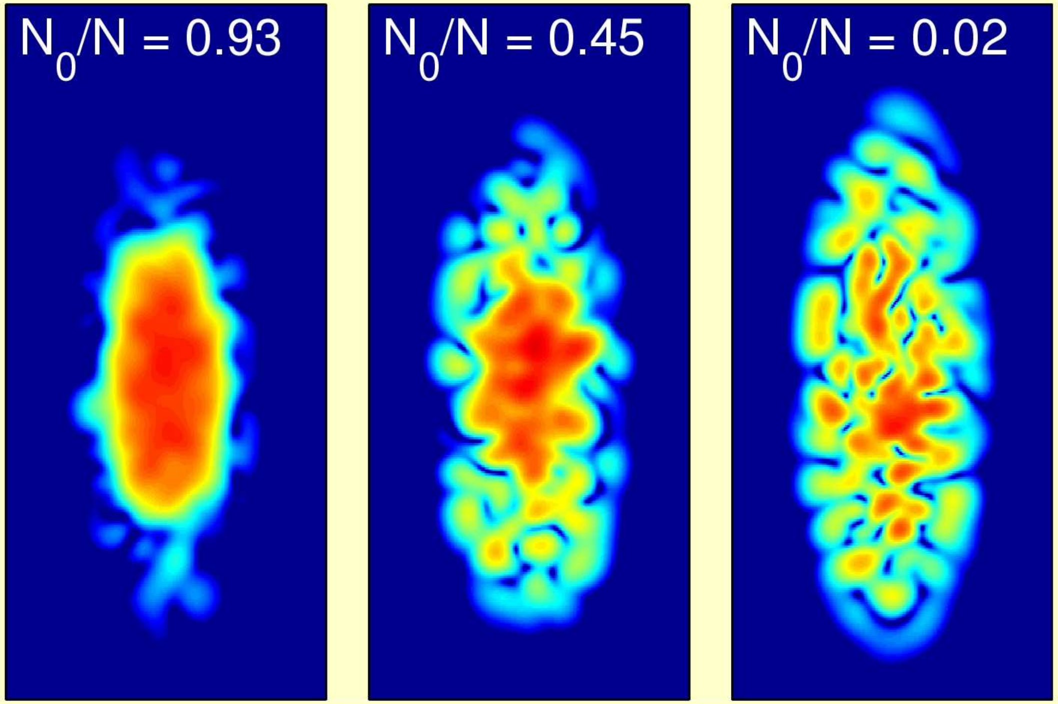
Applying the classical field method to experimental Bose condensed systems



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Overview

- Finite temperature Bose gases.
- Introduction to classical fields.
- Measuring condensate fractions.
- Shift in T_c for interacting Bose gases.
- Summary.

The challenge for theorists

Can we come up with a *practical* non-equilibrium formalism for finite temperature Bose gases?

Desirable features:

- Can deal with inhomogeneous potentials.
- Can treat interactions non-perturbatively.
- Calculations can be performed on a reasonable time scale (say under one week).

Potential applications

Topics of interest include:

- Condensate formation.
- Vortex lattice formation, dynamics
- Low dimensional systems (fluctuations important)
- Correlation functions
- Atom lasers . . .

Classical field approximation

An example: the classical theory of electromagnetic radiation resulted in the Rayleigh-Jeans law.

Based on the equipartion theorem :

- Each oscillator mode has energy $k_B T$ in equilibrium.

