Nonlinear localization of BEC and vortices in optical lattices

Yuri Kivshar, Elena Ostrovskaya, Tristram Alexander,
Pearl Louis, and Beata Dabrowska

Nonlinear Physics Centre and ACQAO,
Australian National University,
Canberra, Australia

http://wwwrsphysse.anu.edu.au/nonlinear

Outline

- Nonlinearity vs periodic potential: introduction
- BEC solitons with a “twist” - gap vortices in 2D lattices: structure, generation, detection

Model

$$i\frac{\partial \Psi}{\partial t} + \nabla_{\perp}^{2}\Psi - V_L(x,y)\Psi - g_{2D}|\Psi|^2\Psi = 0$$

- Length, energy, time measured in “lattice units”:

$$a_L = d/\pi = k_L^{-1}, \quad E_L = \hbar^2 k_L^2/(2m), \quad t_L = \hbar/E_L$$

- Lattice potential:

$$V_L(x,y) = V_0(\sin^2 x + \sin^2 y)$$

- Pancake geometry:

$$\Psi_{3D} = \Psi(x,y)\psi_{ho}(z)$$

- Repulsive interactions

Linear matter-waves in a 2D lattice

$$i\frac{\partial \Psi}{\partial t} + \nabla_{\perp}^{2}\Psi - V_{\mathrm{L}}(x,y)\Psi = 0$$

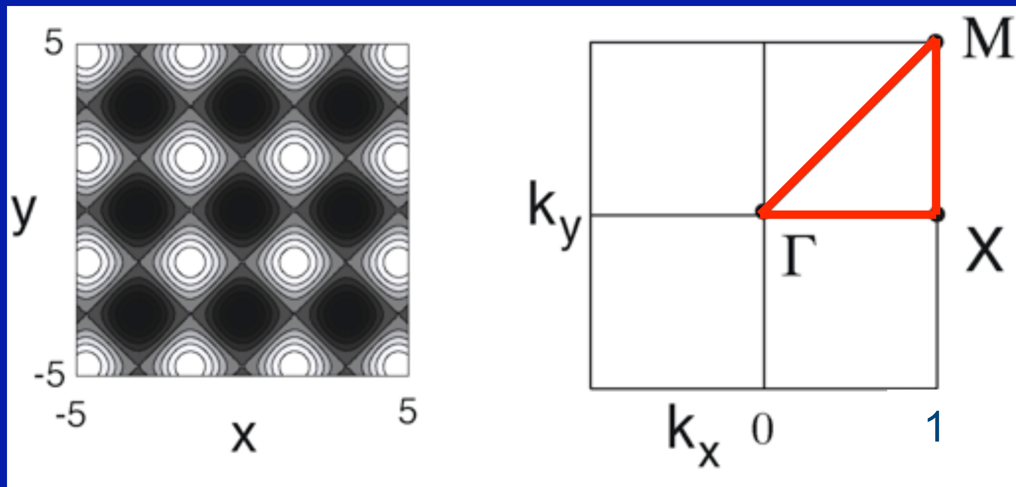
stationary solutions - Bloch waves:

$$\Psi(\mathbf{r};\mu) = B_{\mathbf{k}}(\mathbf{r};\mu)\exp(i\mathbf{kr})\exp\{-i\mu(\mathbf{k})t\}$$

