Matter-wave amplification via dissociation of a molecular BEC

Karén Kheruntsyan

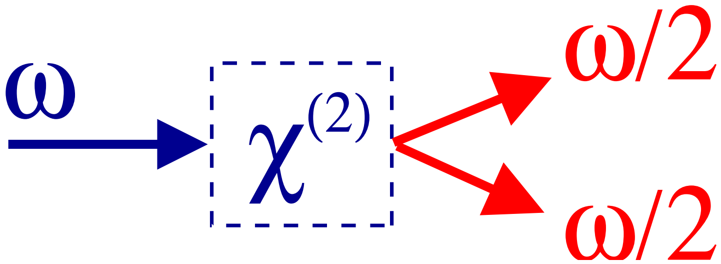
ARC Centre for Quantum-Atom Optics,
University of Queensland, Brisbane, AUSTRALIA

Motivation and Outline

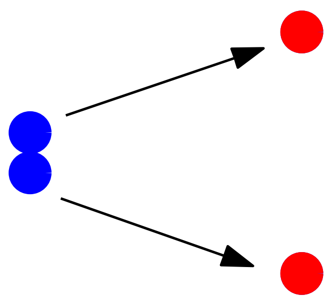
- Coherent control and manipulation of matter waves
- Explore analogies with nonlinear optics and quantum optics $\longrightarrow$ **nonlinear atom optics** and **quantum atom optics**
- Interested in quantum many-body **dynamics** (not in the ground states); **beyond the MFT**
- **Here: propose a scheme for parametric amplification and phase-conjugation of an atomic BEC via stimulated dissociation of a molecular BEC**
- Extend nonlinear atom optics from $\chi^{(3)}$ (four-wave mixing) domain to $\chi^{(2)}$ effects (parametric down-conversion)

Parametric down-conversion in optics

- A photon of frequency ω can be converted into a correlated pair of photons of lower frequencies:
  $\omega = \omega_1 + \omega_2$ ($\omega_1 \simeq \omega_2 \simeq \omega/2$)

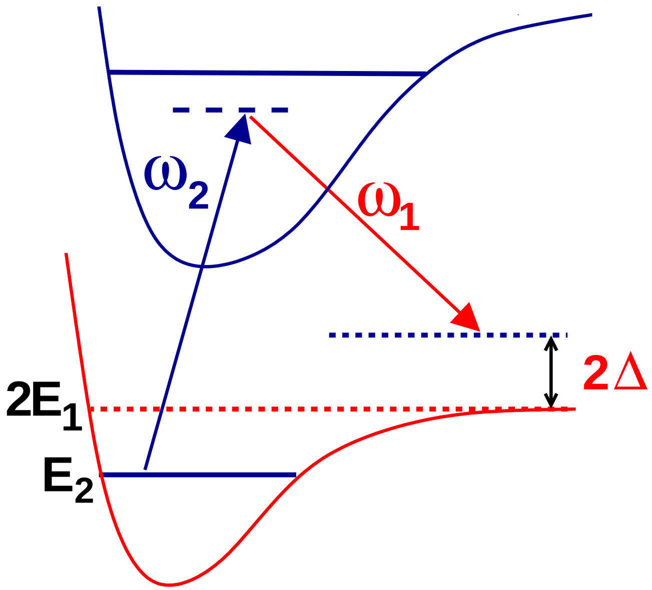


- Atom optics analog: a di-atomic molecule can dissociate into a correlated pair of atoms