Lord Rayleigh

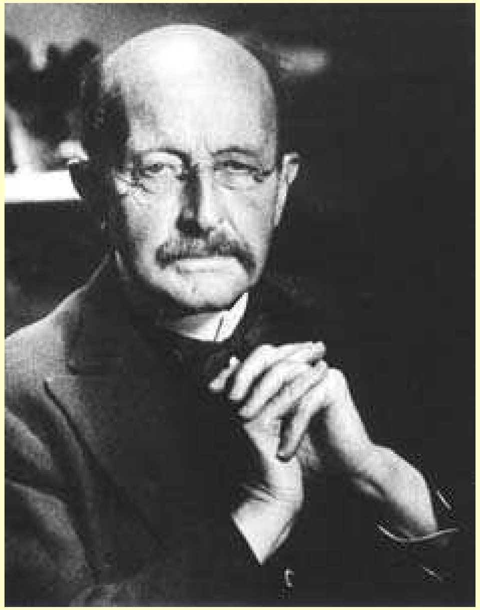
Sir James Jeans

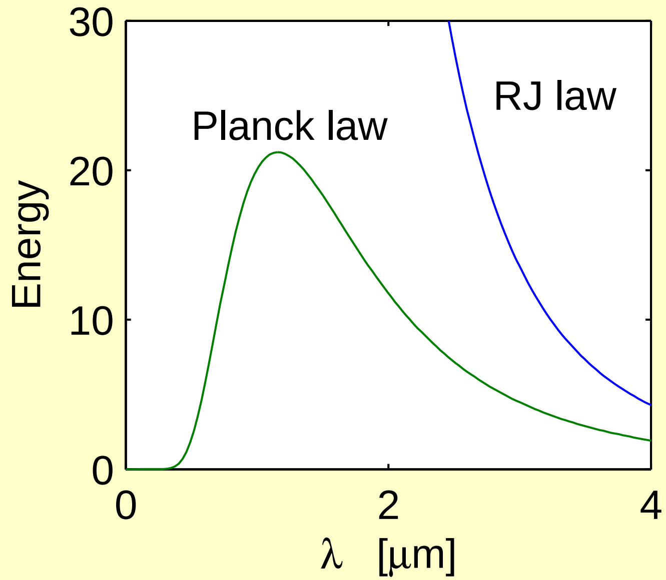
The UV catastrophe

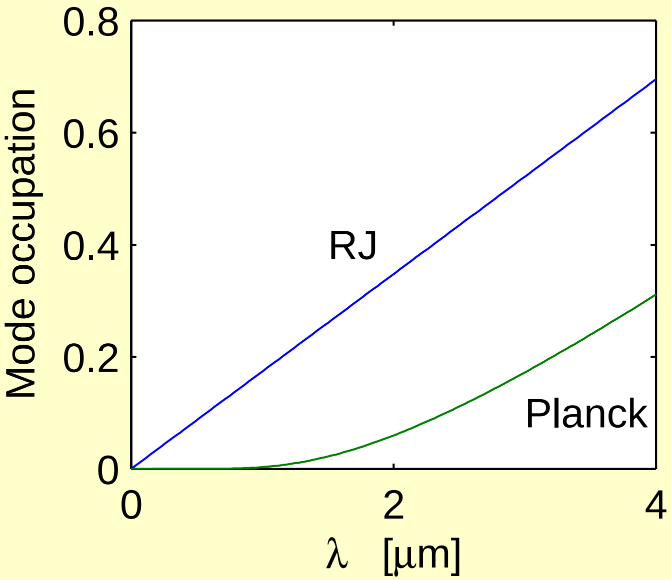
But we all know it doesn't work . . .

So Planck says:

"Classical fi elds are no good"







However . . .

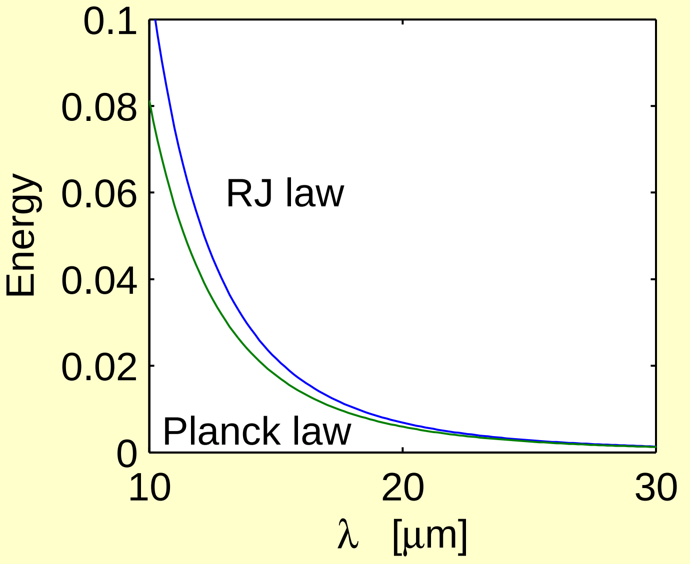
For the infra-red modes the RJ law is a good approximation.

