Nature of laser light

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Is laser light in a coherent state?

Traditional answer: Laser light is in a coherent state $|\alpha\rangle$, with $\alpha = |\alpha| e^{i\theta}$, of definite, but unknown, phase. Thus represent by a mixture:

$$\hat{\rho}_F = \int_0^{2\pi} |\alpha\rangle\langle\alpha| \frac{d\theta}{2\pi}$$

This can be derived by standard treatments.

However we can also write as a mixture:

$$\hat{\rho}_F = e^{-|\alpha|^2} \sum_n \frac{|\alpha|^{2n}}{n!} |n\rangle\langle n|$$

- a state of definite, but unknown, photon number.

Does it matter what state it is in?

Yes, if we wish to use the traditional assumption to infer from the results of an experiment that we have e.g. prepared a squeezed state (**Mølmer**, PRA, **55**, 3195, who conjectured that the coherent state description is a "convenient fiction"). Same applies to continuous variable teleportation (Rudolph and Sanders PRL, **87**, 077903).

Can we tell whether it is a coherent state of unknown phase or a photon number state of unknown number? Not without more information.

Can use *measurement* information or *preparation* information.

Measurement information

Do experiments on the light.

Photon counting? No, clearly does not distinguish.

Split beam and let beams interfere (interferometer)? No, can be described in terms of interfering amplitudes for photon paths.

Excite atoms, (e.g. $\pi/2$ pulse), making use of Rabi precession? *No*, get Rabi precession with a number state.

Interfere with another laser? *See later*.

Disrupt phase of laser? (Photon number states have random phase anyway, so should not be affected. Coherent states would have $\pi/2$ pulse disrupted.) *No* (surprisingly).

Preferred ensemble fallacy

In general we measure probabilities and expectation values. These depend only on $\hat{\rho}$ and not on its partitioning.

$$\langle A \rangle = \mathrm{Tr}(\hat{\rho}\hat{A})$$

This may indicate that no measurement information can tell us whether the laser is in a coherent state or not. So we must turn to preparation information.

Preparation information

Alice prepares a spin-1/2 system in +z or –z state with equal probability. The density operator for this is

$$\hat{\rho} = \tfrac{1}{2}\left|+z\right\rangle\left\langle+z\right| + \tfrac{1}{2}\left|-z\right\rangle\left\langle-z\right| \quad (a)$$

$$= \tfrac{1}{2}\hat{1}$$

$$= \tfrac{1}{2}\left|+y\right\rangle\left\langle+y\right| + \tfrac{1}{2}\left|-y\right\rangle\left\langle-y\right| \quad (b)$$

In (*a*), the coefficients are *preparation probabilities*; in (b), they are not.

Without sufficient knowledge of the preparation we might describe the state as either a state of definite but unknown *z or* a state of definite but unknown *y.*

If we find out that the magnetic field in Alice's Stern-Gerlach apparatus is in the *z*-direction, then we would know that the latter description is a *fiction*, even though it may be convenient to use this description for predicting results of future experiments on the system.

Can we prepare light in a coherent state?

From a classical source? *Yes.*

From a quantum oscillator? *Yes,* if oscillator is in a coherent state itself.

From a group of excited atoms in a cavity? *Yes,* if the atoms are in an eigenstate of $\hat{c}$ which is a linear combination of terms of the type

$$|g\rangle_{i\ i}\langle e|$$

(lasing transition states of *i* th atom)

Does a normal laser prepare such an atomic state?

No (extremely unlikely)

Perhaps a small accidental approximately coherent state is amplified by the laser?

No – amplification adds phase noise, cannot achieve a state with well-defined phase appropriate for an intense coherent state.

A coherent state from a laser does appear to be a fiction which cannot be exposed by future measurements.

What state is prepared by a laser?

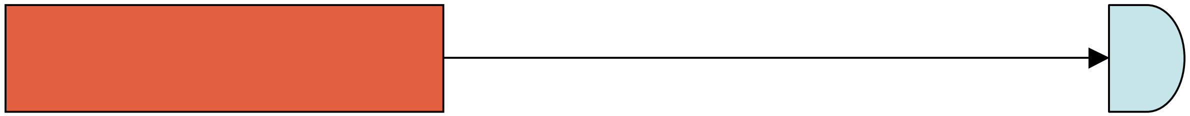
An entangled state of the field and the source (atoms + excitation mechanism) with random optical phase.

