Quantum imaging and information extraction

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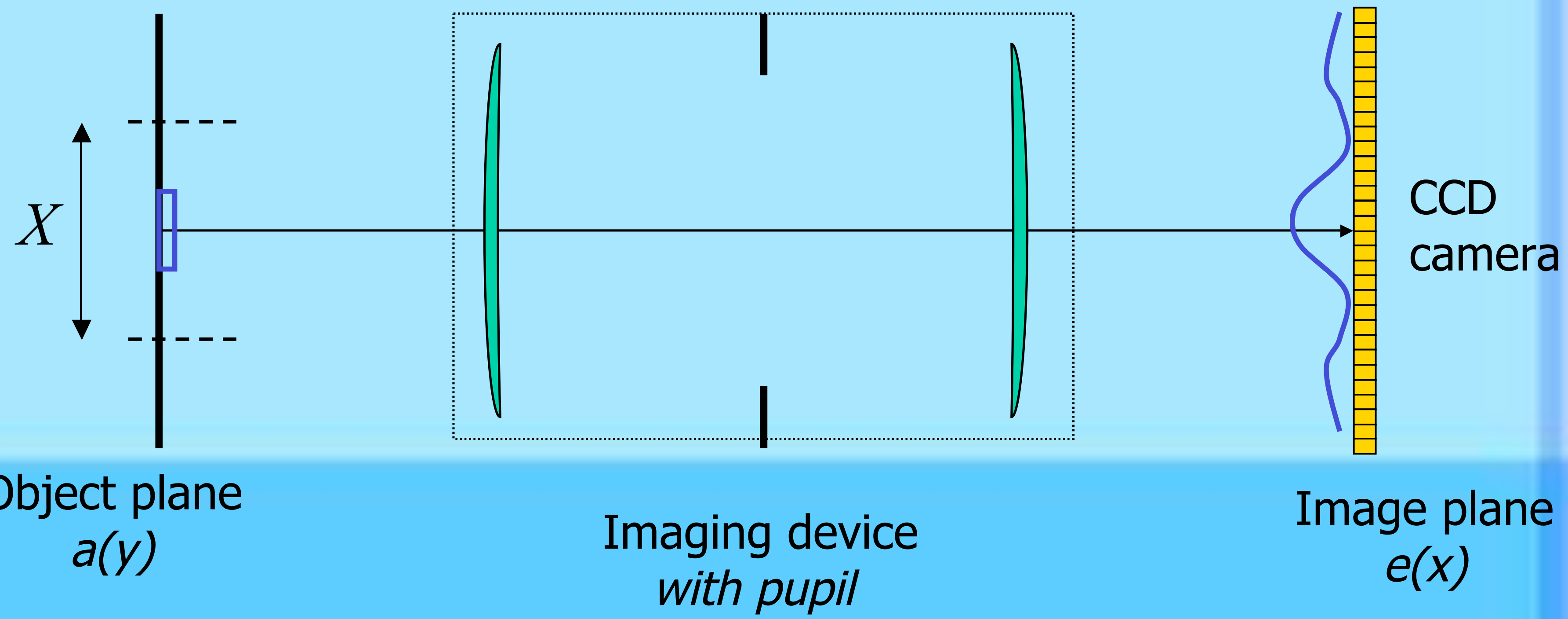
Imaging apparatus



Quality of the detectors : size, number of pixel, response,…
Classical noise (vibrations, thermal noise,…) ⇨ **Technical limits**

Quantum nature of light (quantum noise) ⇨ **Fundamental limit**

Optical resolution vs. information extraction

Optical resolution

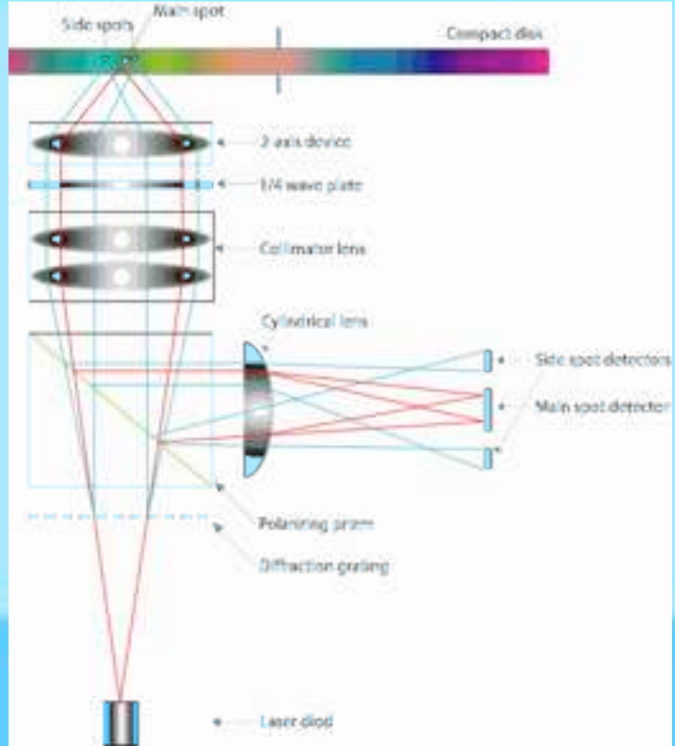
No a-priori information on the image : smallest details measurable.

- In many practical cases : the Rayleigh criteria.
- Crossing the standard quantum limits requires very multimode quantum light, i.e. many resources.

M. I. Kolobov and C. Fabre, Phys. Rev. Lett. 85, 3789 (2000).

Optical resolution vs. information extraction

Information extraction



Optical read out

A lot of a-priori information : presence and/or modification of a given pattern.

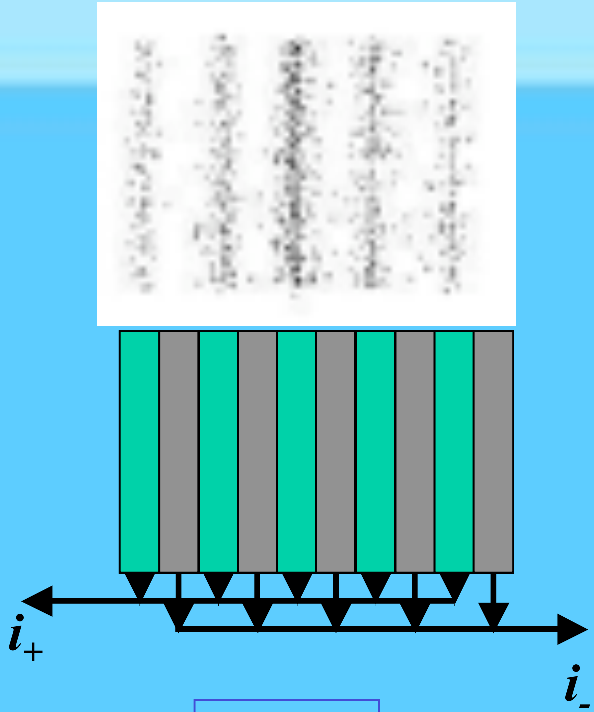
- Quantum limit is easily reached : orders of magnitude smaller than the Rayleigh criteria.
- We will show that crossing the standard quantum limit requires a limited amount of resources.

Some applications

Measuring a slight modification from a known pattern.

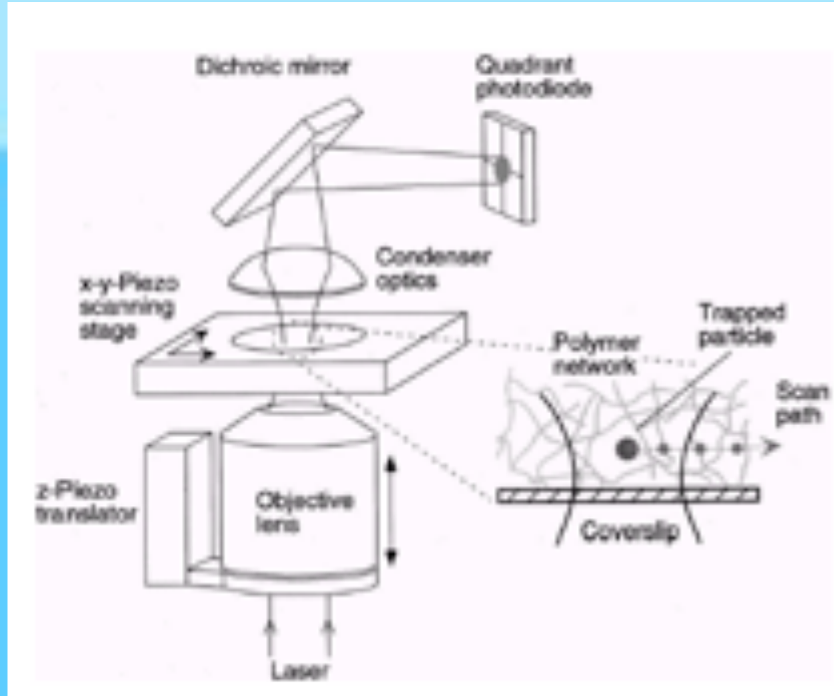
Examples

- Pattern recognition



$$I = i_+ - i_-$$

- Optical tweezers



C. Tischer et al, Appl. Physics Letters, **79**, 3878 (2001).

Outline

- Single mode versus multimode light
- Detection mode associated with any linear measurement
- Optimising both signal and noise