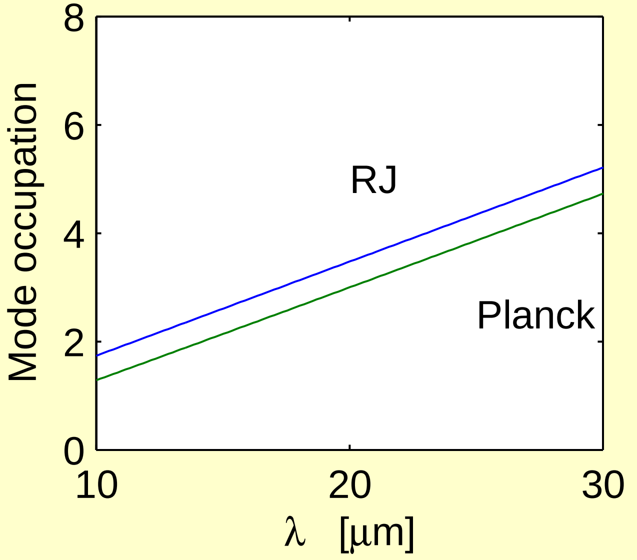
Quantum and classical results are similar for

$$E_{\text{photon}} \leq k_B T$$

Thus we require

- High occupancy per mode.
- An energy cutoff.





Classical fields for matter waves

The Projected Gross-Pitaevskii equation:

$$i\frac{d\psi(\mathbf{x})}{d\tau} = H_{\rm sp}\psi(\mathbf{x}) + C_{\rm nl}\mathcal{P}\left\{|\psi(\mathbf{x})|^2\psi(\mathbf{x})\right\}, \qquad C_{\rm nl} = \frac{8\pi a N}{L}.$$

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All modes assumed to be highly occupied.

Projection prevents higher energy modes becoming occupied :

$$\mathcal{P}\{F(\mathbf{x})\} = \sum_{k\in C}\phi_k(\mathbf{x})\int d^3\mathbf{x}'\ \phi_k^*(\mathbf{x}')F(\mathbf{x}')$$ — prevents UV catastrophe.

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Advantages: 1. Relatively easy (i.e possible!) to simulate in 3D.

2. Method is non-perturbative.

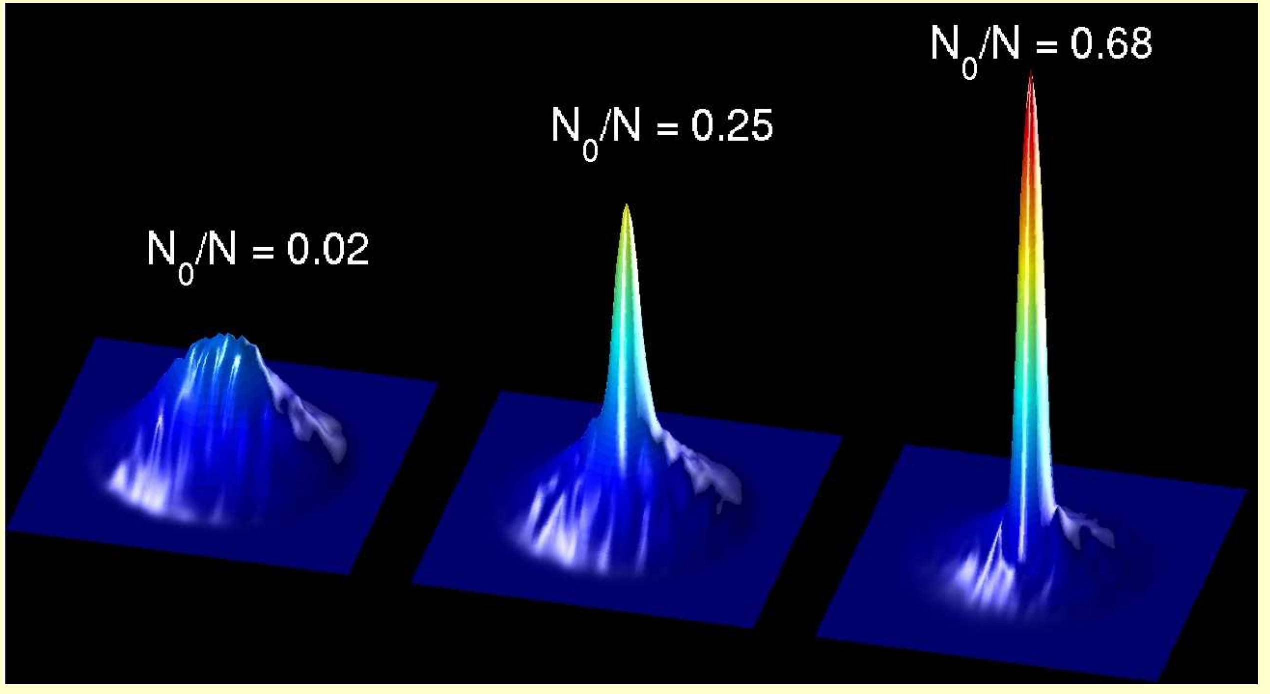
However: Atoms above cutoff necessary for real calculations.