periodic function

quasimomentum

chemical potential

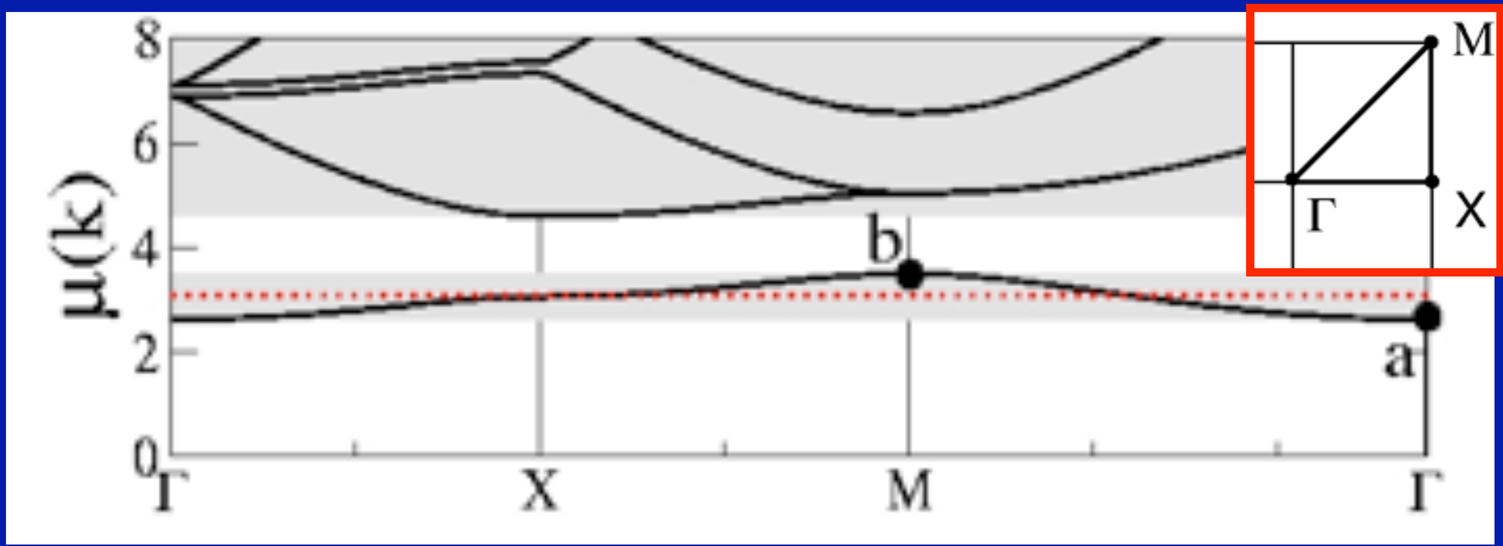


lattice potential

1st BZ

Bloch wave band-gap spectrum

$V_0 = 3$

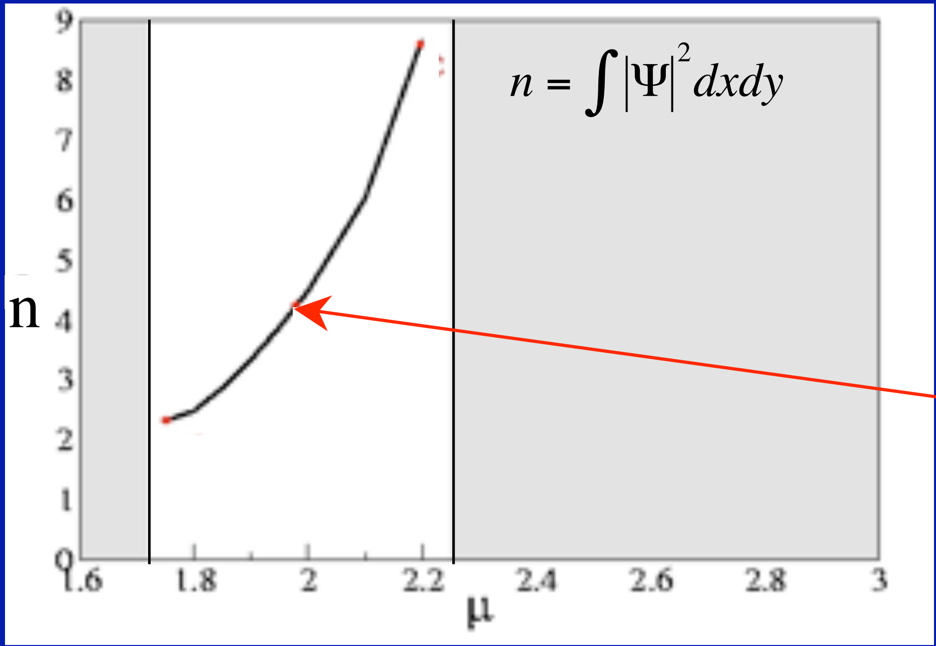


Effective diffraction of a BEC wavepacket:

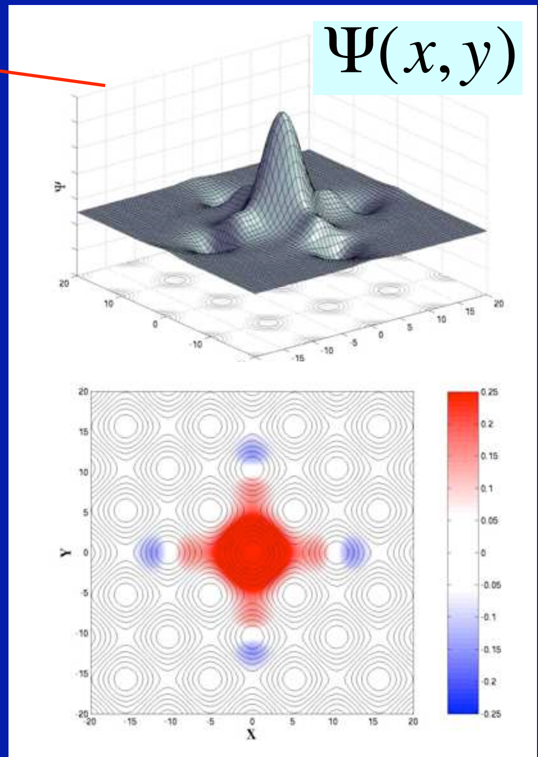
Point **a**: $D_{x,y} = \partial^2 \mu / \partial \mathbf{k}^2 > 0$ normal

Point **b**: $D_{x,y} = \partial^2 \mu / \partial \mathbf{k}^2 < 0$ anomalous

Effect of repulsive nonlinearity



- Self-induced defect states appear in the complete spectral gap

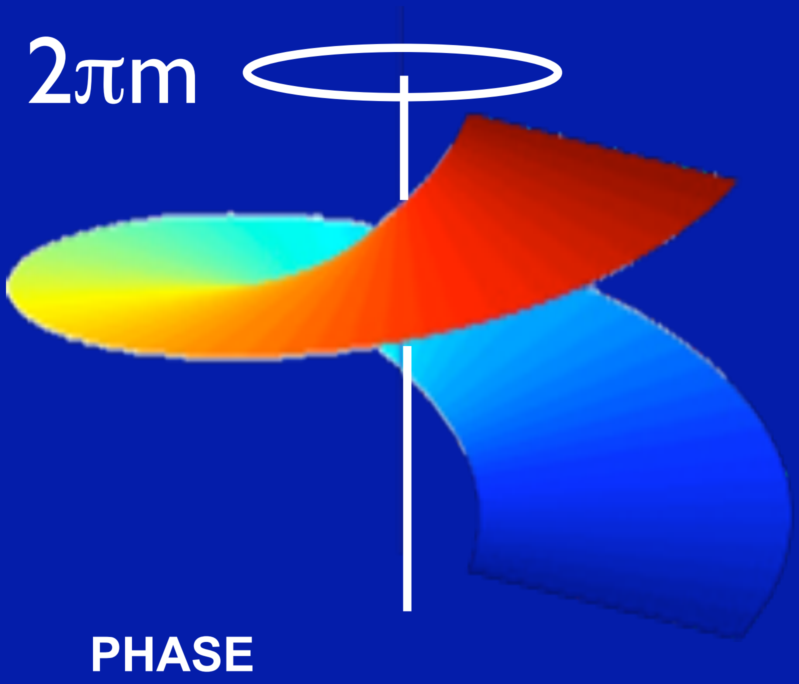


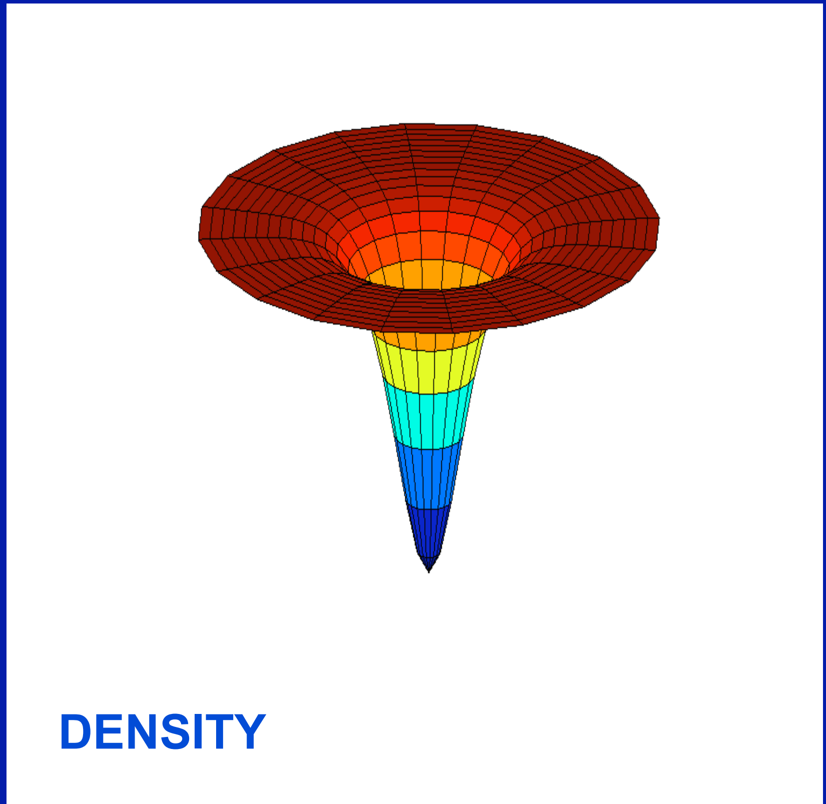
- The balance of repulsive nonlinearity and anomalous effective diffraction supports **bright gap solitons**