Single mode vs. multimode : classical approach

Electric field distribution

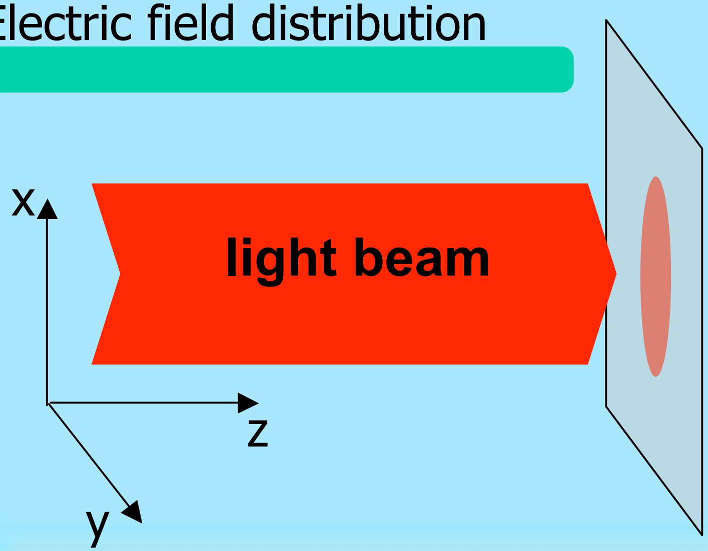


Image = electric field distribution in the transverse plane

$E(\vec{r})$ where $\vec{r} = (x, y)$ is the transverse coordinate

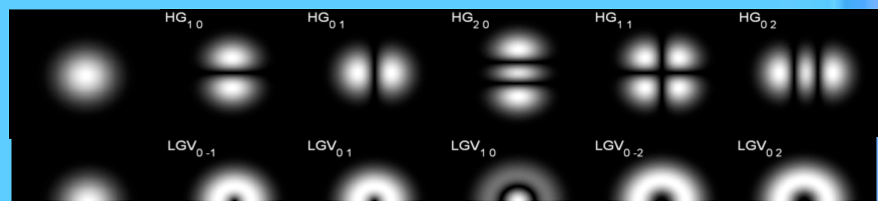
Modal decomposition

Can be decomposed a transverse modes basis $\{u_i\}$ such as :

$$\int u^*_i(\vec{r}) u_j(\vec{r}) d^2 r = \delta_{ij} \quad \text{orthogonal}$$

$$\sum_i u^*_i(\vec{r}) u_i(\vec{r}) = \delta(\vec{r} - \vec{r}\,') \quad \text{complete}$$

$$E(\vec{r}) = \sum_i \alpha_i u_i(\vec{r})$$



One can choose any basis : LG, HG, …

Number of modes involved depends

Single mode vs. multimode : classical approach

Single mode basis

$$v_0(r) = \frac{E(r)}{\|E(r)\|} = \frac{1}{\sqrt{\sum_i |\alpha_i|^2}} \sum_i \alpha_i u_i(\vec{r}),$$

v_0 is the first element of a transverse mode basis : $v_0, v_1, v_2, \ldots$

In that basis

$$E(\vec{r}) = \sqrt{\sum_i |\alpha_i|^2}\, v_0(\vec{r})$$

If the field is a coherent superposition of modes (not a statistical one)

↓

It is single mode at the classical level

No intrinsic definition of multimode for a coherent superposition of modes

Single mode vs. multimode : quantum approach

Electric field operator

$$\hat{E}(\vec{r}) = \sum_i \hat{a}_i u_i(\vec{r})$$

- $\hat{a}_i$: annihilation operator
- $u_i(\vec{r})$: transverse mode

Electric field quantum state

$$|\psi\rangle = \sum_{n_1,\ldots,n_i,\ldots} C_{n_1,\ldots,n_i,\ldots} |n_1,\ldots,n_i,\ldots\rangle$$

- n_i : number of photons in the mode i

DEFINITION : Single mode beam

It exists a basis $\{\hat{b}_i, v_i\}$ such as

$$|\psi\rangle = |\phi,0,\ldots,0,\ldots\rangle$$

where
- $|\phi\rangle$ is the field state in the first mode
- the other modes are populated by vacuum

Single mode vs. multimode : quantum approach

Criteria for a Single mode beam

A quantum field is single mode if and only if the action of all the annihilation operators $\boldsymbol{\hat{a}_i}$ give collinear vectors.

$|\psi\rangle$ monomode $\Longleftrightarrow$ $\exists|\varphi\rangle, \quad \forall i, \quad \hat{a}_i|\psi\rangle = k_i|\varphi\rangle$

- If it is true for a given basis it is true for any annihilation operator

Single mode :
"whatever the photon you remove from the field, it always come from the same mode"

Possible single mode fields

- $|\psi\rangle = |\phi_0, 0, \ldots, 0, \ldots\rangle$ Single mode whatever the quantum state $|\phi_0\rangle$
- A factorized state : $|\psi\rangle = |\phi_0, \ldots, \phi_i, \ldots, \phi_j, \ldots\rangle$

⇒ Single mode if and only if all the states are coherent states

Single mode vs. multimode : quantum approach

'Eigenbasis' description

Basis in which the field is single mode at the classical level : $\langle \psi | \hat{E}(\vec{r}) | \psi \rangle = \alpha_0 u_0(\vec{r})$

The other modes : contribute only to the noise, not to the mean field.
→ state can be vacuum or non-classical vacuum

modes	single mode light	multimode light
$u_0(\vec{r})$	any state of mean value α_0	any state of mean value α_0
$u_1(\vec{r})$	vacuum	At least one is a non-classical vacuum state
⋮	⋮	
$u_n(\vec{r})$	vacuum	

A beam is spatially multimode : one of the 'vacuum' modes is non-classical
A beam is spatially squeezed : one of the 'vacuum' modes is squeezed

Summary on single mode vs. multimode

- Coherent superposition of modes is single mode at the classical level.
- The same beam can be multimode at the quantum level : superposition of a coherent and a non-classical beam is sufficient
- Criteria to define a single mode beam can be extended to a n-mode beam : exactly compute the number of modes
- A suitable basis for description is the 'eigenbasis'.

→ Predict the resources for a particular beam realization

Can be applied to any physical dimension (frequency, time,...)

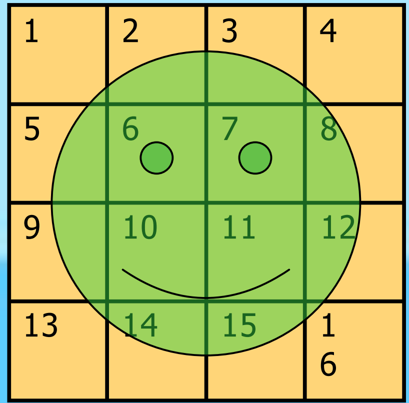
Outline

- Single mode versus multimode light
- Detection mode associated with any linear measurement
- Optimising both signal and noise

Linear measurement of an image

Pixel-like configuration

- Image incident on a CCD camera



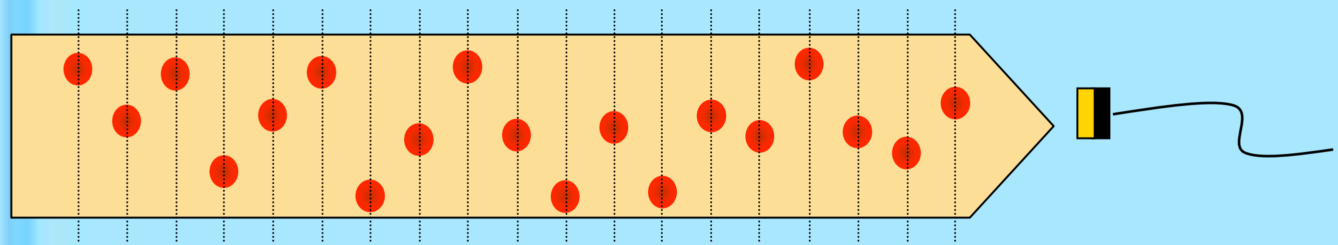
- Linear measurement
  - Intensity on each detector : ***N(D$_i$)***
  - Gain on each detector : σ_i
  - One measurement defined by :

$$N(\{\sigma_i\}) = \sum_i \sigma_i N(D_i)$$

- Image is known
  Measurement : a function of the gains

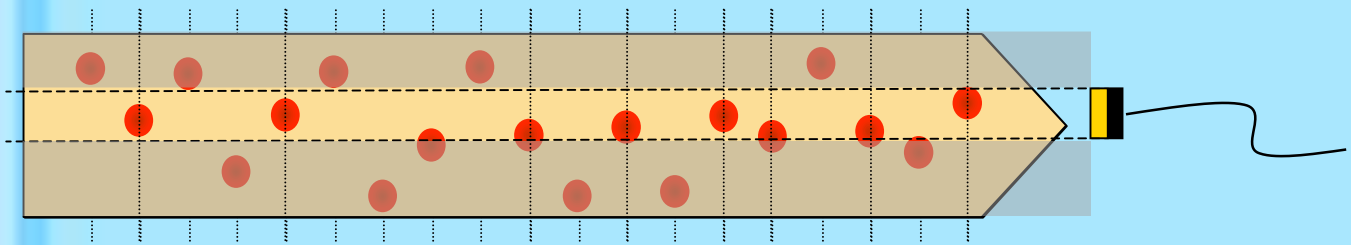
Single mode squeezed light

Partial detection of a squeezed beam

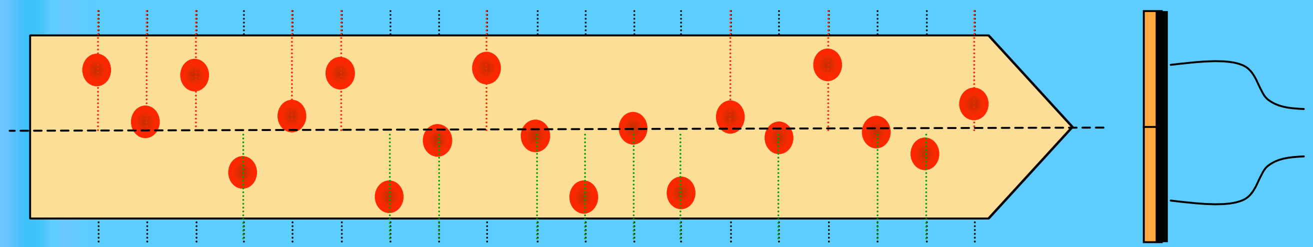


Single mode squeezed light

Partial detection of a squeezed beam



Partial detection is equivalent to a loss : no spatial order.