Ergodicity

Begin simulations with random initial conditions

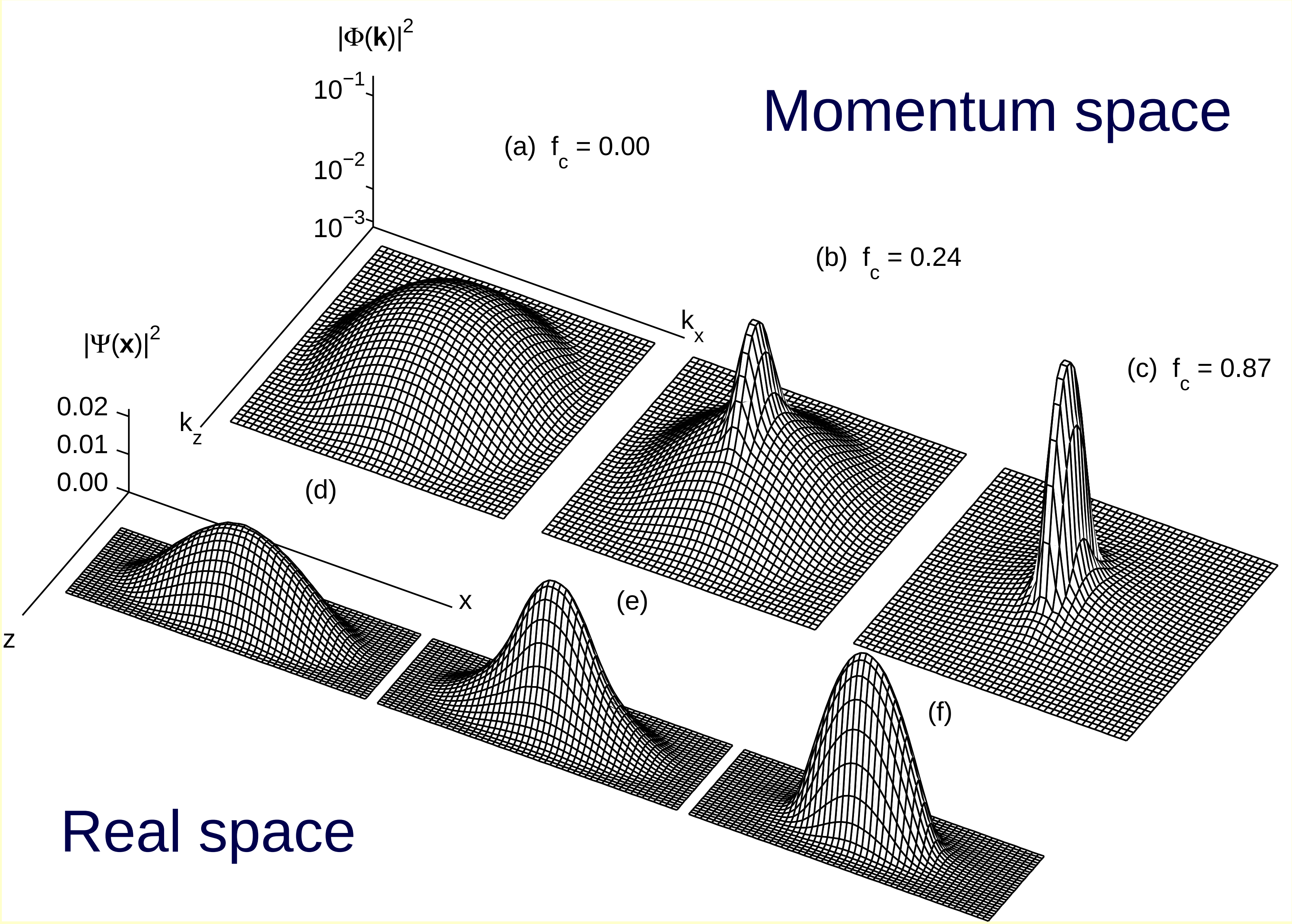
$\Rightarrow$ Result is **thermal equilbrium**

System is **ergodic**: time average $\equiv$ ensemble average



Time-averaged column densities in momentum space, TOP trap

Time-averaged column densities



Theorists' criterion for BEC: Penrose-Onsager

$\Rightarrow$ Single-particle density matrix has a macroscopic eigenvalue.