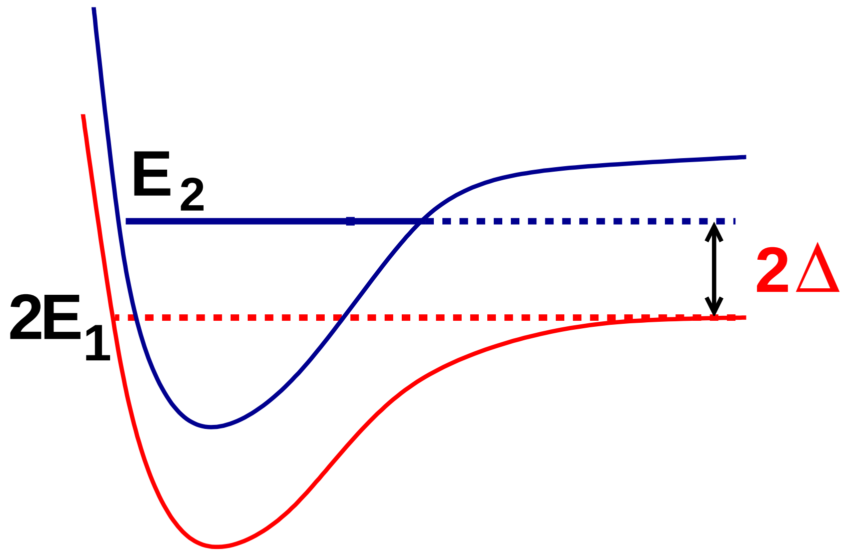


Molecule dissociation: Mechanisms

Raman photo-dissociation



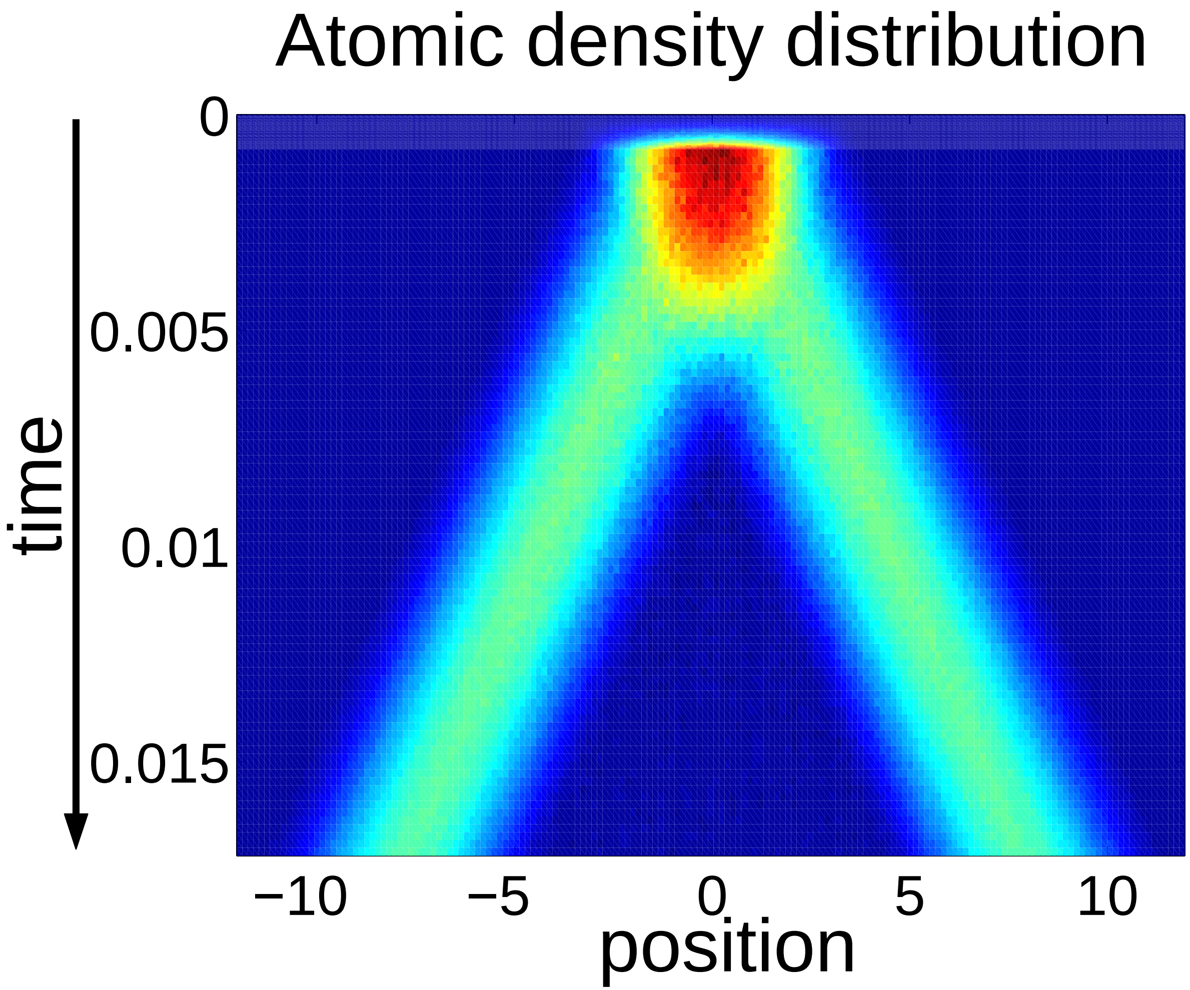
Feshbach resonance



- The excess of energy $2\hbar\Delta$ is released into the kinetic energy of correlated atom pairs ($0 = k_1 + k_2 \Longrightarrow k_1 = -k_2 \equiv k_0$):

$$2\hbar\Delta = 2\frac{\hbar^2 k_0^2}{2m_1} \quad \Longrightarrow k_0 = \pm\sqrt{2m_1\Delta/\hbar}$$

Dissociation of a BEC of molecular dimers (1D)

