality, proving a superposition of the order of the standard deviation;
- Can be used as proof of macroscopic EPR correlations

→ *Either macroscopic realism is false or QM is incomplete*

Thank You!