Pu et al., PRA, 67,43605 (2003);
Ostrovskaya & Kivshar, PRL, 90, 160407 (2003)

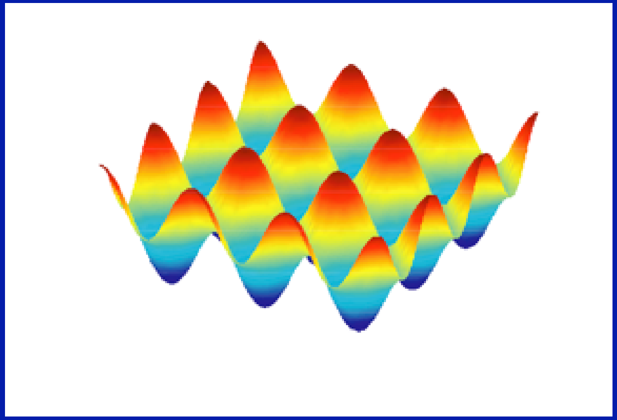
BEC vortex

- Topologically nontrivial state
- Low density core
- Screw-like phase dislocation
- Quantized circulation



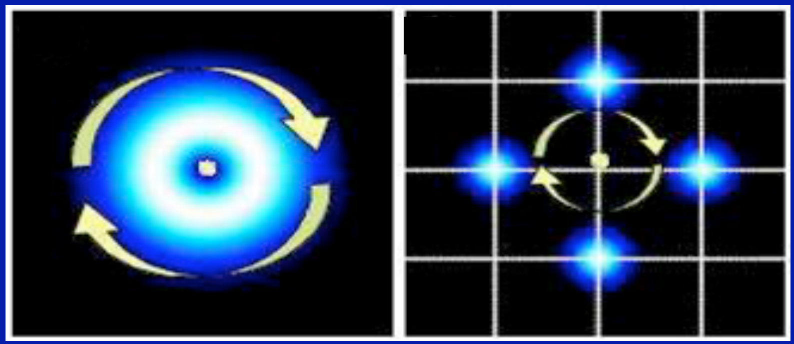


Can vortices exist in a lattice?

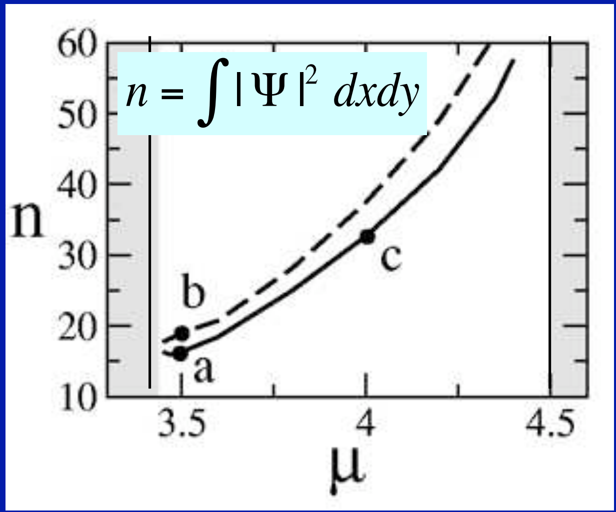


- BEC in a 2D optical lattice
- angular momentum not conserved
- will a vortex survive?

- the answer is “yes”
- circular energy flow persists
- density strongly modulated by lattice



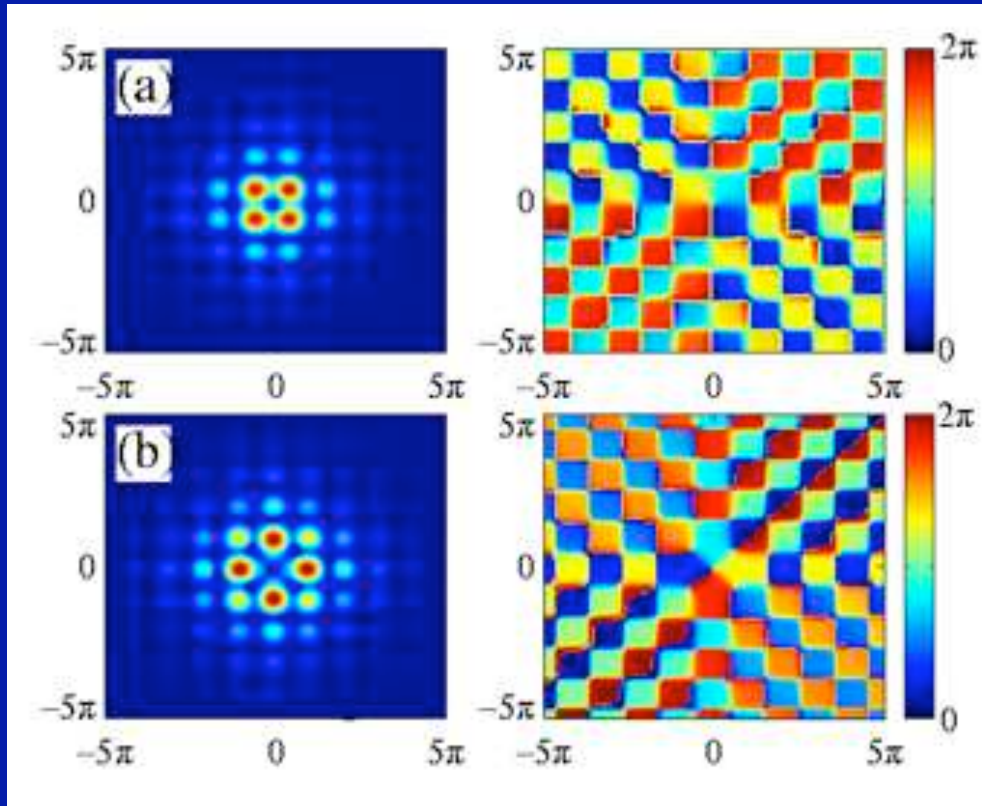
Localized in-gap vortices



Two symmetry types:

(a) off-site vortex

(b) on-site vortex



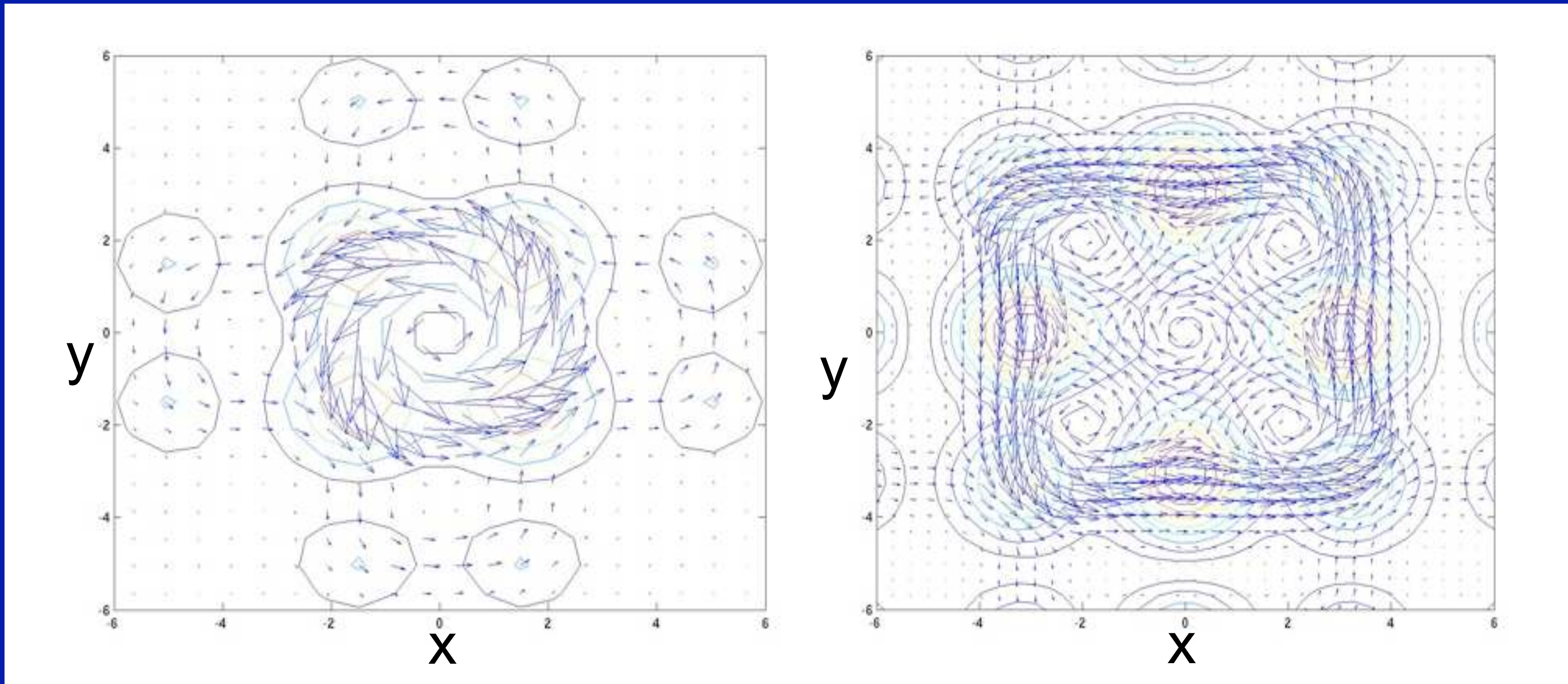
Key features:

- "Bright" localized core
- Phase singularity
- Dynamically stable

Ostrovskaya & Kivshar, PRL, 93, 160405 (2004)

Structure of singularity

$$j = \mathrm{Im}(\psi^{*}\nabla\psi)$$



Off-site

On-site

How to quantify vorticity?

Angular momentum: $\vec{M} = \mathrm{Im}\int \psi^*(\vec{r}\times\nabla\psi)d\vec{r}$

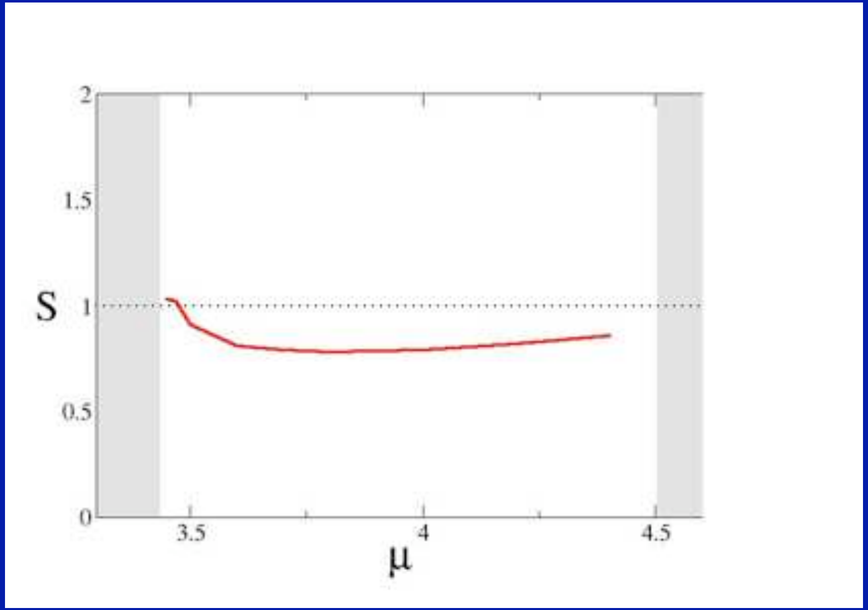
For a conventional vortex: $\psi = \psi_0(r)\exp(i\varphi)$

$$M = \int_0^\infty \psi_0^2 r dr \int_0^{2\pi} \frac{\partial\varphi}{\partial\theta} d\theta = nS$$

