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Optical Measurement of microscopic torques

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Abstract

In recent years there has been an explosive development of interest in the measurement of forces at the microscopic level, such as within living cells [1, 2, 3], as well as the properties of fluids and suspensions on this scale [4], using optically trapped particles as probes. The next step would be to measure torques and associated rotational motion [5]. This would allow measurement on very small scales since no translational motion is needed. It could also provide an absolute measurement of the forces holding a stationary non-rotating particle in place. The laser-induced torque acting on an optically trapped microscopic birefringent particle [6] can be used for these measurements. Here we present a new method for simple, robust, accurate, simultaneous measurement of the rotation speed of a laser trapped birefringent particle, and the optical torque acting on it, by measuring the change in angular momentum of the light from passing through the particle. This method does not depend on the size or shape of the particle or the laser beam geometry, nor does it depend on the properties of the surrounding medium. This could allow accurate measurement of viscosity on a microscopic scale.

1 Introduction

Optical torques have been measured previously; two methods have been used. Firstly, if a particle with known birefringent properties has a simple and accurately known size and shape, ideally a flat disc, the torque can be calculated from the beam power [6]. However, particles of more complex shapes will often be used in experiments or encountered in samples; for example, spherical particles are ideal for making measurements of viscosity. Secondly, torques have been determined by measuring the rotation speed in a medium of known viscosity [6, 7]. This method cannot be used if the aim is to measure an unknown viscosity. This method would also fail if there were other torques acting on the particle, or if the viscous drag is affected by nearby walls or other particles, or if the particle is not rotating. Previous methods for measuring rotation speeds, based on the periodic variation of backscattered light [8], can also have problems. Very regular or rotationally symmetric particles provide insufficient variation or variation at an increased frequency. Consideration of the basic physical processes giving rise to the torque gives a new method for measuring the torque and rotation speed that overcomes all of these problems.

Since optical torques and forces are very small, microscopic particles are ideal for the observation and application of optical torques and rotation. Such microscopic particles will typically be confined within a laser trap. Strongly focussed laser light incident on a transparent particle, usually in a liquid medium, will produce a gradient force acting on the particle towards the region of highest irradiance. If this gradient force near the focus is stronger than scattering and absorption forces, the particle will be trapped at the beam focus, where the irradiance is highest. This technique of three-dimensional confinement and manipulation is called laser micro-manipulation, trapping, or optical tweezers.

2 Polarised beams

A monochromatic laser beam can be written as a plane wave in terms of two orthogonal components:

$$\mathbf{E} = (E_x\hat{\mathbf{x}} + E_y\hat{\mathbf{y}})\exp(ikz - i\omega t) \tag{1}$$

where the beam is propagating in the z-direction. In general, the amplitudes E_x and E_y are complex in order to account for the phases of the components.

There are two cases of special interest. The first is when the phase angle between the complex amplitudes E_x and E_y is equal to 0 or π, in which case the total electric field always lies in a single plane resulting in a beam which is linearly polarised. The direction of the x-axis can be chosen to coincide with the plane of polarisation, so the beam can be written as

$$\mathbf{E} = E_p\hat{\mathbf{x}}\exp(ikz - i\omega t) \tag{2}$$

where E_p is the complex amplitude of the linearly polarised light

The second special case is when the phase angle is $\pm\pi/2$, and $|E_x| = |E_y|$. In this case, $E_y = E_x \exp i\theta$, with $\theta = \pm\pi/2$. The total electric field has a constant magnitude, with the direction varying with the optical frequency ω so that the beam is circularly polarised. When $\theta = +\pi/2$, the electric field has a positive, or right-handed, helicity. Such a beam is here called left circularly polarised. When $\theta = -\pi/2$, the beam has negative helicity and is called right circularly polarised. A circularly polarised beam can always be written as

$$\mathbf{E} = (E_c\hat{\mathbf{x}} \pm iE_c\hat{\mathbf{y}})\exp(ikz - i\omega t) \tag{3}$$

with the sign depending on whether the beam is left or right circularly polarised, and E_c is the complex amplitude.