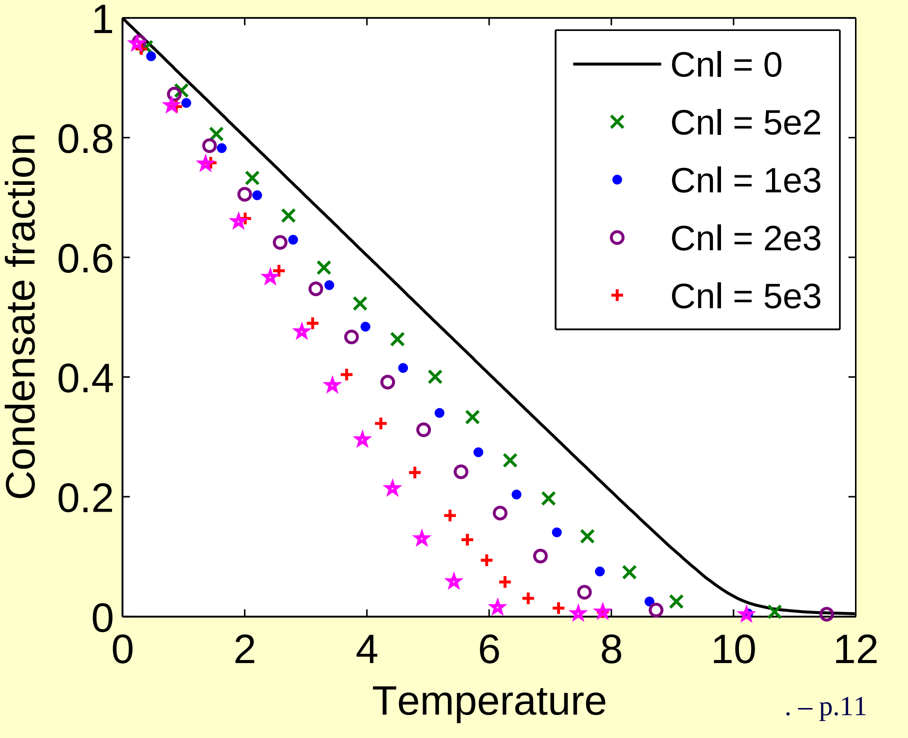
Given $\psi(\mathbf{x},t) = \sum_k c_k(t)\phi(\mathbf{x})$ we can calculate

$$\rho_{ij} = \langle c_i^* c_j \rangle \approx \lim_{T\to\infty} \frac{1}{T}\int_0^T c_i^*(t) c_j(t) dt$$

Typically have ~ 2000 states below cutoff

This can easily be diagonalized on a workstation

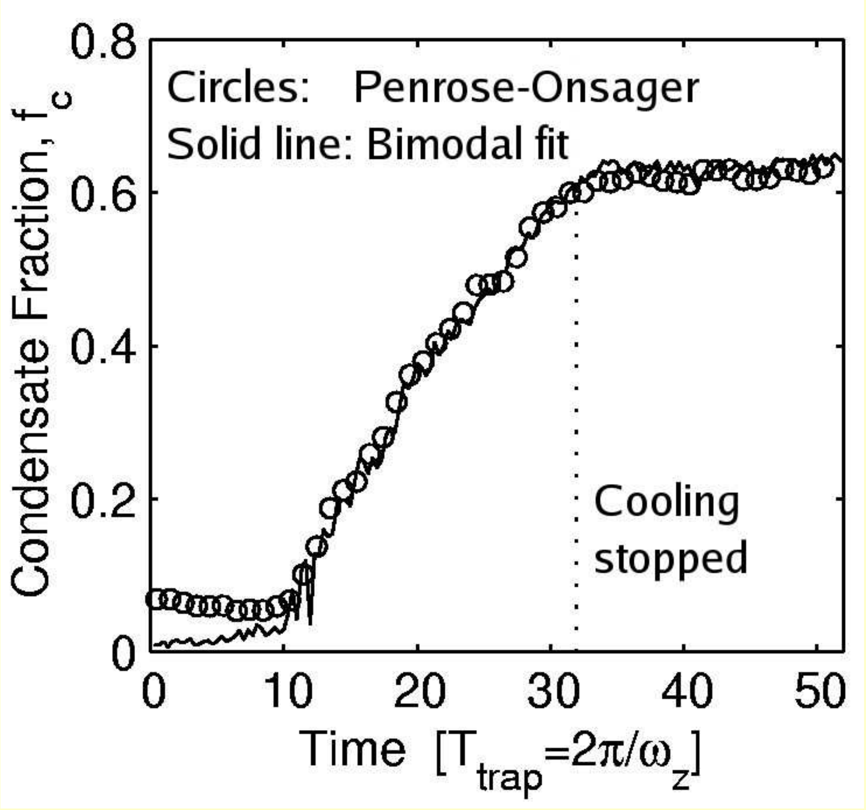
[Also have a microcanonical measure of temperature]