Can we reduce this state by tracing over the source states?

Yes, then we obtain the density operator $\hat{\rho}_F$

However $\hat{\rho}_F$ is itself a fiction, indeed one that can be exposed by measurement information (measuring the atom states).

How can we explain interference effects?

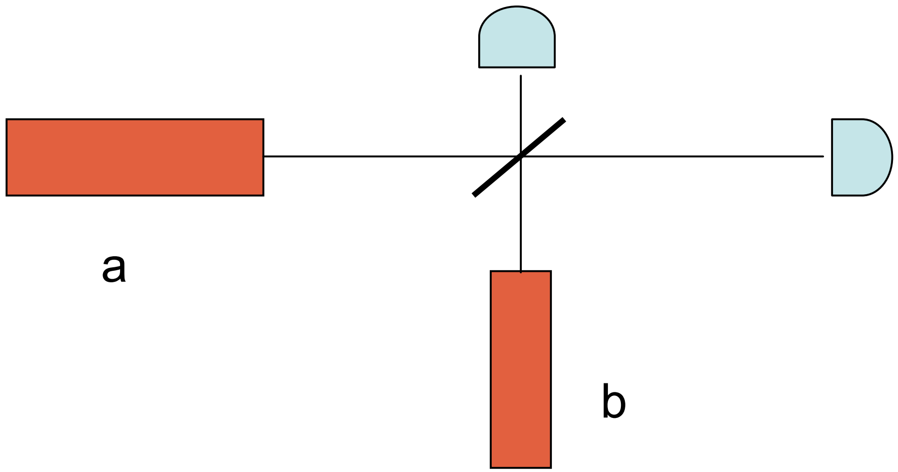


Leaking mirror, interaction Hamiltonian terms

$\propto \hat{a}_o^+ \hat{a}$ and $\hat{a}_o \hat{a}^+$ (energy conserving).

Initial state $\hat{\rho}\left|0\right\rangle_{o\,o}\left\langle 0\right|$ evolves after short time to include a term $\varepsilon \hat{a}\hat{\rho}\hat{a}^+ \left|1\right\rangle_{o\,o}\left\langle 1\right|$

Detection of one photon thus reduces intra-cavity state from $\hat{\rho}$ to $\hat{a}\hat{\rho}\hat{a}^+$ (unnormalised)



One photocount registered. Measured state is $\left| f \right\rangle_o =$

$$\tfrac{1}{\sqrt{2}}\left(\left|1\right\rangle_{oa} \left|0\right\rangle_{ob} + e^{-i\gamma} \left|0\right\rangle_{oa} \left|1\right\rangle_{ob} \right) = \tfrac{1}{\sqrt{2}}\left(\hat{a}_o^+ + e^{-i\gamma}\hat{b}_o^+ \right)\left| vac \right\rangle_o$$

when evolved back to just outside cavities.

Interaction Hamiltonian term:

$$\hat{a}_o^+\hat{a} + \hat{b}_o^+\hat{b} = \tfrac{1}{2}[(\hat{a}_o^+ + e^{-i\gamma}\hat{b}_o^+)(\hat{a} + e^{i\gamma}\hat{b}) + (\hat{a}_o^+ - e^{-i\gamma}\hat{b}_o^+)(\hat{a} - e^{i\gamma}\hat{b}_o)]$$

Evolves from $\hat{\rho}\left|vac\right\rangle_{o\ o}\left\langle vac\right|$ to include term

$$\hat{a}_o^+\hat{a}+\hat{b}_o^+\hat{b}=(\hat{a}_o^+ + e^{-i\gamma}\hat{b}_o^+)\left|vac\right\rangle_o(\hat{a}+e^{i\gamma}\hat{b})\hat{\rho}(\hat{a}_o^+ + e^{-i\gamma}\hat{b}_o^+)\ _o\left\langle vac\right|(\hat{a}+e^{i\gamma}\hat{b}_o)$$

$$\propto \left|f\right\rangle_o(\hat{a}+e^{i\gamma}\hat{b})\hat{\rho}(\hat{a}^+ + e^{i\gamma}\hat{b}^+)\ _o\left\langle f\right|$$