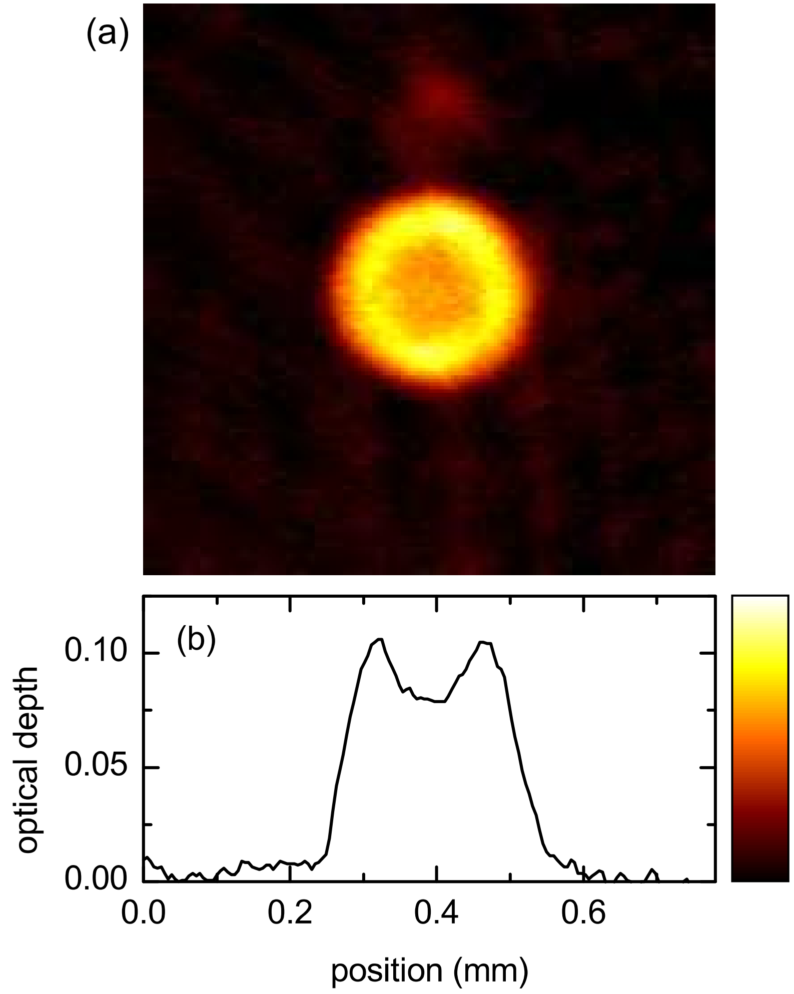


- Twin (number-correlated) atom laser beams [Kheruntsyan, Drummond, PRA **66**, (R)031602 (2002)]

First experiments using Feshbach resonance

- Dissociation of $^{87}Rb_2$ dimers via rapid crossing through the resonance
- Mono-energetic wave of atoms on the surface of a hollow sphere
- **Our proposal: do this in a cigar-shaped trap** – obtain directionality due to gain-guiding

[Rempe, PRA **70** (2004)]



Why amplification?