S **- integer spin or topological charge**

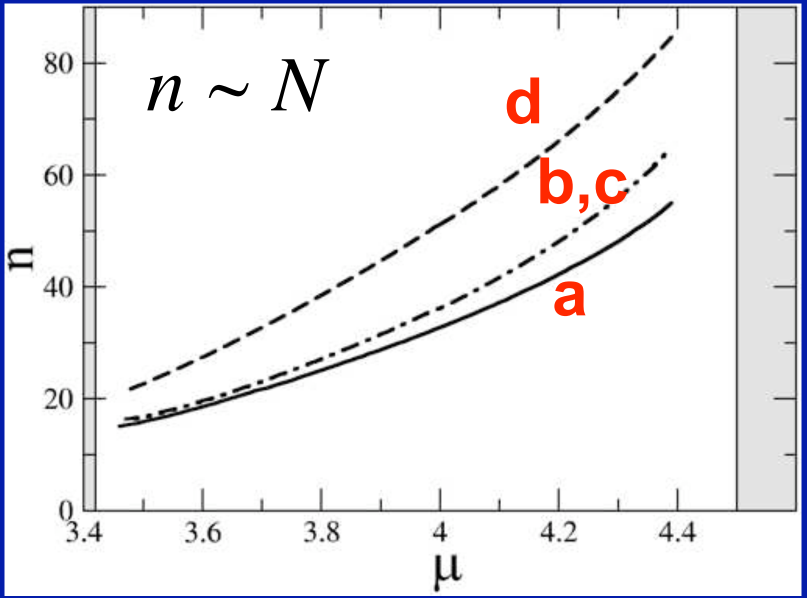
Spin of a gap vortex:

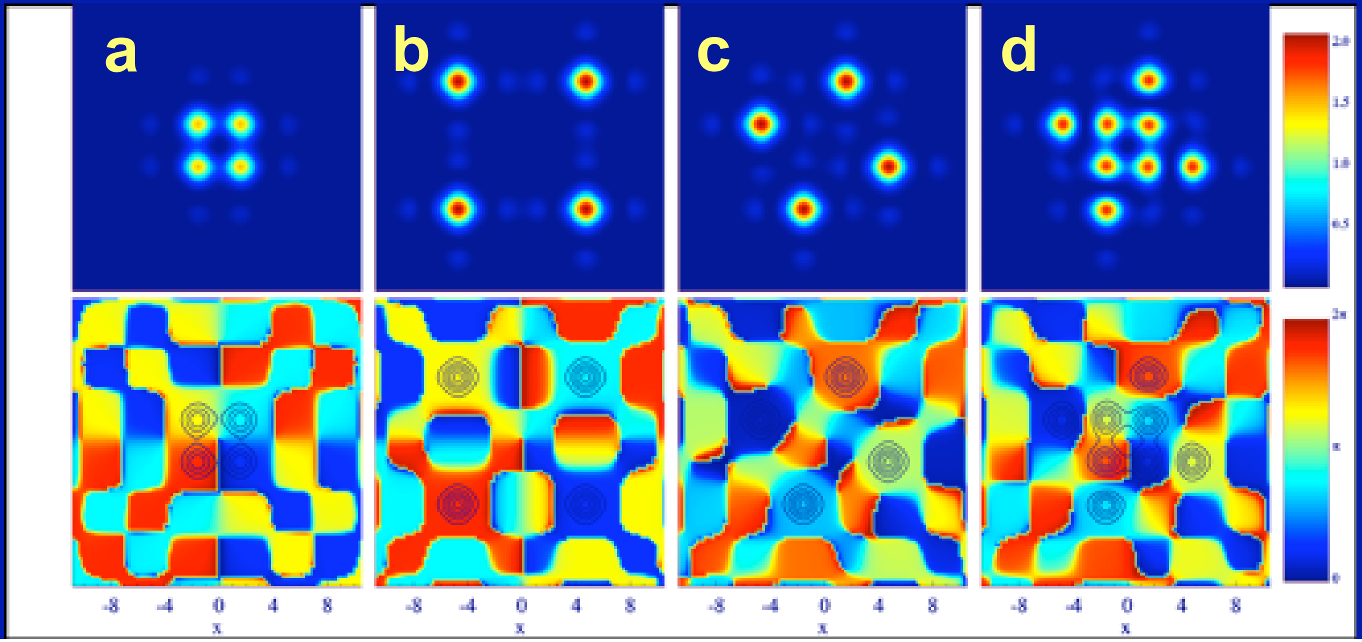
$$S = M/n, \quad n = \int |\psi|^2 d\vec{r}$$



Off-site vortex families

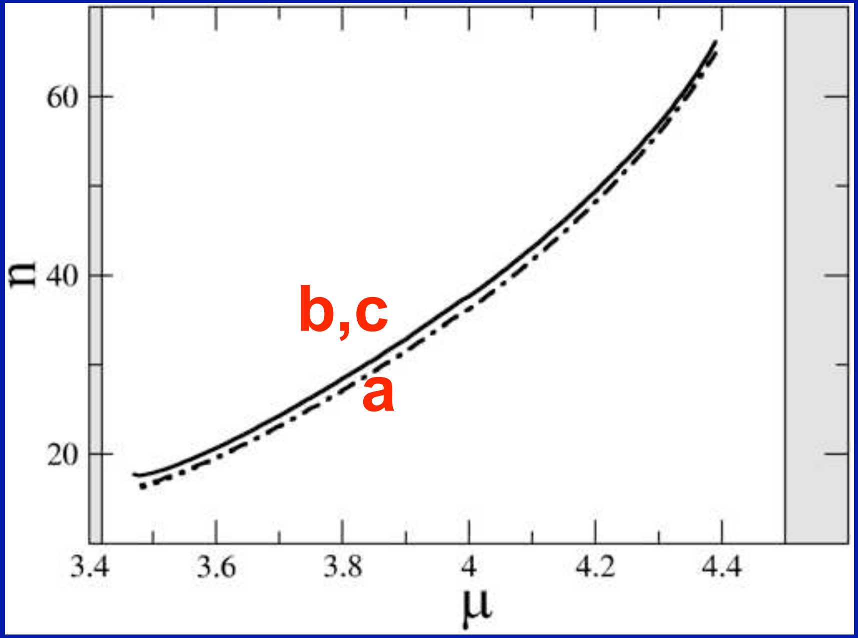
- Off-site vortices of different radii
- Almost degenerate in atom number