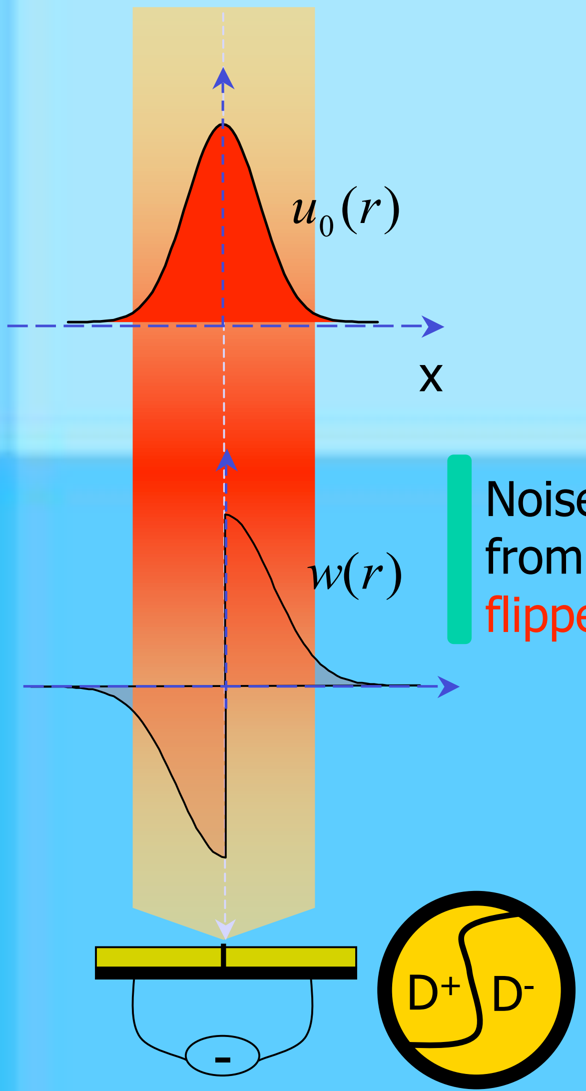


Partial measurement

No spatial squeezing

Noise in a difference measurement

Split detection

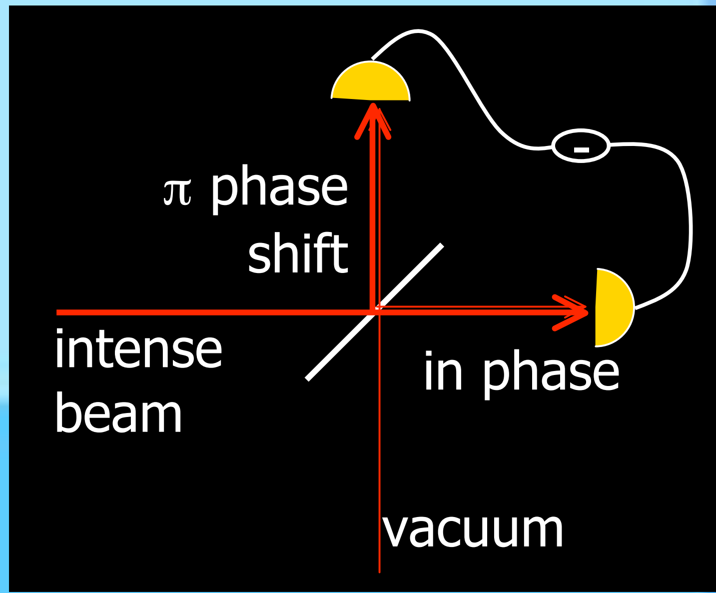


Noise come from the flipped mode

$$\hat{N} = \hat{N}(D^+) - \hat{N}(D^-)$$

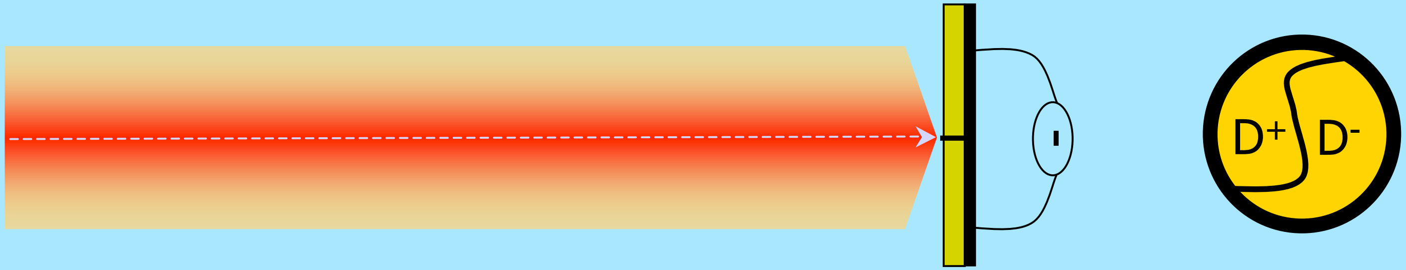
$$\langle \hat{N} \rangle = 0$$

Balanced detection



Noise come from the vacuum port

Noise in a difference measurement



Electric field operator

$$\hat{E}(\vec{r}) = \sum_i \hat{a}_i u_i(\vec{r})$$

Mean field

$$\langle \hat{E}(\vec{r}) \rangle = \alpha_0 u_0(\vec{r})$$

Measurement

$$\hat{N} = \hat{N}(D^+) - \hat{N}(D^-)$$

NOISE

$$\langle \delta \hat{N}^2 \rangle = N_0 \langle (\delta \hat{X}_w^+)^2 \rangle$$

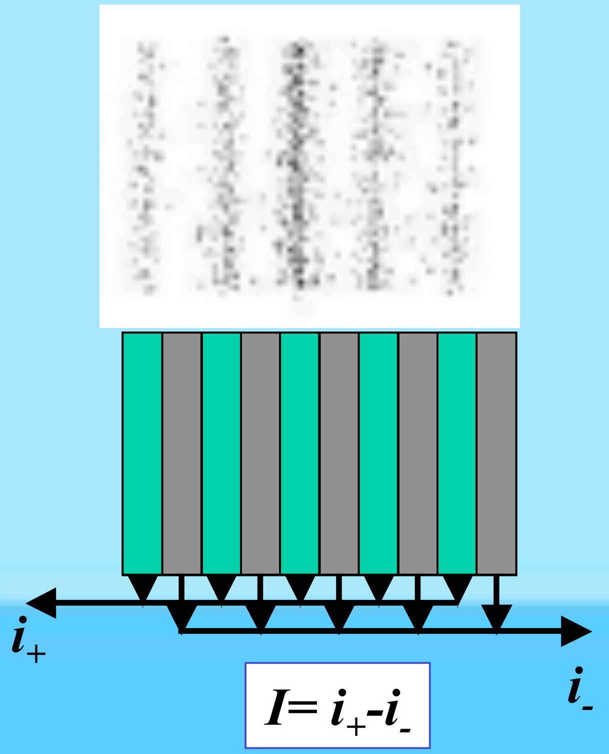
Noise of mode *w*

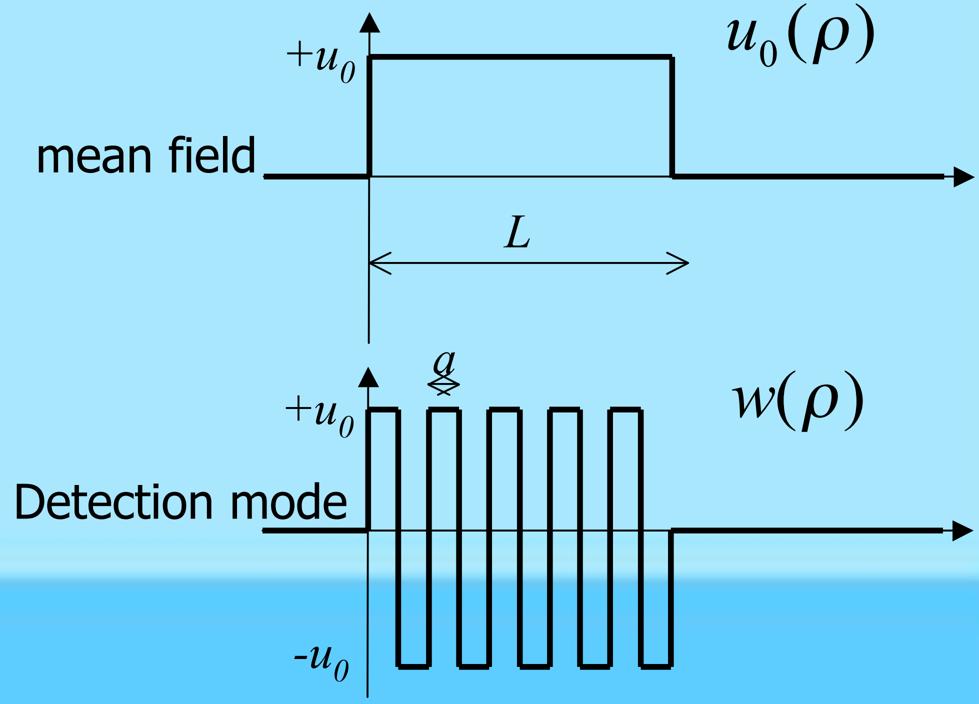
Where *w* is the flipped mode :