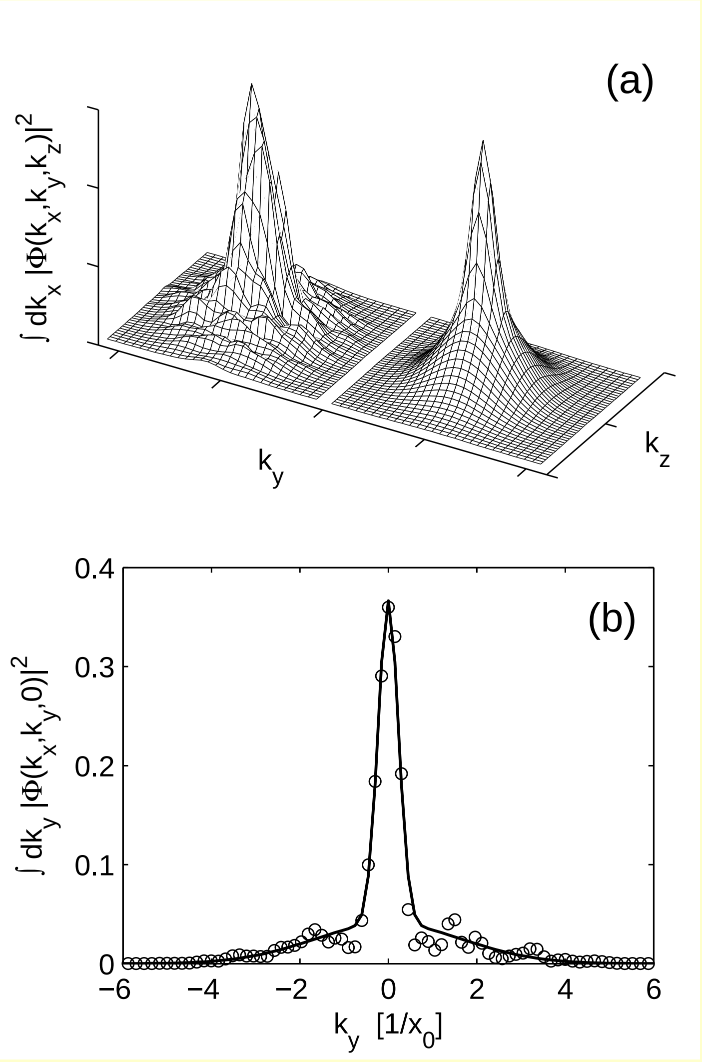


Experimentalists’ measure of BEC

⇒ Fit a bimodal distribution to column density.

Compare the two measures from an evaporative cooling calculation.