- Molecular dimers made of bosonic atoms are **short lived**; spontaneous dissociation (no atoms present initially) may have practical limitations

- Can we speed up molecule dissociation, and still get correlated atomic beams?

- **Proposal: use a "seed" (small) atomic BEC – gives matter-wave parametric amplification and phase conjugation; correlation can still be strong**

Effective quantum field theory (in 1D)

$$H_0 = \sum_{i=1,2} \int dx \left[\frac{\hbar^2}{2m_i} |\nabla \hat{\Psi}_i|^2 + \hbar\Delta \hat{\Psi}_1^\dagger \hat{\Psi}_1 + \hbar V(x) \hat{\Psi}_2^\dagger \hat{\Psi}_2 \right]$$

$$H_s = \sum_{ij} \frac{\hbar U_{ij}}{2} \int dx \hat{\Psi}_i^\dagger \hat{\Psi}_j^\dagger \hat{\Psi}_j \hat{\Psi}_i$$

$$H_{M \rightleftarrows A+A} = \frac{\hbar\chi}{2} \int dx \left[\hat{\Psi}_2^\dagger \hat{\Psi}_1 \hat{\Psi}_1 + H.c. \right]$$

- $\hat{\Psi}_{1,2}(t,x)$ – atomic/molecular field operators
- $2\hbar\Delta$ – energy detuning, $V(x)$ – trapping potential
- U_{ij} – s-wave scattering interactions
- χ – **atom-molecule coupling** ($M \rightleftarrows A + A$)

Approximations

- Limit to short dissociation times: number of atoms produced small (~ 150); can neglect atom-atom scattering U_{11}
- Large detuning: $\Delta \gg |U_{12}| \left\langle \hat{\Psi}_2^\dagger \hat{\Psi}_2 \right\rangle$; atom-molecule s-wave scattering U_{12} is also negligible
- Molecular condensate is initially in a coherent state
- **We do take into account:**
  – molecule-molecule s-wave scattering, U_{22}
  – molecular field depletion
  – non-uniform multi-mode structure
  – possible one-body losses of atoms and molecules