$$w(\vec{r}) = u_0(\vec{r}) \quad if \ \vec{r} \in D^+$$

$$w(\vec{r}) = -u_0(\vec{r}) \quad if \ \vec{r} \in D^-$$

Noise in a general measurement





Variance of the noise

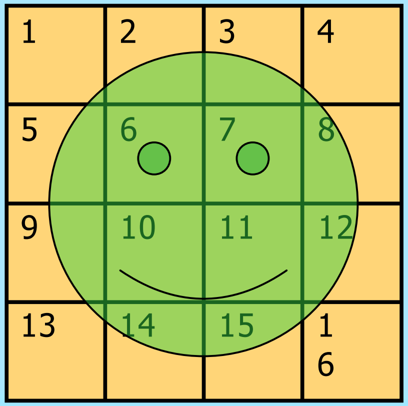
$$V(\hat{N}) = N V(\hat{X}_w^1)$$

Transverse modes description

Same as for the differential measurement.

Noise in a general measurement

General measurement



$$\hat{N}(\{\sigma_i\}) = \sum_i \sigma_i \hat{N}(D_i)$$

Mean field mode : any shape
What is the detection mode ?

Detection mode

It exists a detection mode *w* such as $if\ \vec{\rho} \in D_i, w(\vec{\rho}) = \frac{1}{f} \sigma_i u_0(\vec{\rho})$

Variance of the noise

$$V(\hat{N}) = f^2 N V(\hat{X}_w^1)$$

N. Treps, V. Delaubert, A. Maître, J.M. Courty and C. Fabre, to be published in Phys. Rev. A, quant-ph/0407246

Simultaneous measurements

N Independent measurements

One measurement: one set of gains $\{\sigma_i\}_k$

one flipped mode $w_k(\vec{r}) = \frac{1}{f}\sigma_i u_0(\vec{r}), if\ \vec{r} \in D_i$

N independent measurements : none of them is a linear combination of the others.

The corresponding detection modes are orthogonal !

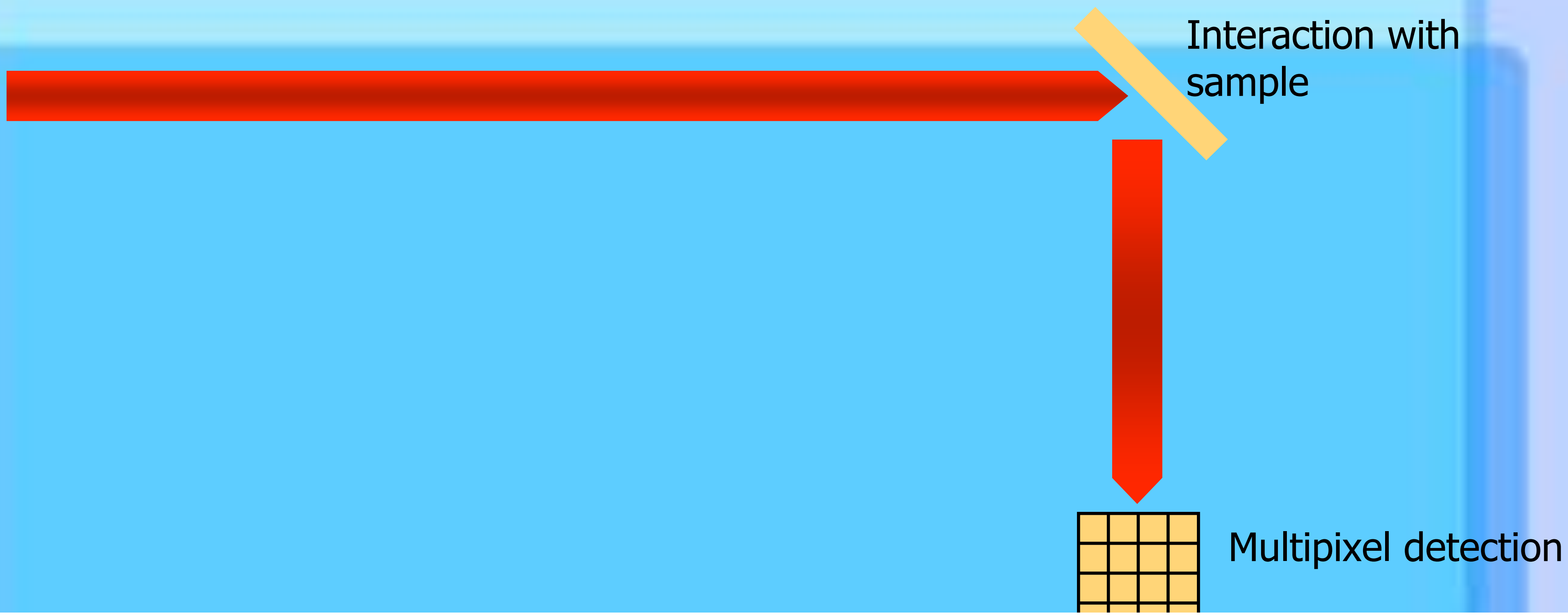
Field description

	mode	state
	$u_0(\vec{r})$	any state of mean value α_0
1st flipped mode :	$w_1(\vec{r})$	squeezed vacuum
	⋮	⋮
nth flipped mode :	$w_n(\vec{r})$	squeezed vacuum

All the measurements are improved simultaneously if and only if all the flipped modes are populated with squeezed vacuum.