Thus effect of detecting first photocount is to change $\hat{\rho}$ to $(\hat{a}+e^{i\gamma}\hat{b})\hat{\rho}(\hat{a}^+ + e^{i\gamma}\hat{b}^+)$ (unnormalised), giving

$$\hat{\rho}_1=\frac{(\hat{a}+e^{i\gamma}\hat{b})\hat{\rho}_a\hat{\rho}_b(\hat{a}^+ + e^{-i\gamma}\hat{b}^+)}{\overline{n}_a+\overline{n}_b}$$

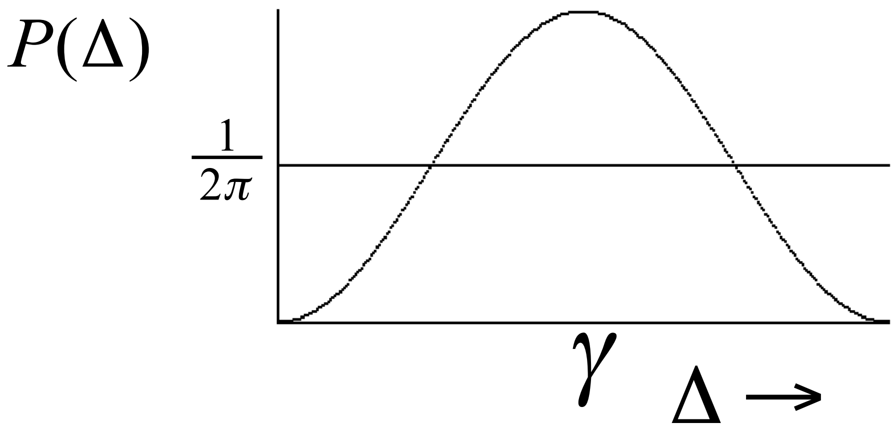
$P(\Delta)$ changes from 1/2π (random) to

$$P(\Delta)=\frac{1}{2\pi}+\frac{1}{\pi}\frac{\overline{n_a^{1/2}}\,\overline{n_b^{1/2}}}{\overline{n}_a+\overline{n}_b}\cos(\Delta-\gamma)$$

First photocount sharpens phase difference distribution.

For sharp *number state* distribution and

$$\overline{n}_a = \overline{n}_b :$$



First photocount also increases probability that second photocount will be at same detector.

$$\frac{P_{12}}{P_{11}} = \frac{\overline{n_a^2} + \overline{n_b^2} - \overline{n}_a - \overline{n}_b}{\overline{n_a^2} + \overline{n_b^2} - \overline{n}_a - \overline{n}_b + 4\overline{n}_a\overline{n}_b}$$

Narrow number state distributions:

$\overline{n}_a = \overline{n}_b >> 1$: $P_{12}/P_{11} = \frac{1}{3}$

$\overline{n}_a = \overline{n}_b = 1$: $P_{12}/P_{11} = 0$ (c.f. Hong-Ou-Mandel “dip”)

The measurement itself *creates* the quantity being measured, that is, the well-defined phase difference between the two laser beams.

For coherent states, the measurement does not alter the states and they retain their well-defined phase difference. Only retrodiction is involved.

This gives rise to the illusion that the laser beams are in coherent states.

Disrupt phase during a π-pulse

Strong coherent state similar to classical light with definite phase. Thus shift of phase will disrupt π-pulse.

Photon number state has no well-defined phase to disrupt – therefore phase shift will not affect π-pulse?

No – same result in both cases. π-pulse is disrupted.

Before pulse the atom-field state is $|n\rangle|g\rangle$

Phase shift $\exp(i\hat{n}\theta)$ does not physically alter this state.

After pulse the state is $|n-1\rangle|e\rangle$

Phase shift does not physically alter this state.

During pulse atom-field state is in a *superposition* of $|n\rangle|g\rangle$ and $|n-1\rangle|e\rangle$.

Phase shift *does* physically alter this state.

Obtain same result as for coherent state.

Conclusion

Mølmer's conjecture that describing laser light as a coherent state is a "convenient fiction" appears to be correct.

However describing by a number state is also a fiction.

We do not need *either* to predict interference between two lasers. Cause of interference is the measurement process itself.

?

Interpretation of experiments:

Squeezed states

Continuous variable teleportation

Optical homodyne tomography

State measurement

Phase measurement

Coherent state qubits

Quantum state engineering

Measurement by retrodictive state engineering

etc.