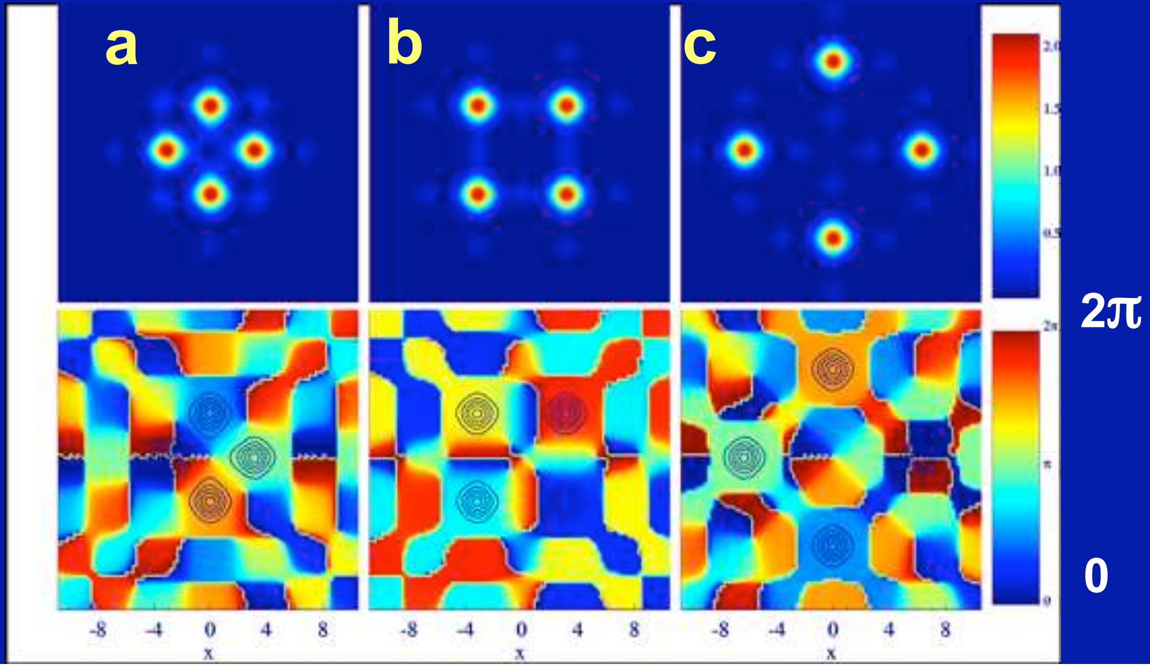




On-site vortex families

- On-site vortex families are even closer in atom numbers

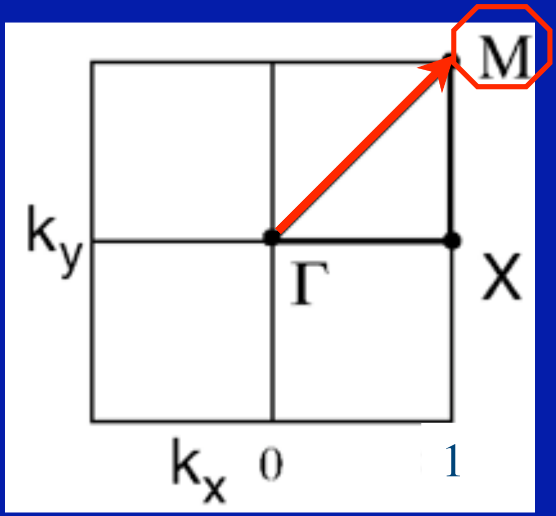




How to generate a gap vortex?

State preparation requirements:

- Negative effective diffraction regime $D_{x,y} < 0$
- Adiabatic drive to the Brillouin zone's edge $t >> \hbar / \Delta E_{gap}$
- Phase ramp imprinting



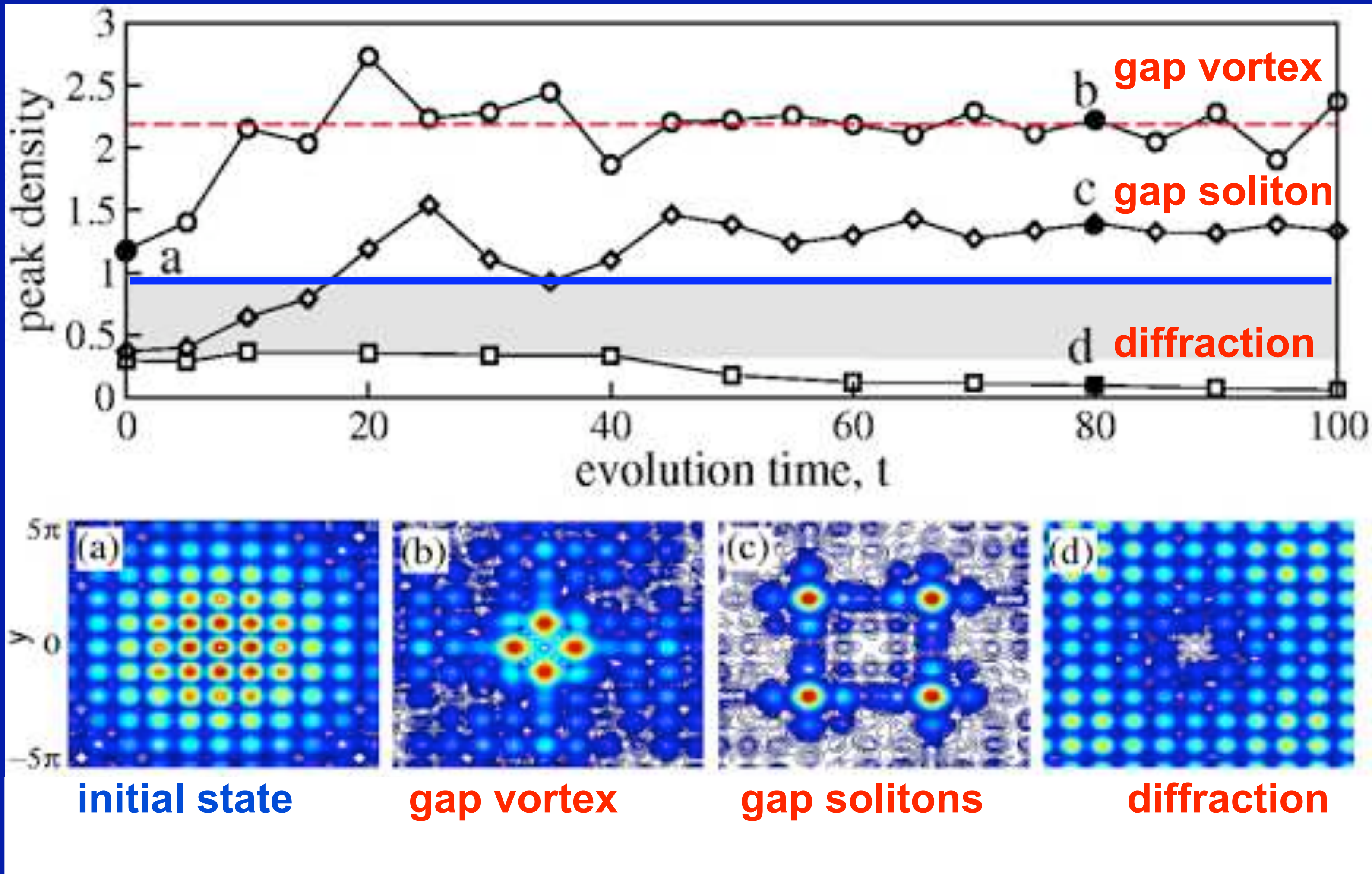
Simulation procedure:

- Start with a broad BEC wavepacket at the correct band edge (with a nontrivial Bloch-wave phase)
- Imprint a charge one vortex phase
- Let evolve in time

Dudarev et al., J.Opt.B, 6, S231 (2004)

Possible outcomes

- Necessary: **atom number** above vortex state threshold
- Sufficient: **peak density** above threshold



Dynamics below threshold

discrete diffraction at below threshold atom number

formation of bright gap solitons at below threshold peak density

Gap vortex generation

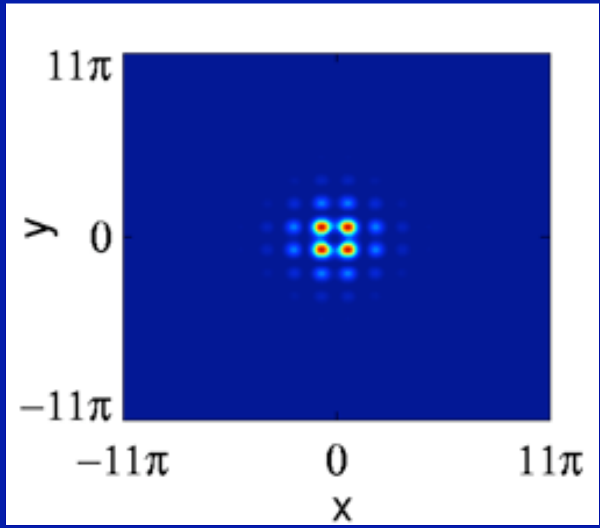
- Gap vortices exhibit exchange between states of different radii within the same symmetry group
- No cross-talk occurs between the different symmetry states

off-site gap vortices **on-site gap vortices**

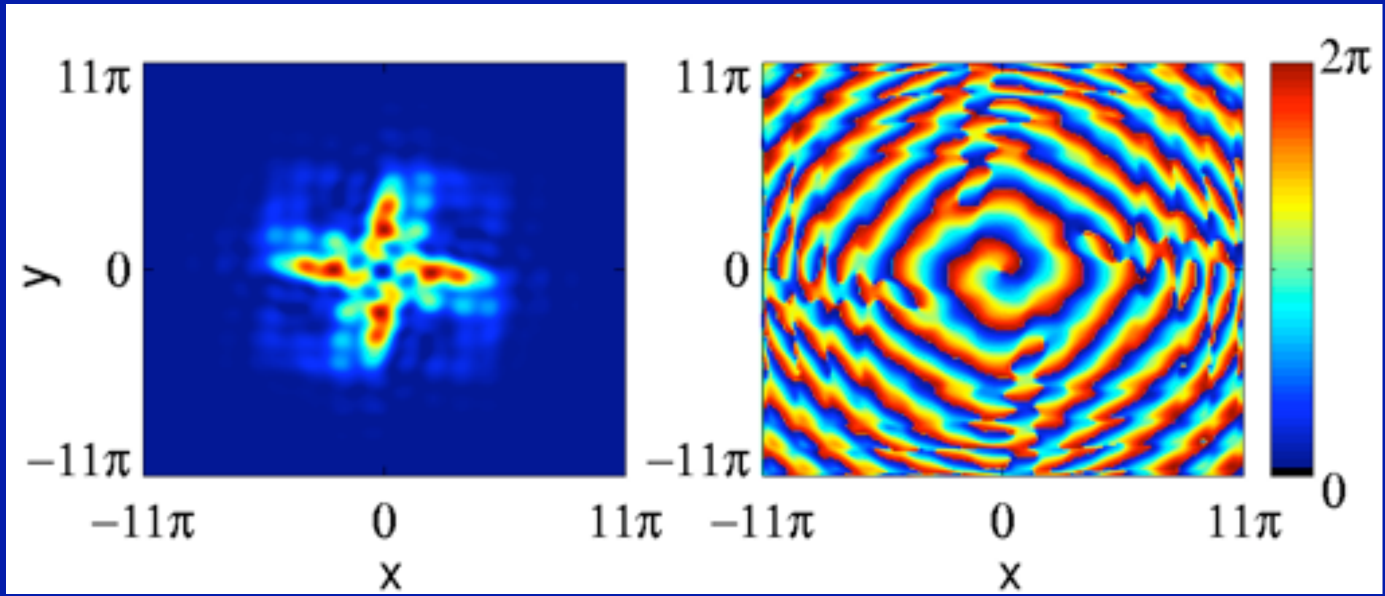
How to detect a gap vortex?

Gap vortex detection

- Confirmation of a localized state
- Confirmation of the phase structure



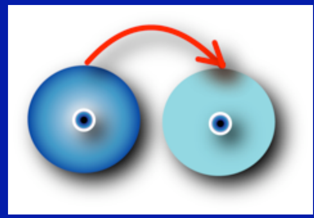
Post-release state after ~1.5 ms expansion

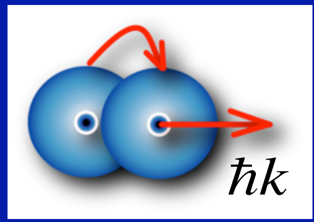


Interferometric phase detection

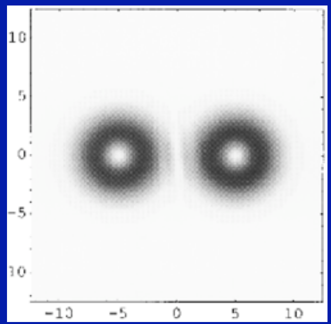
Homodyne

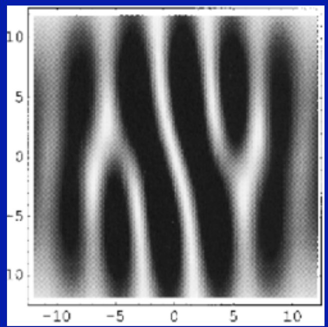
(RF or Bragg coherent splitting)



