FERMIONS IN FLATLAND

MEERA PARISH

Levinsen & MMP, arXiv:1408.2737

Monash University

UQ colloquium, 24 April 2015
Equation that describes everyday world:

\[ i\hbar \frac{\partial}{\partial t} |\Psi\rangle = H |\Psi\rangle \]

\[
H = - \sum_{j}^{N_e} \frac{\hbar^2}{2m} \nabla_j^2 - \sum_{\alpha}^{N_n} \frac{\hbar^2}{2M_\alpha} \nabla_\alpha^2
- \sum_{j}^{N_e} \sum_{\alpha}^{N_n} \frac{Z_\alpha e^2}{|r_j - R_\alpha|} + \sum_{j<k}^{N_e} \frac{e^2}{|r_j - r_k|} + \sum_{\alpha<\beta}^{N_n} \frac{Z_\alpha Z_\beta e^2}{|R_\alpha - R_\beta|}
\]

- Rocks, trees, people, metals etc...

More is different...

- Complexity of problem grows exponentially with number of particles
- The behaviour of many interacting particles can be qualitatively different
  - Phase transitions
  - Broken symmetry
  - Collective excitations

P. W. Anderson, Science 177, 393 (1972)
“Quantum Simulators”

- Proposed by Feynman in 1982

- Basic idea: Simulate a complicated quantum system using a different (ideally simpler) quantum system

- Candidates for quantum simulators:
  - Trapped ions
  - Ultracold atomic gases
  - ...
OUTLINE

- Why consider 2D?
- Ultracold atomic gases
  - The quasi-2D geometry
- Simulating superconductivity
  - Fermion pairing
  - Behaviour in 2D vs 3D
- How 2D is 2D?
  - Effect of the 3rd dimension
The challenge of 2D

• The “marginal” dimension
  • Ordered phases only really exist at zero temperature

• Notoriously difficult to treat theoretically
  • Strong quantum fluctuations, but few exact solutions

• A variety of 2D systems in nature
  • Graphene, electron-hole bilayers, high-temperature superconductors, …

• Many poorly understood materials are quasi-2D, i.e. layered

✦ Idea: Gain insight into 2D phenomena using tunable, idealised systems
The cold-atom system

Bose-Einstein condensation first achieved in 1995 with $^{87}\text{Rb}$

the coldest matter in the universe...

E. Cornell & C. Wieman, JILA
The cold-atom system

- Atoms confined in magnetic or optical traps within a vacuum
- Low particle density: \(10^{13} \text{ cm}^{-3}\) (NB. air has \(10^{19} \text{ cm}^{-3}\))
- Highly tunable experimental parameters
  - Different atomic species (fermions or bosons)
  - Tunable short-range interactions
  - Dimensionality
    - …

![Diagram](image)

- Magnetic field
- \(1/a\) vs. 2-body bound state
- No bound state
- Quasi-2D geometry
- s-wave scattering length
Quasi-2D geometries

- Potential for single layer approx. harmonic: \( V(z) = \frac{1}{2} m \omega_z^2 z^2 \)
- Gas is kinematically 2D when \( k_B T \ll \hbar \omega_z \) and \( \varepsilon_F < \hbar \omega_z \)

\[ l_z = \sqrt{\frac{\hbar}{m \omega_z}} \]
Quasi-2D geometries

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- Gas is kinematically 2D when \( k_B T \ll \hbar \omega_z \) and \( \varepsilon_F < \hbar \omega_z \)
- Attractive interactions:
  - Always have 2-body bound state

\[ l_z = \sqrt{\frac{\hbar}{m \omega_z}} \]

2D limit: \( \varepsilon_B \approx \omega_z \exp(\sqrt{2\pi l_z/a_s}) \)
Pairing of fermions

- Consider simple case of $(\uparrow, \downarrow)$ Fermi gas interactions with attractive s-wave interactions
- Smooth crossover between pairing regimes by varying the interactions at fixed density:
  - Potential relevance to neutron stars, QCD, superconductors ...