In general, however, the phase angle θ will have a value between these limiting values, or even if $\theta = \pm\pi/2$, $|E_x| \neq |E_y|$. In these cases, the beam is elliptically polarised, and the electric field vector $\mathbf{E}$ traces out an ellipse during each optical period.

Recognising that we can rewrite equation (3) for a circularly polarised beam as

$$\mathbf{E} = E_c(\hat{\mathbf{x}} \pm i\hat{\mathbf{y}}) \exp(ikz - i\omega t), \tag{4}$$

we see that any beam can be represented as a sum of two circularly polarised components using the (complex) orthogonal basis vectors

$$\begin{aligned} \hat{\mathbf{e}}_L &= \frac{1}{\sqrt{2}}(\hat{\mathbf{x}} + i\hat{\mathbf{y}}) \\ \hat{\mathbf{e}}_R &= \frac{1}{\sqrt{2}}(\hat{\mathbf{x}} - i\hat{\mathbf{y}}) \end{aligned} \tag{5}$$

as

$$\mathbf{E} = (E_L\hat{\mathbf{e}}_L + E_R\hat{\mathbf{e}}_R) \exp(ikz - i\omega t). \tag{6}$$

The amplitudes of the left and right circular components can be found from the x and y amplitudes in the linear orthogonal representation (equation (1)):

$$\begin{aligned} E_L &= \frac{1}{\sqrt{2}}(E_x - iE_y) \\ E_R &= \frac{1}{\sqrt{2}}(E_x + iE_y) \end{aligned} \tag{7}$$

When $|E_L| = |E_R|$, the beam is linearly polarised, with the plane of polarisation given by the phase angle between the complex amplitudes E_L and E_R. If $E_L = 0$ (and $E_R \neq 0$), the beam is right circularly polarised, and left circularly polarised if $E_R = 0$.

The time-averaged irradiance is given by [9]

$$I = \frac{c\epsilon_0 E_L^\star E_L}{2} + \frac{c\epsilon_0 E_R^\star E_R}{2} = I_L + I_R. \tag{8}$$

Although we have only considered the beam as a classical EM wave so far, the fact that the angular momentum of left and right circularly polarised photons is $\pm\hbar$ can be used to simply find the angular momentum of the beam. Since the energy of a photon is $\hbar\omega$, the photon flux per unit area is

$$N = \frac{I}{\hbar\omega} = \frac{I_L}{\hbar\omega} + \frac{I_R}{\hbar\omega}, \tag{9}$$

giving an angular momentum flux per unit area of

$$L_z = (I_L - I_R)/\omega. \tag{10}$$

Thus, the beam can be considered to have a net circularly polarised component with a power of $|I_L - I_R|$ which contributes to the angular momentum of the beam, and a linearly polarised component of $I - |I_L - I_R| = 2\min(I_L, I_R)$ which does not contribute to the angular momentum of the beam. We can define a coefficient of circular polarisation σ_z by

$$\sigma_z = (I_L - I_R)/I, \tag{11}$$

and write the angular momentum flux density of the beam as

$$L_z = \sigma_z I/\omega. \tag{12}$$

When the irradiance is integrated across the whole beam, the total power can be obtained and will be given by $P = P_L + P_R = \int I_L dA + \int I_R dA$. A suitable average coefficient of circular polarisation can be defined by

$$\sigma_z = (P_L - P_R)/P, \tag{13}$$

with the resulting total angular momentum flux of the beam being

$$L_z = \sigma_z P/\omega. \tag{14}$$

3 Optical torque

If the beam passes through some birefringent material, the polarisation will be affected. In general, σ_z will change. The incident beam will have an initial coefficient of circular polarisation σ_{zin}, and will have an emergent polarisation described by σ_{zout}. Thus the angular momentum of the beam will change, and a reaction torque on the birefringent material will result. The reaction torque is equal to the change in the angular momentum flux:

$$\tau = (\sigma_{zin} - \sigma_{zout}) P/\omega \tag{15}$$

assuming that absorption and reflection can be ignored. Equation (15) is general, and can always be used to find the torque if the coefficients of circular polarisation of the incident and outgoing beams are known or can be found.