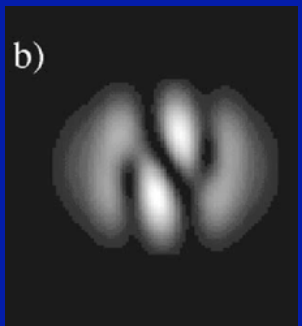


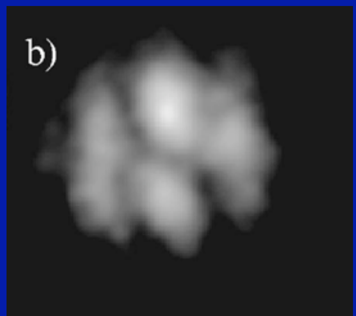
theory: Tempere & Devreese, Solid State Comm. 108, 993 (1998);
Dobrek et al., Phys. Rev. A, R3381 (1999)





experiment: Chevy et al, Phys. Rev. A, 031601(R) (2001)



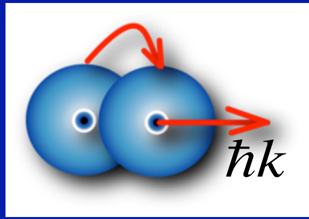


RF splitting into Zeeman states

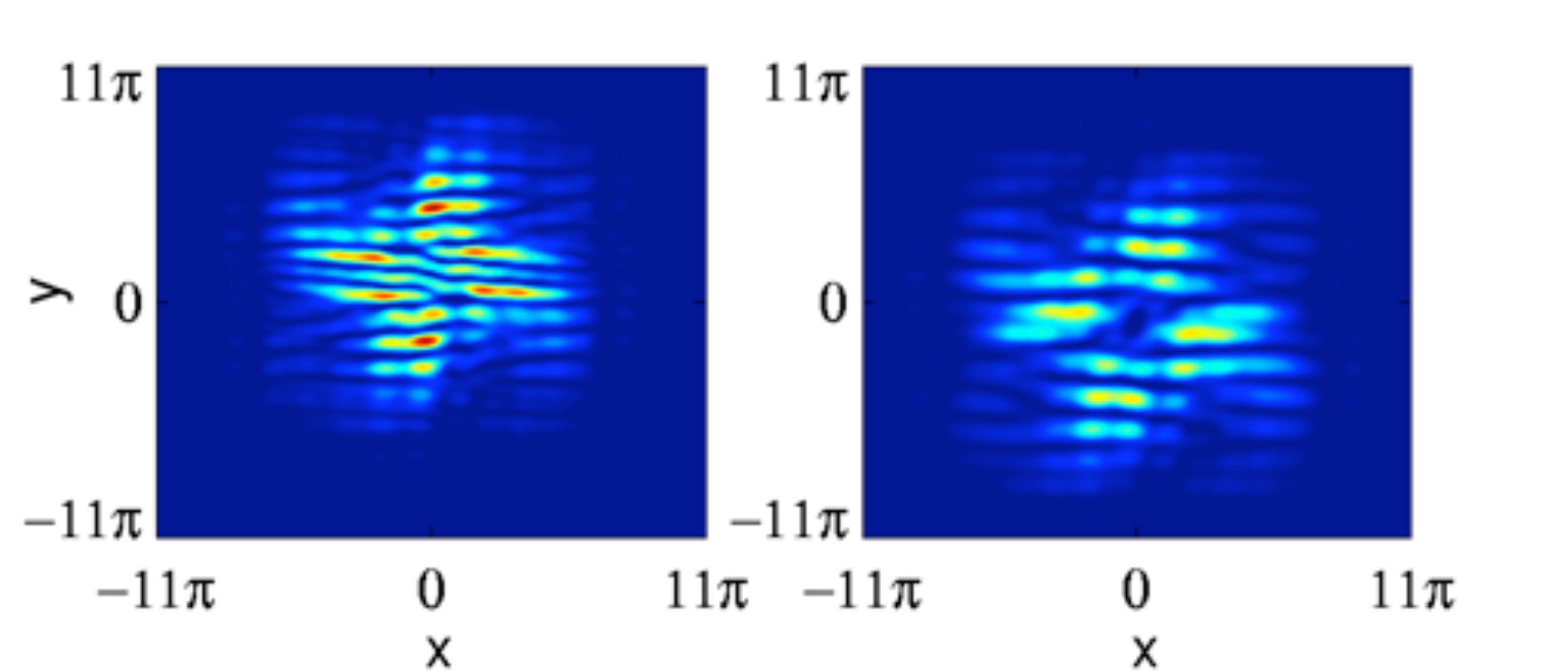
momentum + position displacement

Homodyne detection of a gap vortex

- Bragg-pulse splitting shortly after release, followed by free expansion



After 0.5 ms evolution:



Conclusion

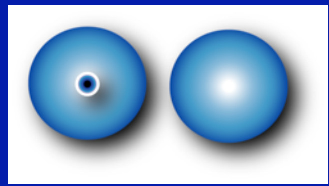
2D Cs BEC in Innsbruck; PRL, 92, 73003 (2004):

D. Rychtarik, B. Engeser, H.-C. Nägerl, and R. Grimm

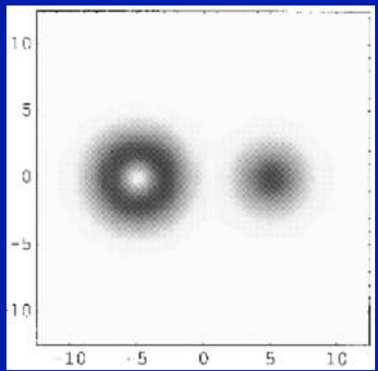
The trapped two-dimensional condensate will allow us to study elementary excitations such as solitons and vortices, the properties of which may exhibit striking differences as compared to the three-dimensional case. Further intriguing possibilities are offered by the creation of optical surface lattices created through the interference of different evanescent waves.

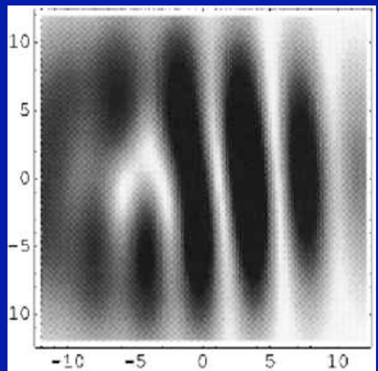
Interferometric phase detection

- Heterodyne (two condensates required)



theory: Tempere & Devreese, Solid State Comm. 108, 993 (1998)
Bolda & Walls, PRL 81, 5477 (1998)





experiment: Inoue et al., PRL 87, 080402 (2001) - vortex pair