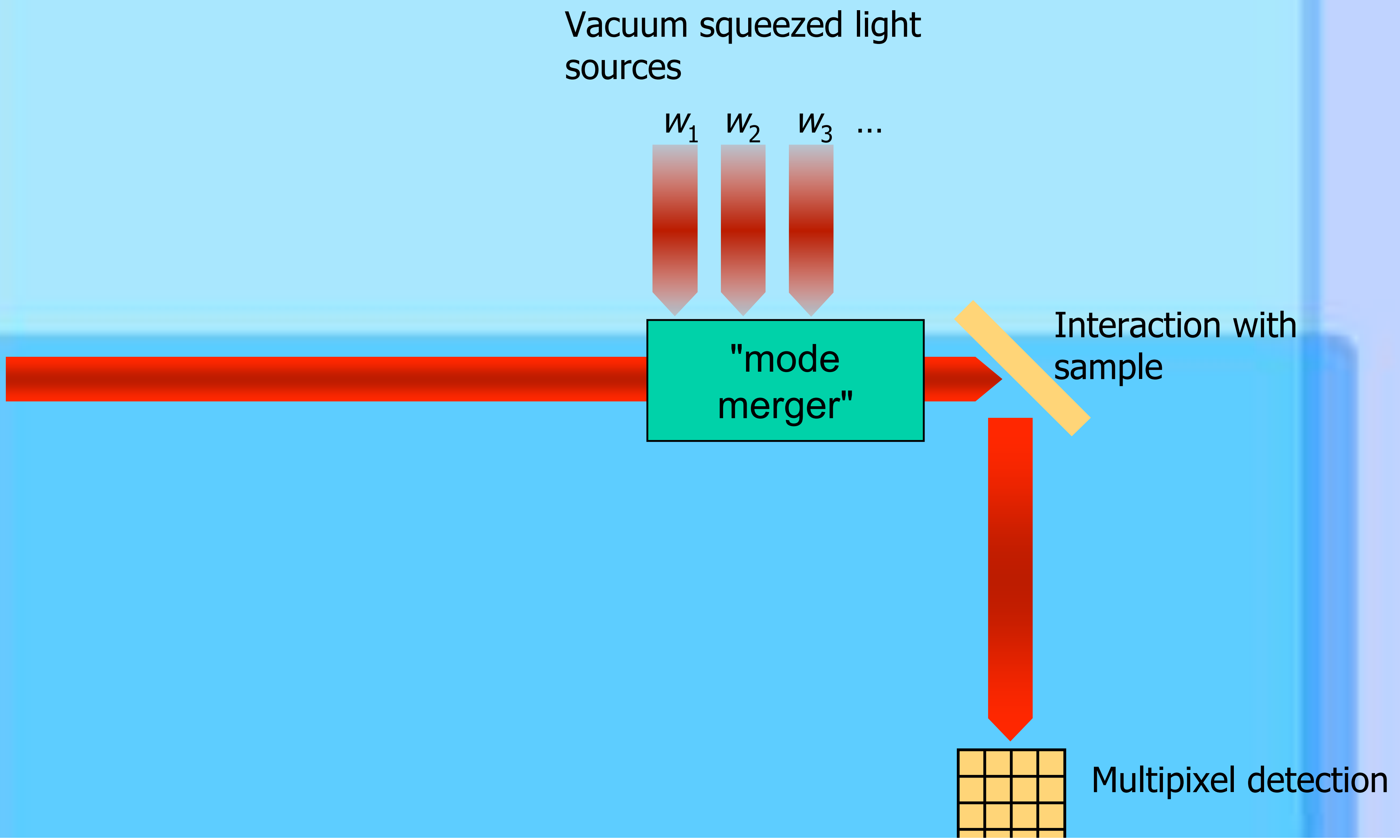
Strategy for optimal measurements

Classical scheme



Strategy for optimal measurements

Quantum noise optimized scheme

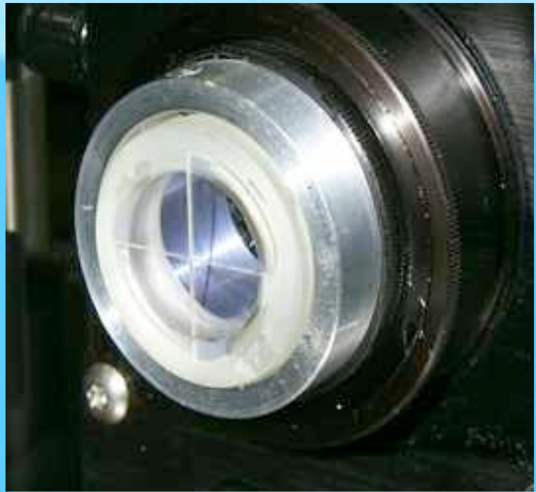


Strategy for optimal measurements

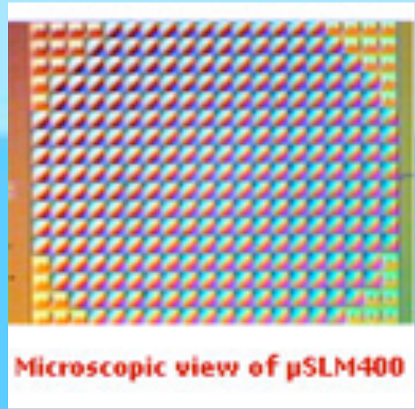
Experimental realization

- Vacuum Squeezing source : Optical Parametric Amplifier

- Producing the transverse modes



Transmission phase plate developed at the Australian National University



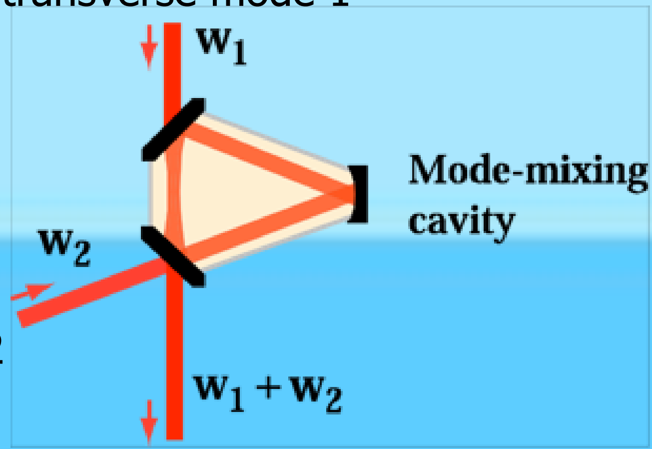
Deformable mirror from *Boston Micromachines Corporation*

Strategy for optimal measurements

Experimental realization

- Mixing two orthogonal transverse modes :

transverse mode 1

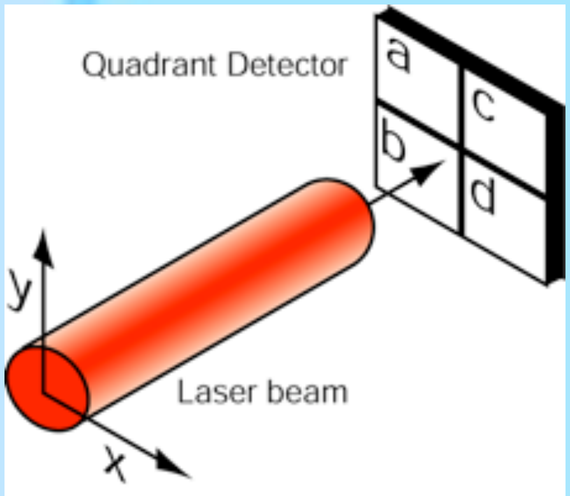


transverse mode 2

transverse modes 1 and 2

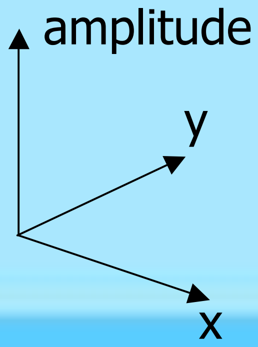
N. Treps, N. Grosse, W. Bowen, C. Fabre, H. Bachor, P.K. Lam, "Nano displacement measurements using spatially multimode squeezed light", J. Opt. B: Quantum Semiclass. Opt. 6 S664-S674 (2004)