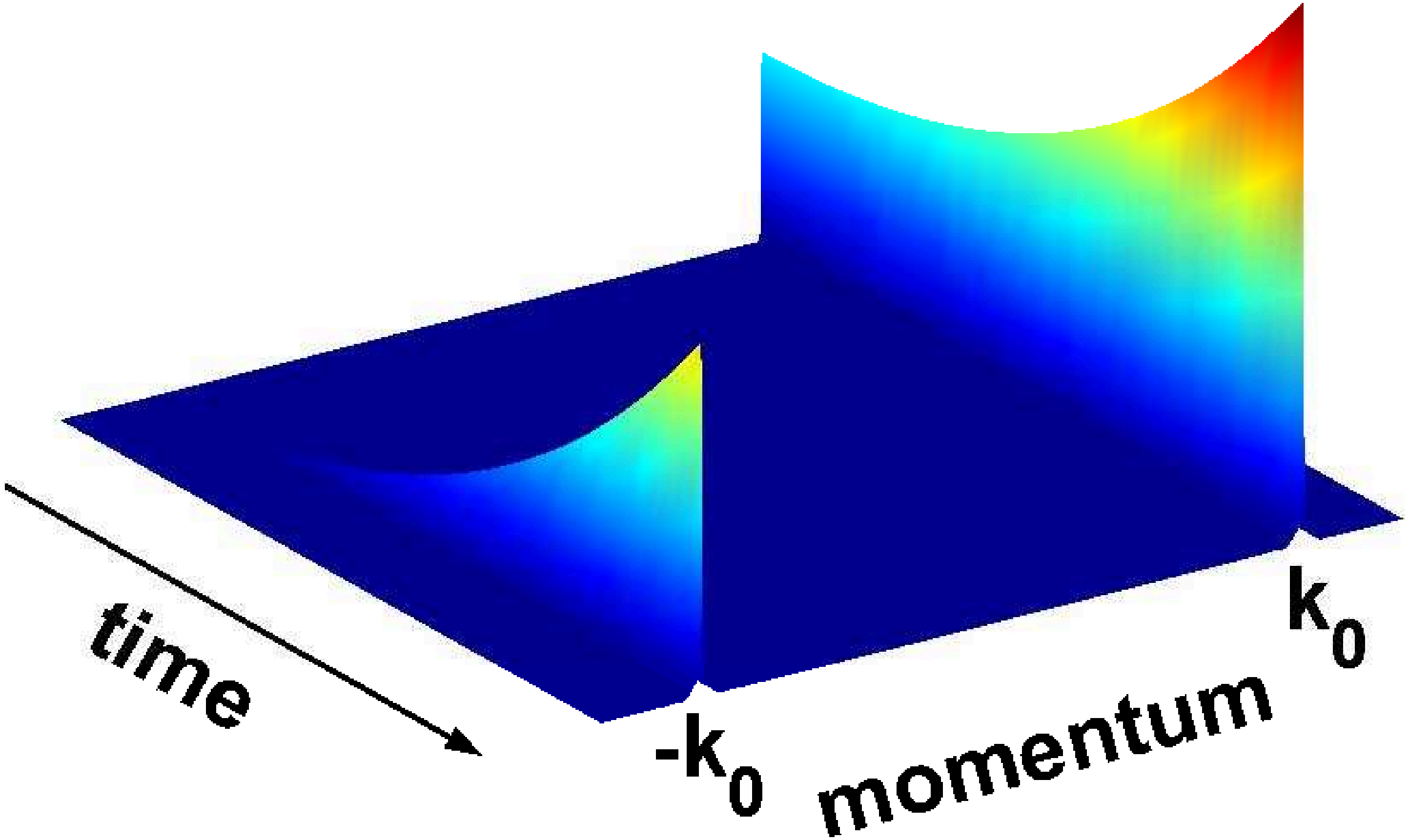
Positive-P stochastic equations

$$\frac{\partial\psi_1}{\partial\tau} = i\frac{\partial^2\psi_1}{\partial\xi^2} - (\gamma_1 + i\delta)\psi_1 + \kappa\psi_2\psi_1^+ + \sqrt{\kappa\psi_2}\eta_1,$$

$$\frac{\partial\psi_2}{\partial\tau} = \frac{i}{2}\frac{\partial^2\psi_2}{\partial\xi^2} - \left[\gamma_2 + iV_2(\xi) + iu_{22}\psi_2^+\psi_2\right]\psi_2 - \frac{\kappa}{2}\psi_1^2 + \sqrt{-iu_{22}}\psi_2\eta_2,$$

- plus two more equations like these, for ψ_1^+ and ψ_2^+
- $\eta_i(\xi,\tau)$ – independent δ-correlated Gaussian noises
- $\left\langle (\hat{\Psi}_i^\dagger)^m (\hat{\Psi}_j)^n \right\rangle = \left\langle (\Psi_i^+)^m (\Psi_j)^n \right\rangle_{\mathrm{stoch}}$ over many stochastic trajectories
- To solve these SDEs: **Go XMDS !** – http://www.xmds.org