Shift in critical temperature with interactions

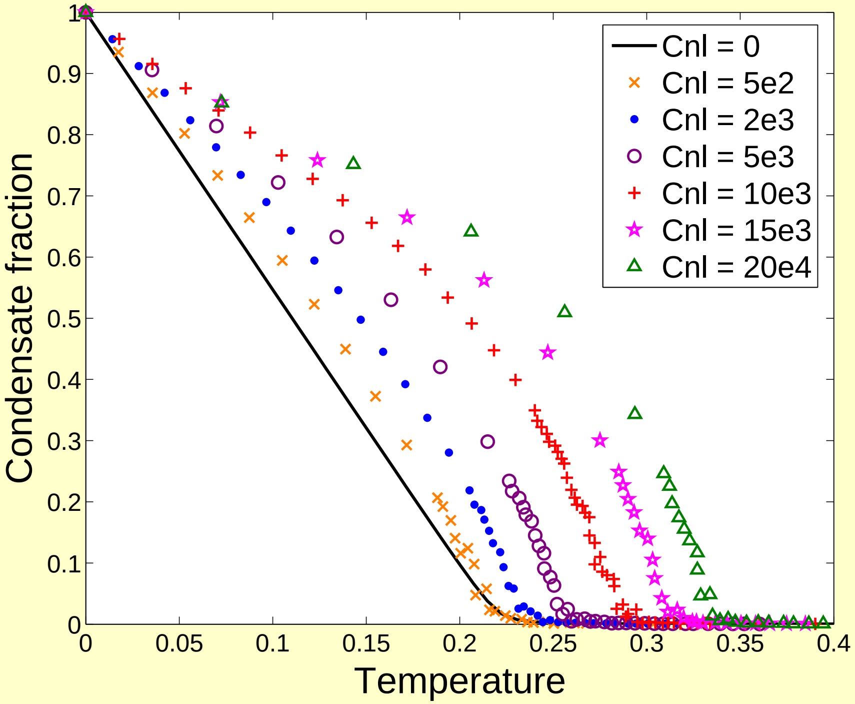
A diffi cult problem: perturbation theory doesn't work near T_c.

Several competing phenomena:

- Finite size effects (downwards)
- Mean fi eld effects (downwards)
- Critical fluctuations (upwards)

Homogeneous gas, thermodynamic limit: $\Delta T_c / T_{c0} = c a n^{1/3}$.

We fi nd $c = 1.3 \pm 0.4$ — agrees with Monte Carlo calculations.



P. Arnold and G. Moore, PRL **87**, 120401 (2001);
V. A. Kashurnikov et al., PRL **87**, 120402 (2001).

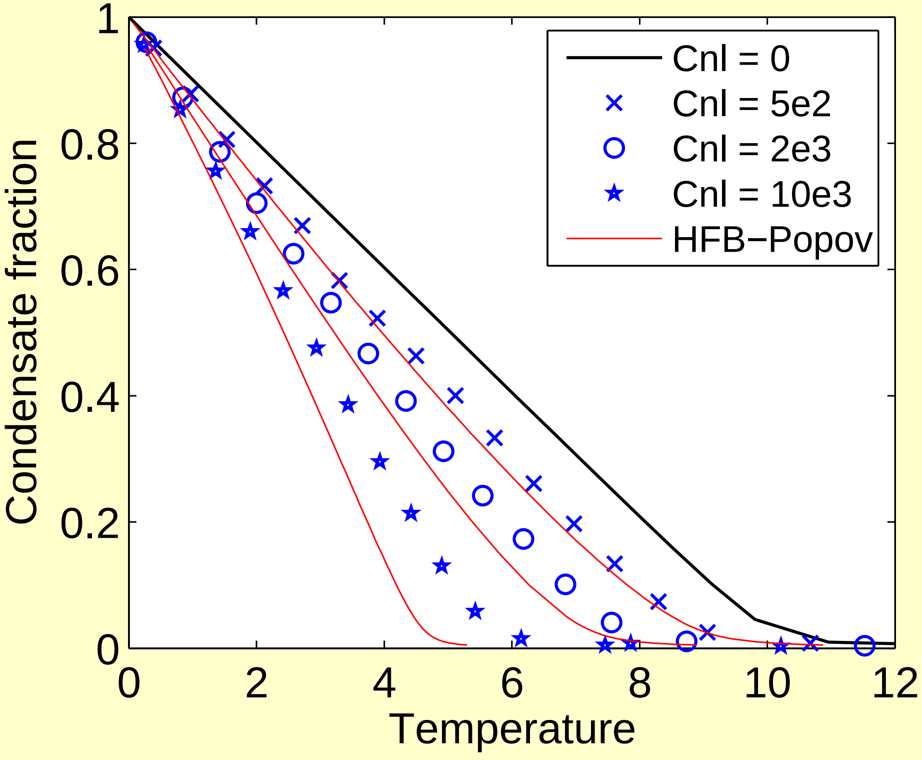
Critical temperature for trapped gas

Giorgini et al. estimate downwards shift in T_c due to mean fi eld.

$$\frac{T_c}{T_{c0}} \approx 1.33 N^{1/6} \frac{a}{a_{ho}}.$$

Are critical fluctuations important?