Although equation (15) applies in general, it is instructive to carry through a detailed calculation for a simple case: a uniform sheet of birefringent material, for example calcite.

A uniaxial birefringent material such as calcite can be described by two refractive indices: an *ordinary refractive index* n_o for electric fields normal to the optic axis, and an *extraordinary refractive index* n_e for electric fields parallel to the optic axis. For calcite, $n_o = 1.66$ and $n_e = 1.49$. Consider a thickness d of uniaxial birefringent material with the optic axis in the xy plane, at an angle of θ to the x-axis. The front face of the material is at $z = z_0$, and the rear face is at $z = z_0 + d$. If the electric field of the incident beam at the front surface of the material is given by equation (1), we can express this in terms of unit vectors $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ parallel to and normal to the optic axis:

$$\begin{aligned} \mathbf{E} \;=\; & [(E_x \cos\theta + E_y \sin\theta)\hat{\mathbf{i}} + (-E_x \sin\theta + E_y \cos\theta)\hat{\mathbf{j}}] \\ & \times \exp(ikz_0 - i\omega t). \end{aligned} \tag{16}$$

In terms of circular components, this gives

$$\begin{aligned}\mathbf{E} &= \frac{1}{\sqrt{2}}[(E_x - iE_y)\exp(i\theta)\hat{\mathbf{e}}'_L + (E_x + iE_y)\exp(-i\theta)\hat{\mathbf{e}}'_R] \\ &\quad \times \exp(ikz_0 - i\omega t) \qquad (17)\end{aligned}$$

where $\hat{\mathbf{e}}'_L = \frac{1}{\sqrt{2}}(\hat{\mathbf{i}}+i\hat{\mathbf{j}})$ and $\hat{\mathbf{e}}'_R = \frac{1}{\sqrt{2}}(\hat{\mathbf{i}}-i\hat{\mathbf{j}})$ The coefficient of circular polarisation is given by

$$\begin{aligned}\sigma_{zin} &= \frac{E_L^\star E_L - E_R^\star E_R}{E_L^\star E_L + E_R^\star E_R} \\ &= \frac{i(E_x E_y^\star - E_x^\star E_y)}{E_x^\star E_x + E_y^\star E_y}. \qquad (18)\end{aligned}$$

After passing through the thickness d, the field will be

$$\begin{aligned}\mathbf{E} &= [(E_x\cos\theta + E_y\sin\theta)\exp(ikdn_e)\hat{\mathbf{i}} \\ &\quad +(-E_x\sin\theta + E_y\cos\theta)\exp(ikdn_o)\hat{\mathbf{j}}]\exp(ikz_0 - i\omega t), \qquad (19)\end{aligned}$$

which we can express in terms of circular components

$$\begin{aligned}E_L &= \frac{1}{\sqrt{2}}[(E_x\cos\theta + E_y\sin\theta)\exp(ikdn_e) \\ &\quad -i(-E_x\cos\theta + E_y\sin\theta)\exp(ikdn_o)] \\ E_R &= \frac{1}{\sqrt{2}}[(E_x\cos\theta + E_y\sin\theta)\exp(ikdn_e) \\ &\quad +i(-E_x\cos\theta + E_y\sin\theta)\exp(ikdn_o)] \qquad (20)\end{aligned}$$