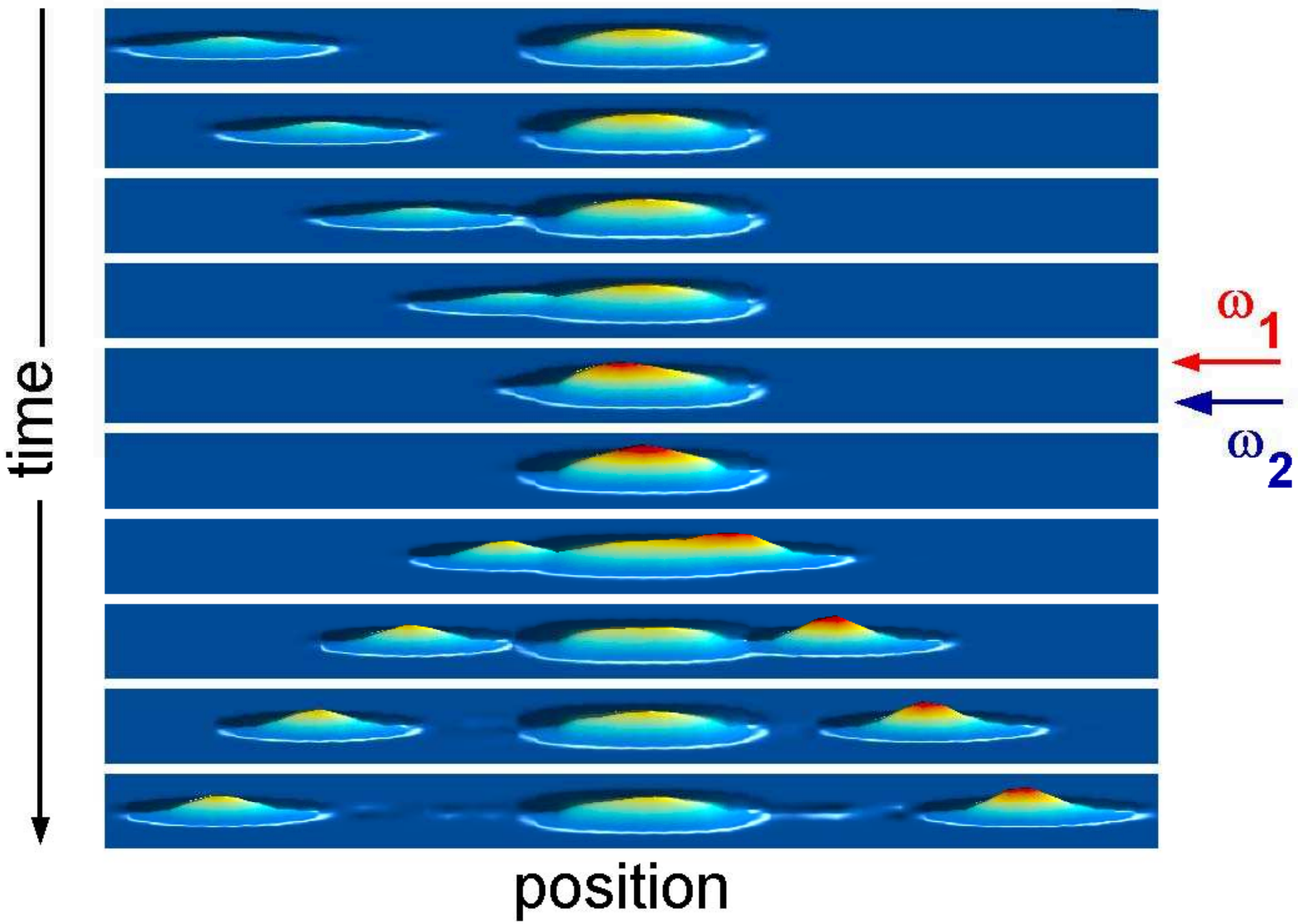
Atomic momentum distribution



$$\pm k_0 = \pm\sqrt{2m_1\Delta/\hbar}$$

Density distribution: Amplification and phase-conjugation

"Seed" BEC Molecular BEC



Atom number correlation

- Define atom number operators for the "right" and the "left" beams, $\hat{N}_+$ and $\hat{N}_-$:

$$\hat{N}_{+(-)}(\tau) = \int_{0(l/2)}^{l/2(0)} \hat{\psi}_1^\dagger(\xi,\tau)\hat{\psi}_1(\xi,\tau)d\xi$$