Small displacements measurement

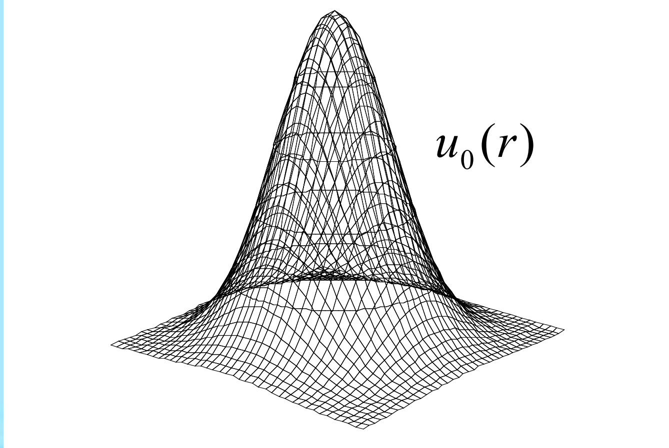


$$I_x = (I_a + I_b) - (I_c + I_d)$$

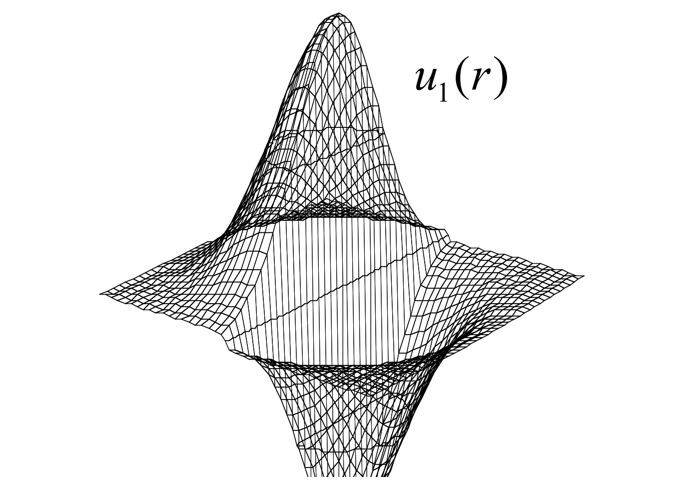
$$I_y = (I_a + I_c) - (I_b + I_d)$$



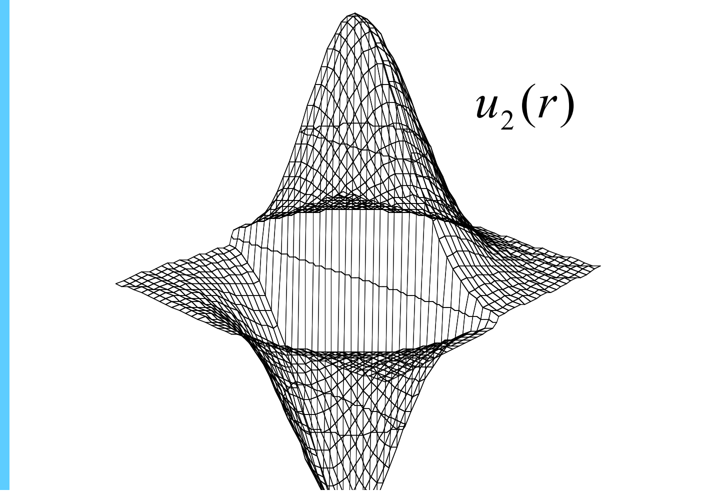
Beam shape



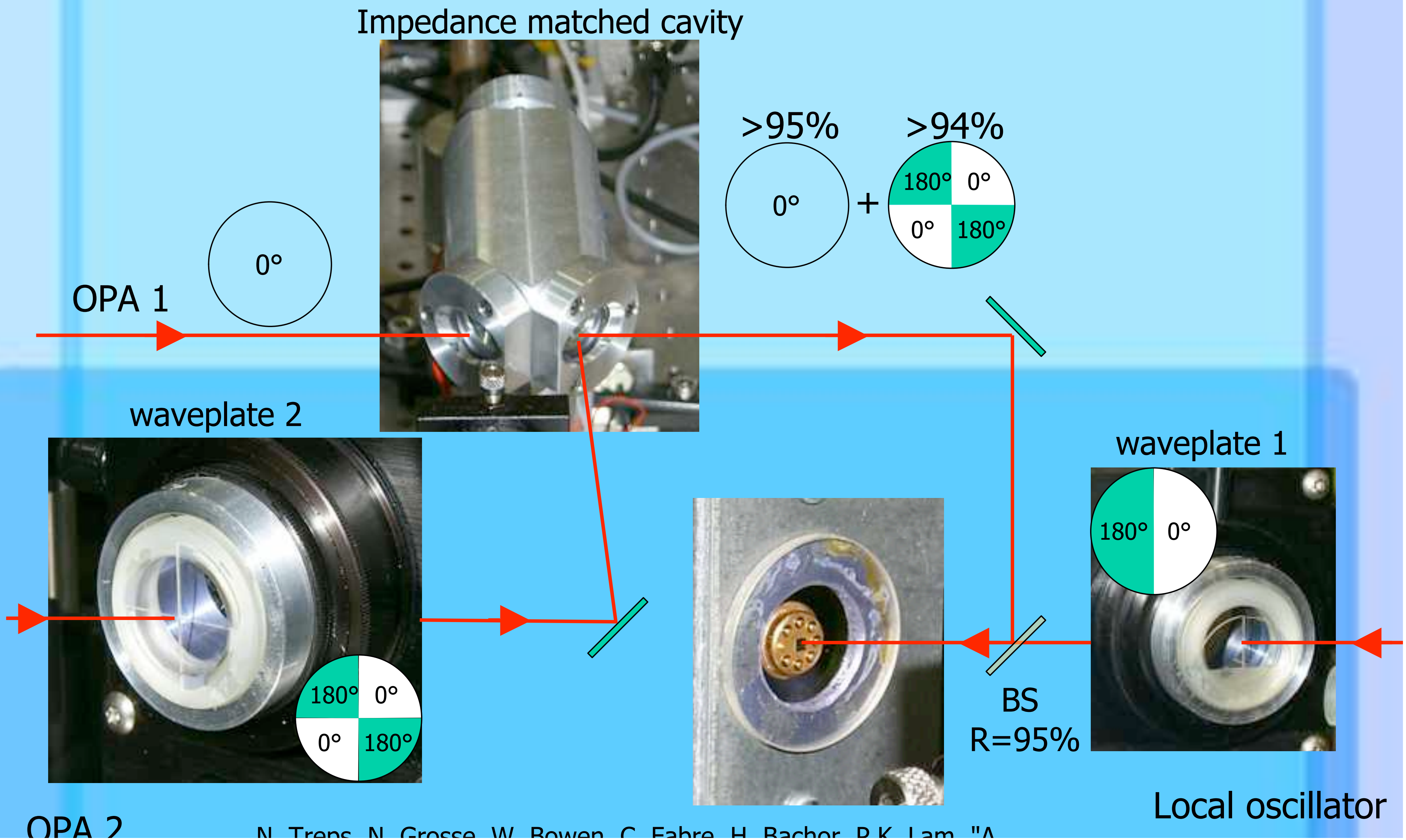
x flipped mode



y flipped mode



Small displacements measurement



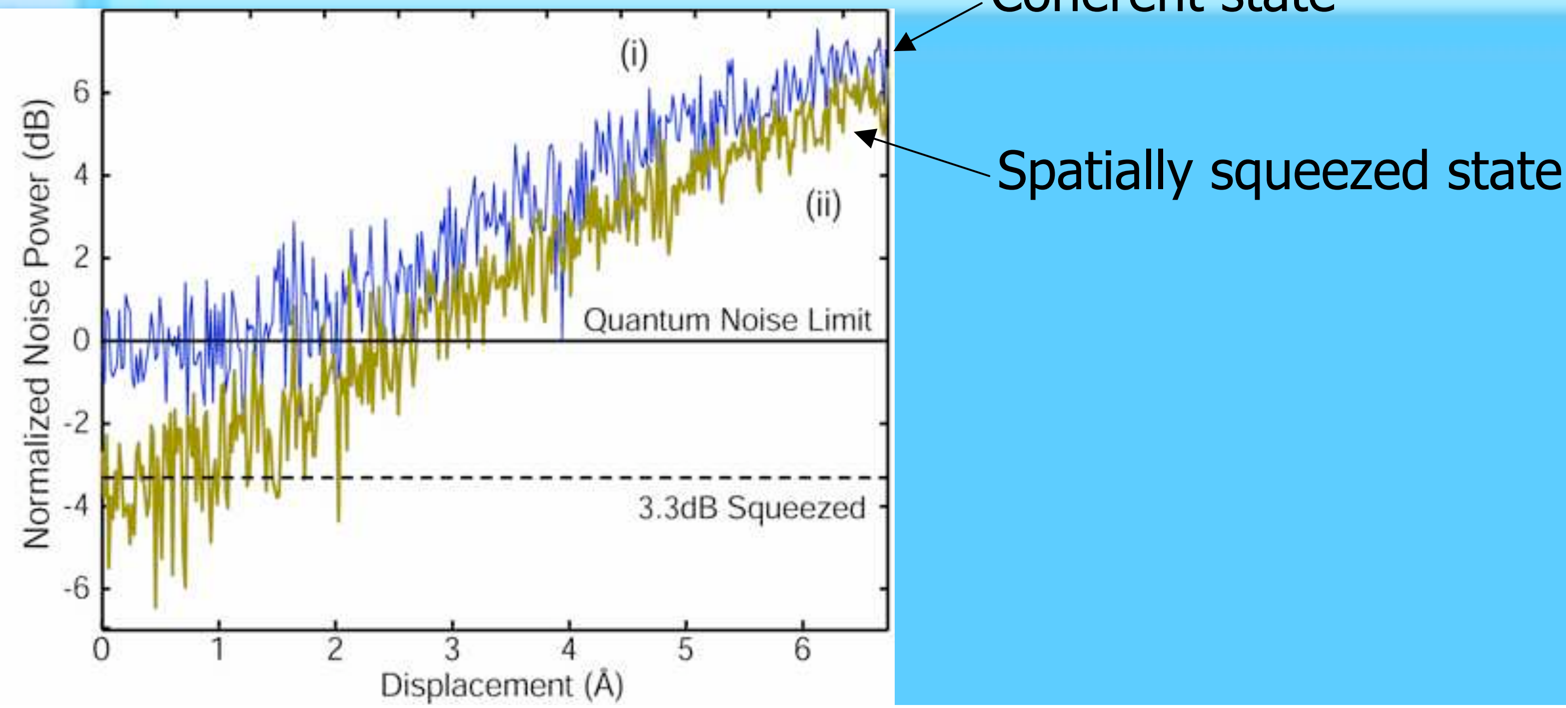
N. Treps, N. Grosse, W. Bowen, C. Fabre, H. Bachor, P.K. Lam, "A

Small displacements measurement

Oscillation at 4.5 MHz : mirror on a piezo-electric crystal.

Oscillation amplitude is linearly increased with time.

Signal measured



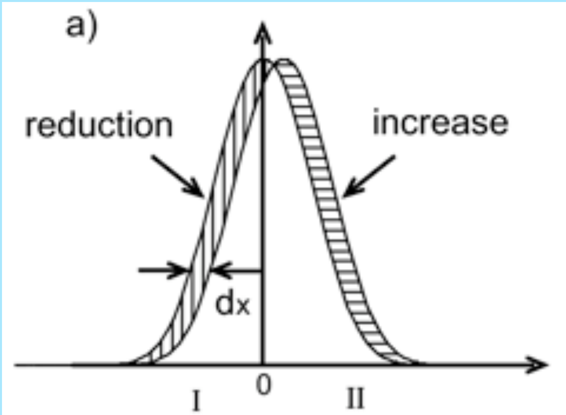
Outline

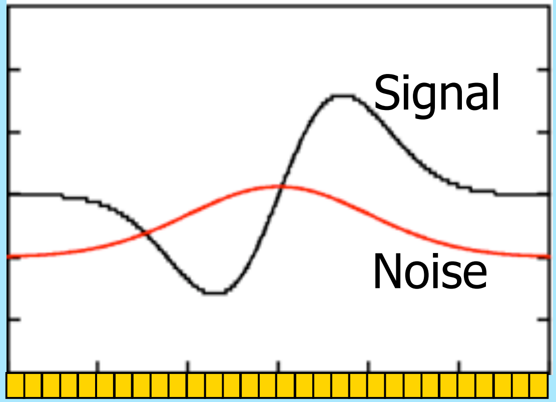
- Single mode versus multimode light
- Detection mode associated with any linear measurement
- Optimising both signal and noise

Optimum displacement measurement

Is there better than the split detector ?

One can use a CCD camera, and optimise the gains.





Pixel-like detector

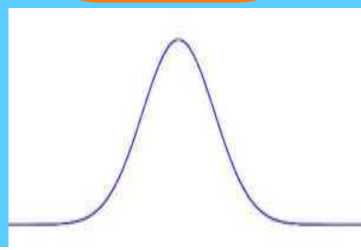
Study of the effect of displacement

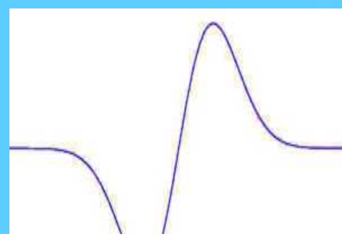
Original beam $$E(x,y) = \alpha_0 u_0(x,y)$$

Displaced beam $$E(x+dx,y) \approx \alpha_0 \left[u_0(x,y) + \frac{\partial u_0(x,y)}{\partial x} dx \right]$$

TEM_{00} TEM_{01}

A displaced TEM_{00} is a TEM_{00} + TEM_{01} !!