Experiments.: MIT, JILA, Rice, Innsbruck, ENS, …

- Can also vary density with fixed interactions
  - must always have a 2-body bound state
2D Fermi gases

- Interaction regimes
  \[ \frac{\varepsilon_B}{\varepsilon_F} \ll 1 \quad \text{BCS} \quad \log(k_Fa_{2D}) \gg 1 \]
  \[ \frac{\varepsilon_B}{\varepsilon_F} \gg 1 \quad \text{BEC} \quad \log(k_Fa_{2D}) \ll -1 \]

- Mean-field theory
  - Order parameter \( \Delta = g \sum_k \langle c_{-k\downarrow}c_{k\uparrow} \rangle = \sqrt{2\varepsilon_F\varepsilon_B} \)
  - Chemical potential \( \mu = \varepsilon_F - \varepsilon_B/2 \)

- “Crossover point” at \( \mu = 0 \)
  - Change in quasiparticle dispersion:
    \[ E_k = \sqrt{(\epsilon_k - \mu)^2 + \Delta^2} \]

(Randeria et al. 1990)

Loss of the Fermi surface
2D Fermi gases

- Critical temperature for superfluidity:
Pseudogap phenomena

- Loss of spectral weight (a “gap”) at the Fermi surface observed above $T_c$ in the cuprates

- **Q:** can this be produced by pairing above $T_c$? *Loktev et al., Phys. Rep. 2001*

- Existence of pseudogap regime in 3D Fermi gases currently under debate
  
  *Magierski et al. PRL 2009; Gaebler et al., Nature Phys. 2010; Schneider & Randeria, PRA 2010 …*

- What about 2D?

*Pairing gap observed, but no Fermi surface?* *Feld et al., Nature 2011*
Pseudogap

The pseudogap regime is often synonymous with "pairing above $T_c$" in the cold-atom literature. However, such a scenario is trivially achieved in a classical gas of diatomic molecules, where the gap in the spectrum corresponds to the dimer binding energy. To reproduce the phenomenology of high-$T_c$ superconductors, one requires the presence of a Fermi surface, since the pseudogap in these systems manifests itself as a loss of spectral weight at the Fermi surface.

Indeed, it is not a priori obvious that such a phenomenon can be replicated with an attractive Fermi gas: a large attraction will surely lead to a pronounced pairing gap above $T_c$, but it will also destroy the Fermi surface. It is therefore reasonable to assume that any pseudogap regime must have $\mu > 0$ in addition to pairing, as schematically depicted in Fig. 15.

Fig. 18. The occupied part of the spectral function at $\ln(k_F a^2D) = 0$. (left) Measured momentum-resolved photoemission signal, taken from Ref. 10. (right) Theoretical prediction at $T/T_F = 1$ from the virial expansion up to second order.

The white dashed line marks the edge of the band of bound dimers (the incoherent part of the spectrum) and corresponds to the free atom dispersion shifted by the two-body binding energy, i.e., $\varepsilon_k = b_k$.

High-temperature effect?

- Comparison of experiment and theory for strong attraction
PSEUDOGAP REGIME?

- Experiments too hot and too far in Bose regime

![Graph showing the phase diagram throughout the BCS-Bose crossover. The critical temperature for superfluidity is represented by the solid line, and corresponds to an interpolation between the known limits. The dashed lines correspond to the onset of pairing. The graph shows the variation of \( T_c \) with \( \ln(k_F a_{2D}) \) and \( \mu \approx 0 \).]
**EQUATION OF STATE**

- Qualitatively different behaviour in normal state $T > T_c$

\[ \beta \equiv \frac{1}{k_B T} \]

**3D unitary Fermi gas**

Ku et al, Science 335, 563 (2012)

**2D Fermi gas**

Bauer, MMP & Enss, PRL 112, 135302 (2014)
Equation of state - Pressure

FIG. 2 (color online). Normalized local pressure vs interaction parameter. Scale-invariant assumption except finite temperature, are corrected for as explained in text. Data in table form are in Ref. [17].

Ideal Fermi liquid at $a^2 \sqrt{n_2} = 0.1$.

The pressure of an ideal Fermi gas is $\frac{P_{2}}{P_{2\text{ideal}}}$. This estimate is of sufficient precision to agree with a model of a finite-temperature locally homogeneous and mesoscopic regimes. According to this model, the system is ferromagnetic with $\mu_0 > E_F$.

The temperature in the units of the local Fermi energy $\omega_F$ is 19% higher. Unexpectedly, the data agree with this. However, neither an unaccounted population of excited states nor a pairing gap would be evenly distributed in the range $a^2 \sqrt{n_2} = 0.1$.

The temperature $T$ is larger than the rms cloud size $p_{2\text{ideal}}$ such that $\pi \exp \left( \frac{\mu_0}{T} \right)$ is slow.

The pressure calculations are made for each datum as $\frac{P_{2}}{P_{2\text{ideal}}} = \frac{\ln \left( 1 + \frac{\mu_0}{\omega_F} \right)}{\pi \exp \left( \frac{\mu_0}{\omega_F} \right)}$. In further discussion, the borders are attached with a thicker border.

In the deep Bose regime, the pressure on average is 10% above $\frac{P_{2}}{P_{2\text{ideal}}} = \frac{\ln \left( 1 + \frac{\mu_0}{\omega_F} \right)}{\pi \exp \left( \frac{\mu_0}{\omega_F} \right)}$. In the data, the scaling coefficient is 19% higher. Unexpectedly, the data agree with this model.