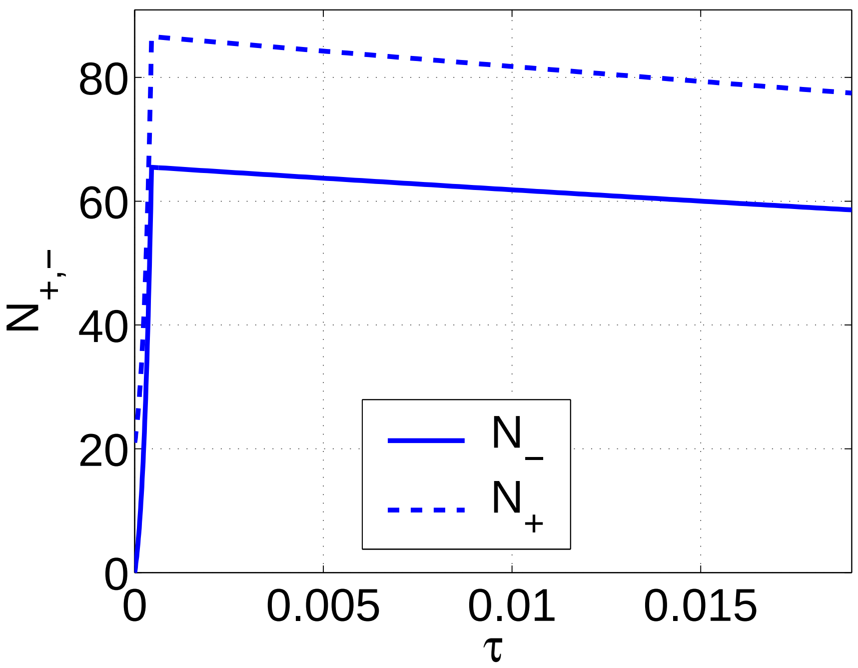
- Fluctuations in the atom number difference $(\hat{N}_+ - \hat{N}_-)$ can be characterized by the normalized **variance**:

$$V(\tau) = \frac{\langle(\hat{N}_+ - \hat{N}_-)^2\rangle - (\langle\hat{N}_+\rangle - \langle\hat{N}_-\rangle)^2}{\langle\hat{N}_+\rangle + \langle\hat{N}_-\rangle}$$

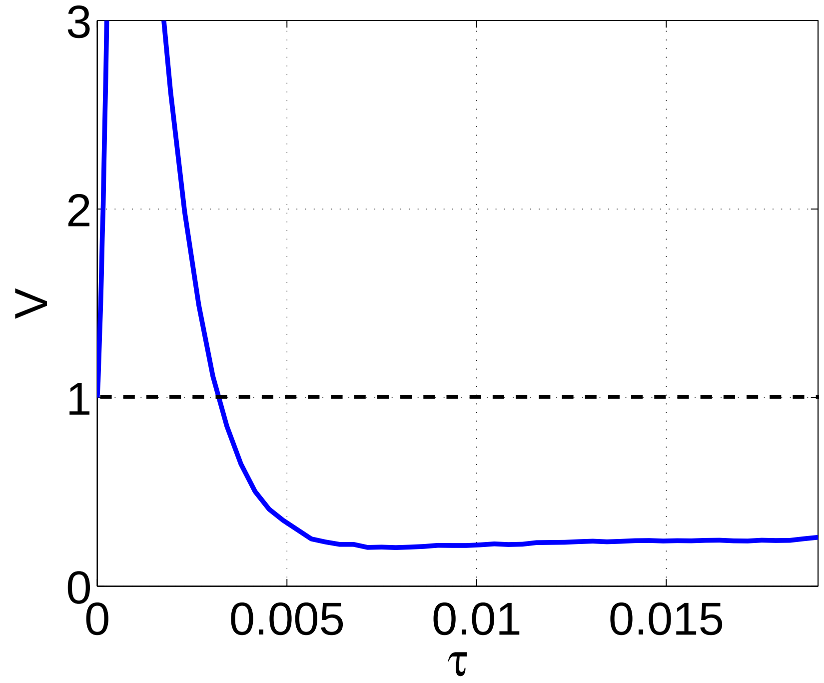
- $V < 1$ – **squeezing** due to strong nonclassical correlation between $\hat{N}_+$ and $\hat{N}_-$ [for coherent states, $V = 1$]

Results

Total number in N_+ and N_-



Variance in $N_+ - N_-$



- Final variance: $V(\tau_f) \simeq 0.25$, or $\sim 75\%$ squeezing
- Obtained on much shorter time scales than in spontaneous dissociation

Summary

- Scheme for matter-wave amplification and phase-conjugation
- Gives two correlated atomic beams with squeezed atom number-difference fluctuations
- Required timescales are shorter than in spontaneous dissociation (can make use of short-lived molecules)
- The two beams possess EPR correlations in quadratures, with large numbers of massive particles [see poser by Murray Olsen]

[cond-mat/0406209; cond-mat/0407363]