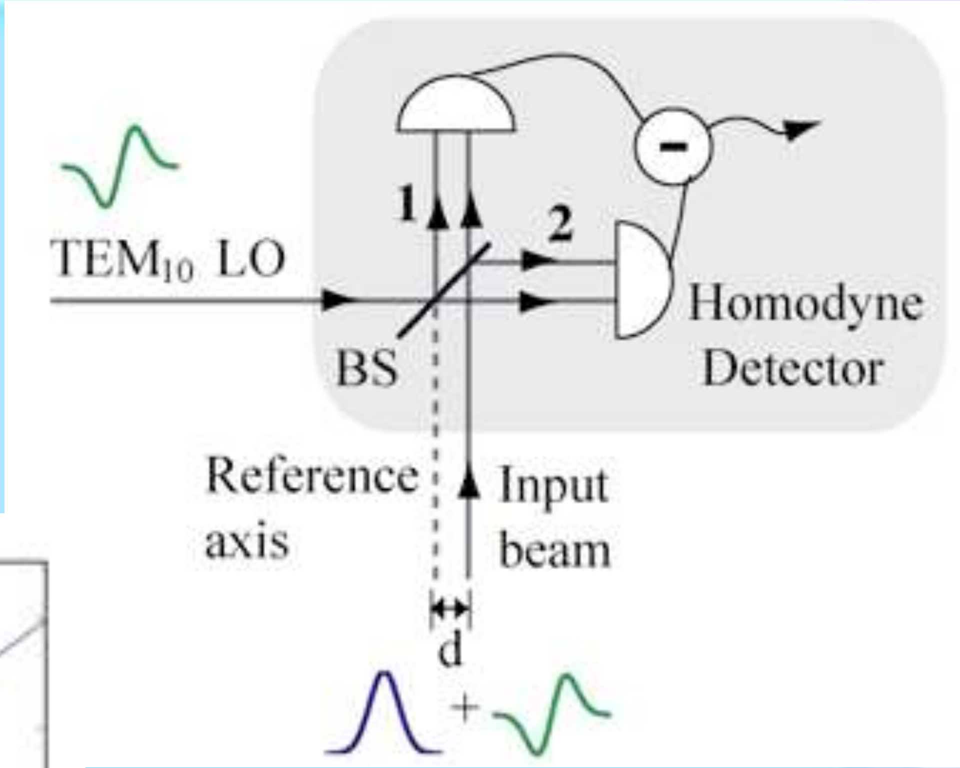


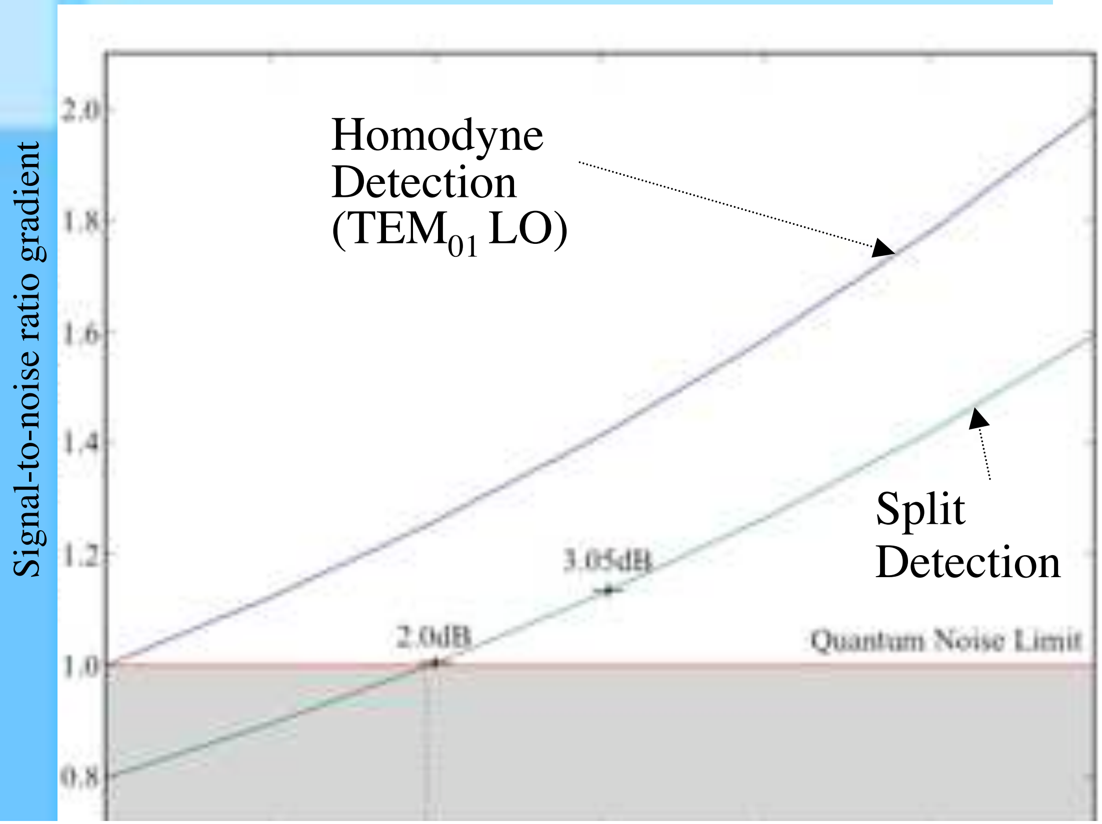


M. T. L. Hsu, V. Delaubert, P. K. Lam, W. P. Bowen Optimum small optical beam displacement measurement, *quant-phys* 0407209