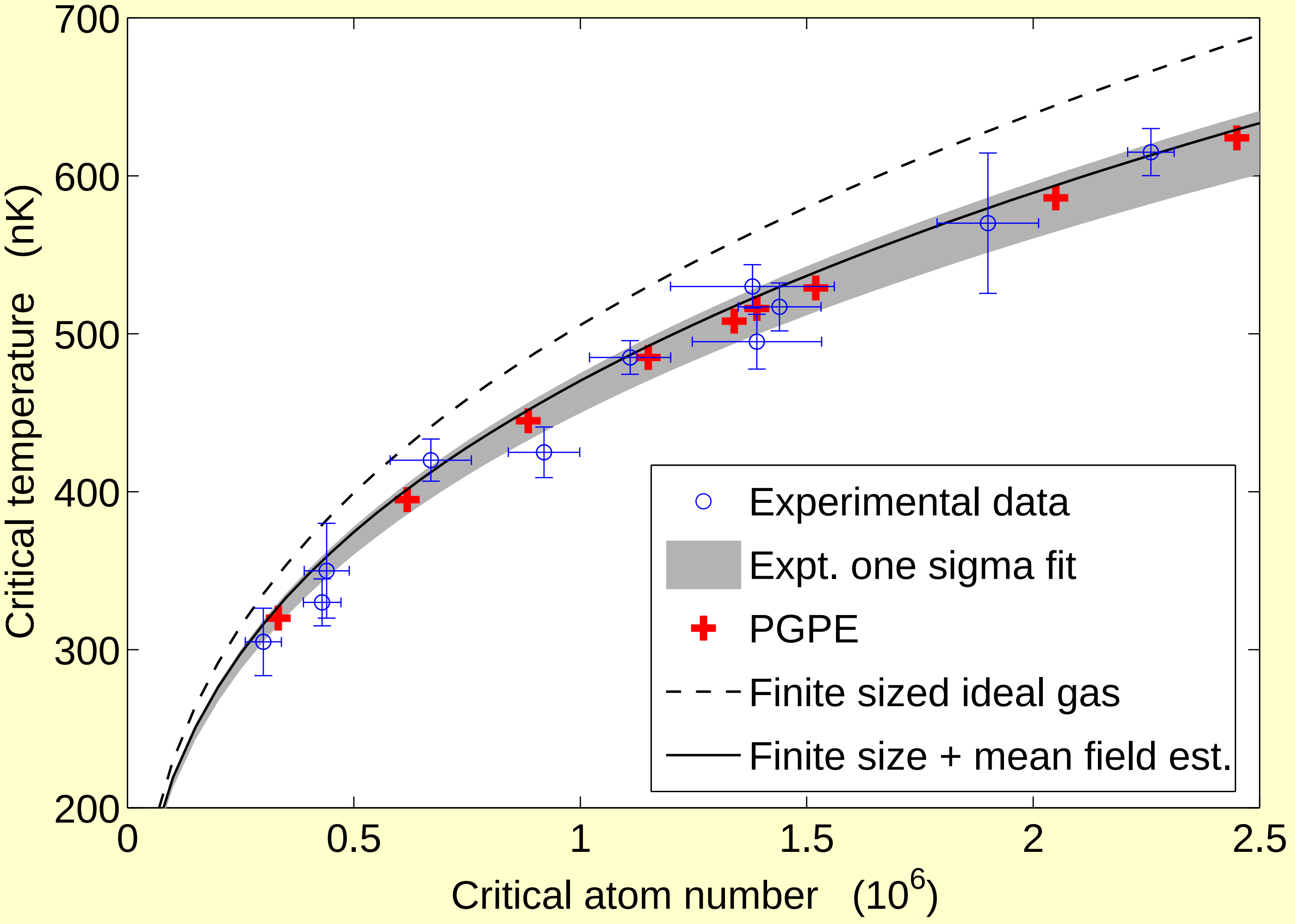
We compare PGPE calculations for a TOP trap to mean-fi eld HFB-Popov calculations for the same basis set.



Answer: maybe!

Comparison with experiment

Careful measurements by Gerbier et al. Phys. Rev. Lett. **92**, 030405 (2004).



Analytic result is an estimate of mean fi eld shift. We will calculate this numerically.

Other current topics

- Formation of vortices at the phase transition:

  $\Rightarrow$ Kibble / Zurek scenario for BECs?

  $\Rightarrow$ Only phenomenological time-dependent Landau-Ginzburg theory to date.

- Vortices in 2D

  $\Rightarrow$ Pairing / Kosterlitz-Thouless transition?

- Trapped Bose gases with angular momentum.

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That's all, folks!