For a weakly attractive Fermi gas, the gap $\epsilon_0$ increases from 0 to 0.08, which gives an $a^2 \sqrt{n_2} = 0.1$ confidence interval.

The pairing energy is even larger than the rms cloud size $p_{2\text{ideal}}$ such that $\pi \exp \left( \frac{\mu_0}{T} \right)$ is slow.

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Equation of state - Pressure

- 2D theory consistent with experiment in weakly attractive regime
Equation of state - Pressure

Experiment: $\varepsilon_F \approx \hbar \omega_z$

Makhalov et al., PRL 2014
Quasi-2D effects?
Quasi-2D problem

- Consider energy levels of harmonic confinement
  - Effect of interactions:
Quasi-2D problem

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\[ \hat{H} = \sum_{\mathbf{k}, n, \sigma} (\epsilon_{\mathbf{k}n} - \mu) c_{\mathbf{k}n\sigma}^\dagger c_{\mathbf{k}n\sigma} + \sum_{\mathbf{k}, n_1, n_2, \mathbf{k}', n_3, n_4, q} \langle n_1 n_2 | \hat{g} | n_3 n_4 \rangle c_{\mathbf{k}n_1\uparrow}^\dagger c_{q-kn_2\downarrow}^\dagger c_{q-k'n_3\downarrow} c_{k'n_4\uparrow} \]

\[ g \sum_{N, \nu, \nu'} \phi_{\nu}(0) \phi_{\nu'}(0) \langle n_1 n_2 | N \nu \rangle \langle N \nu' | n_1 n_2 \rangle \equiv g \sum_N V_{N}^{n_1 n_2} V_{N}^{n_3 n_4} \]

\[ \hat{H}_{\text{MF}} = \sum_{\mathbf{k}, n, \sigma} (\epsilon_{\mathbf{k}n} - \mu) c_{\mathbf{k}n\sigma}^\dagger c_{\mathbf{k}n\sigma} + \sum_{\mathbf{q}, N} \left( \Delta_{\mathbf{q}N} \sum_{\mathbf{k}, n_1, n_2} V_{N}^{n_1 n_2} c_{\mathbf{k}n_1\uparrow}^\dagger c_{q-kn_2\downarrow} + \Delta_{\mathbf{q}N}^* \sum_{\mathbf{k}', n_3, n_4} V_{N}^{n_3 n_4} c_{q-k'n_3\downarrow} c_{k'n_4\uparrow} - \frac{|\Delta_{\mathbf{q}N}|^2}{g} \right) \]

\[ \text{Assumption: } \Delta_{\mathbf{q}N} = \delta_{q0} \delta_{N0} \Delta_0 \]

(See also: Martikainen & Torma, PRL 2005)
Critical temperature

\[ \frac{1}{g} = \frac{1}{2} \sum_{k,n,m} (V_0^{nm})^2 \frac{\tanh(\beta_c \xi_{kn}/2) + \tanh(\beta_c \xi_{km}/2)}{\xi_{kn} + \xi_{km}} \]

\[ \xi_{kn} = \epsilon_k + n\hbar\omega - \mu \]

\[ \beta_c \equiv \frac{1}{k_B T_c} \]

\[ \varepsilon_B/\varepsilon_F = 0.05 \]

\[ \varepsilon_B/\varepsilon_F = 0.01 \]

2D limit

Fischer & MMP, PRB 90, 214503 (2014)
Observation of $T_c$

\[ \varepsilon_F \approx \hbar \omega_z \]

Graph showing $T/T_F$ vs. $\ln(k_F a_{2D})$ with color representing $N_q/N$. Points indicate data, and lines are fits to the regimes above and below the phase transition.

Higher $T_c$ due to system being quasi-2D?

Ries et al, arXiv:1409.5373
Concluding remarks

- Idealised 2D Fermi systems can be investigated with cold atoms
  - Gain insight into 2D phenomena
- BCS-BEC crossover displays qualitatively different behaviour from that in 3D
- “Quasi-2D-ness” can significantly impact pairing even for weak interactions
  - Perturbing away from 2D appears to enhance $T_c$
- Outlook: spin imbalance and frustrated pairing
Acknowledgements

- Jesper Levinsen, Monash
- Andrea Fischer, UCL
- Wave Ngampruetikorn, OIST
- Marianne Bauer, Munich
- Tilman Enss, Heidelberg

Bauer, MMP & Enss, PRL 112, 135302 (2014)
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Ngampruetikorn, Levinsen & MMP, PRL 111, 265301 (2013)
Acknowledgements

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