Optimum displacement measurement

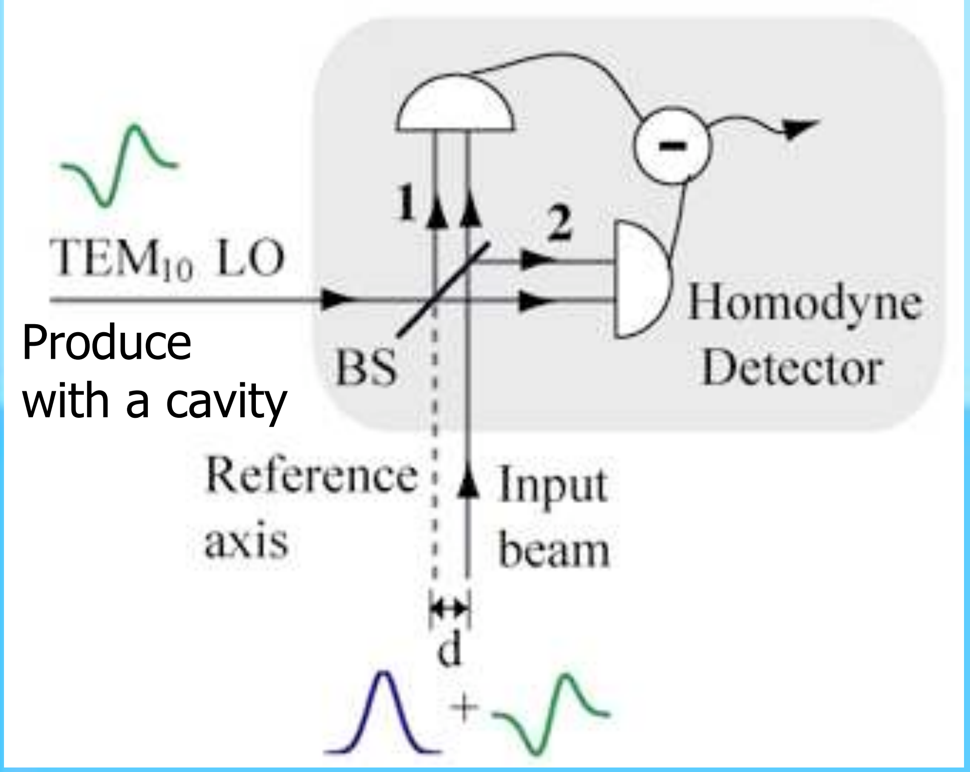
Homodyne detection





Optimum displacement measurement

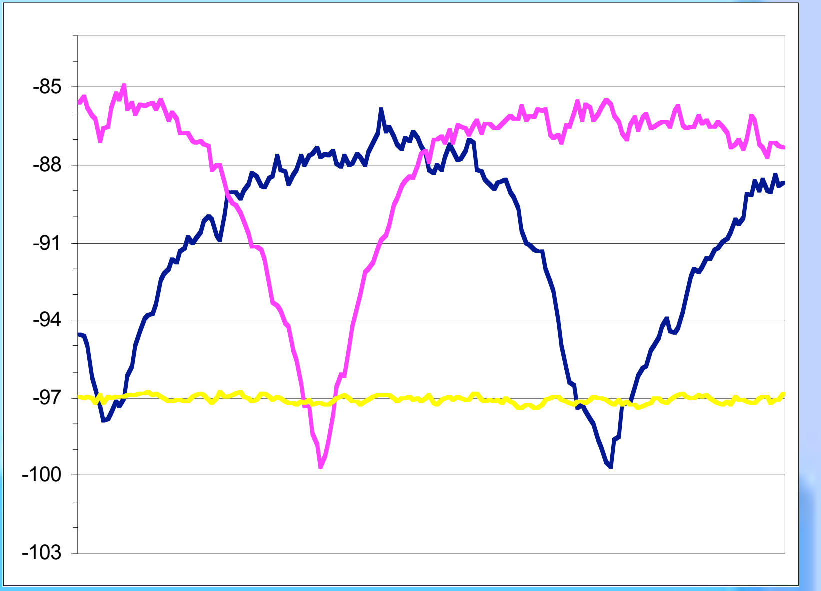
Experimental setup

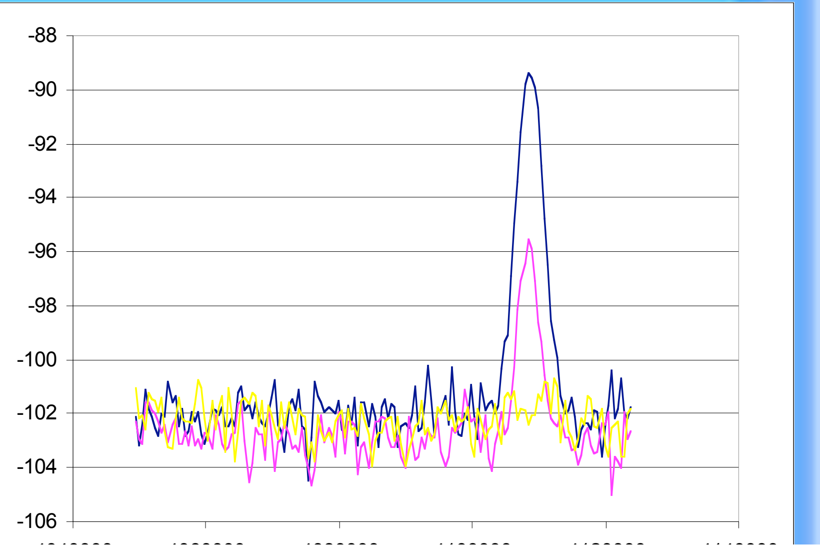


Produce
with a cavity

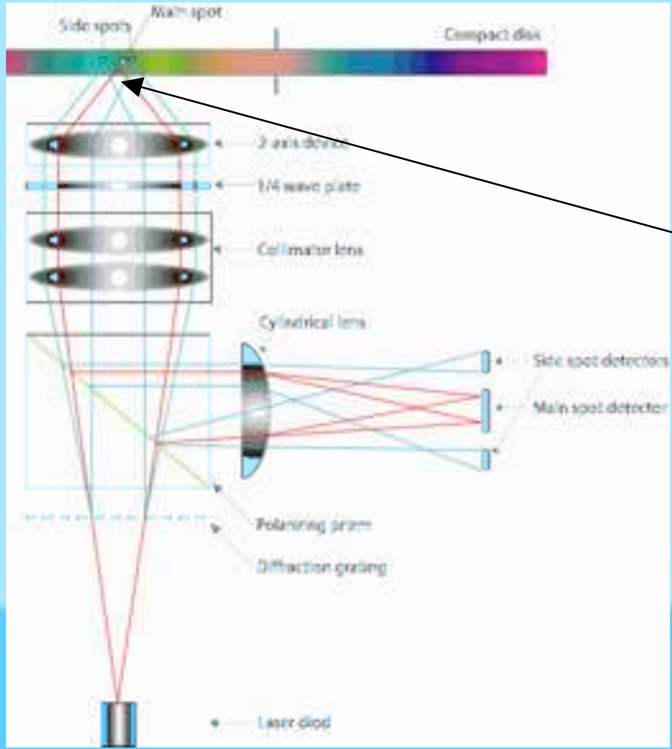
Mixing of the TEM_{00} mode and TEM_{01} squeezed vacuum via a Mach Zehnder interferometer.

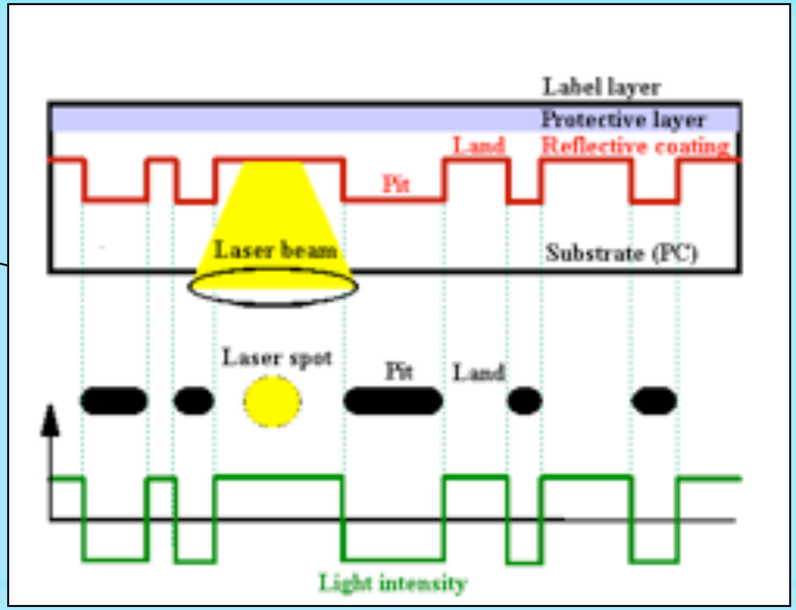
First results !!!





Other optimum measurement : optical read-out

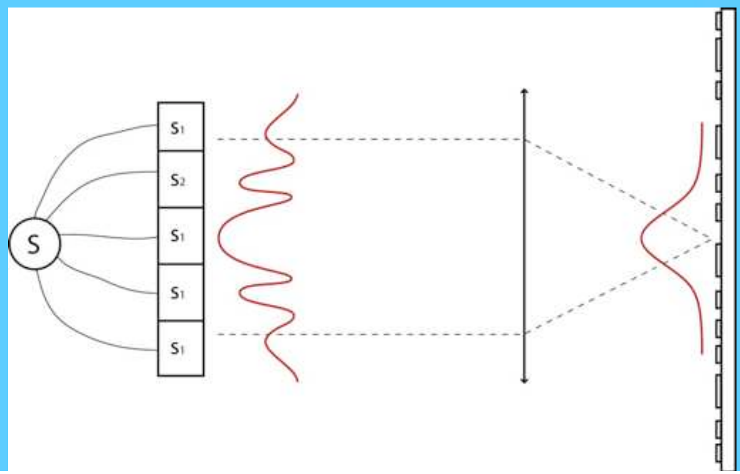




Bit density is limited by the wavelength : on bit per focal point

Read the reflected beam with an array detector.

- To each bit sequence : an appropriate gain settings
  → mean value is zero
- Find the corresponding detection mode to reduce the noise



Conclusion

- We have a proper definition of single mode vs. multimode light : compute necessary resources
- For each linear measurements it exists a detection mode : only way to reduce the noise is to squeeze it.
- -For further applications, the general signal study remains to be done.
  -Exploit the large Hilbert space to generate spatial entanglement.

People

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Claude Fabre
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Sylvain Gigan
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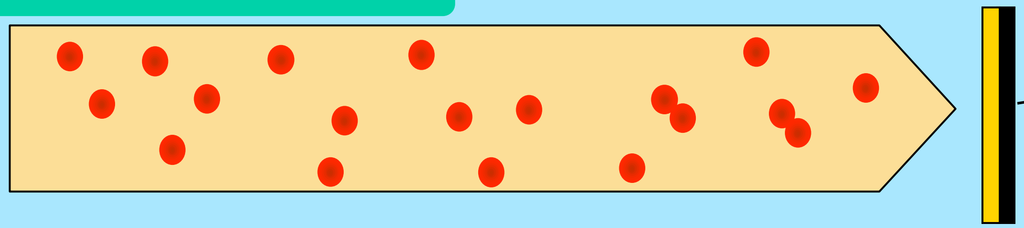
Ping Koy Lam

Warwick Bowen
Nicolai Grosse
Magnus Hsu

Single mode squeezed light

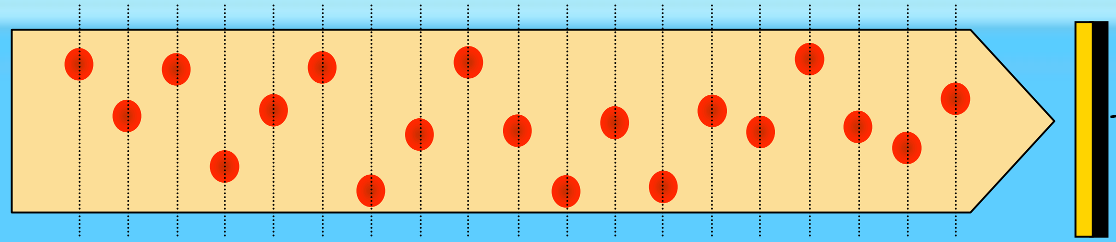
Photon picture



$$\left\langle \left(\delta \hat{X}_0^+\right)^2 \right\rangle = 1$$

Ordering the photons in time

No spatial order



$$\left\langle \left(\delta \hat{X}_0^+\right)^2 \right\rangle < 1$$

Modes picture

$$\hat{E}(\vec{r}) = \sum_i \hat{a}_i u_i(\vec{r})$$

$$\left\langle \hat{E}(\vec{r}) \right\rangle = \alpha_0 u_0(\vec{r})$$

$$\left\langle \delta \hat{N}^2 \right\rangle = N_0 \left\langle \left(\delta \hat{X}_0^+\right)^2 \right\rangle$$

Noise of mode 0