We define the convenient notation

$$\Delta = kd(n_o - n_e) \qquad (21)$$

The coefficient of circular polarisation of the emergent light is

$$\sigma_{zout} = [i\cos\Delta(E_x E_y^\star - E_x^\star E_y)$$

$$
\begin{aligned}
& - \sin\Delta\{(E_x^\star E_x - E_y^\star E_y)\sin 2\theta - (E_x E_y^\star + E_x^\star E_y)\cos 2\theta\}] \\
& /(E_x^\star E_x + E_y^\star E_y)
\end{aligned} \tag{22}
$$

giving a torque per unit area of

$$
\begin{aligned}
\tau &= \frac{c\epsilon_0}{2\omega}[i(E_x E_y^\star - E_x^\star E_y)(1 - \cos\Delta) \\
& + \sin\Delta\{(E_x^\star E_x - E_y^\star E_y)\sin 2\theta - (E_x E_y^\star + E_x^\star E_y)\cos 2\theta\}]
\end{aligned} \tag{23}
$$

If the incident light is linearly polarised ($E_y = 0$), the torque is

$$
\tau = \frac{c\epsilon_0}{2\omega}\sin\Delta E_0^\star E_0 \sin 2\theta, \tag{24}
$$

which acts to align the slow axis of the particle with the plane of polarisation if $n_o > n_e$, or normal to the plane of polarisation if $n_e > n_o$. If the incident light is left circularly polarised ($E_y = iE_x$), the torque is

$$
\tau = \frac{c\epsilon_0}{\omega} E_0^\star E_0 (1 - \cos\Delta) \tag{25}
$$

which is independent of the orientation of the birefringent material.

If the birefringent material is of a uniform thickness, the total torque can be simply calculated from this [6]. In general, a laser trapped birefringent particle will have a varying thickness, and direct calculation of the torque will not be feasible. Also, if the orientation of the birefringent particle is different, so the the optic axis does not lie in the xy plane, the calculation will be further complicated. Equation (15), however, is general, and will still apply, and the torque acting on the particle can be deduced from the change in polarisation of the light.

4 Optical torque measurement

Consider a circularly polarised laser beam used to trap a microscopic particle composed of a uniaxial birefringent material such as calcite or a suitable polymer. If the optical torque is large enough to overcome forces holding the particle in place, the particle will rotate at a speed determined by the equilibrium between the optical torque and other forces such as viscous drag. In this way, a probe particle can be used to measure viscosity on a microscopic scale. If the particle does not rotate, the optical torque can be used to determine the torque due to static forces acting on the particle.

The maximum torque and rotation rate will occur when the incident beam is completely circularly polarised (ie $\sigma_{zin} = 1$). The torque in this case will also be constant as well as maximal [6], and we will only consider this case here. The torque τ acting on the trapped particle is given in equation (15) by the difference between the incident and outgoing angular momentum fluxes, and in this case, assuming no reflection or absorption, is

$$\tau = (1 - \sigma_{zout})P/\omega. \tag{26}$$

Measurement of the outgoing polarisation σ_{zout} and beam power P gives an absolute measurement of the torque, which does not depend on the mechanical properties of the surrounding medium or the particular size or shape of the particle or laser beam.

We can also note that the plane of polarisation of the linearly polarised component of the outgoing beam exiting a rotating birefringent particle will be rotating at the same rotation rate Ω as the particle. If the outgoing beam is a (rotating) purely plane-polarised beam, as would occur if the particle acted as a quarter-wave plate, rotating at Ω, and of power P, and is passed through a linear polariser, the measured power P_m will be $P_m = (1 + \cos 2\Omega t)P/2$ (with variation at a frequency of 2Ω since a rotation of 180° rotates the plane of polarisation onto itself) [10]. By measuring this power, the rotation rate Ω of the trapped particle can be simply determined. This will still be the case for an elliptically polarised beam, as the same variation at a frequency of 2Ω will be observed. The angular momentum associated with this rotation of the plane of polarisation will be negligible as $\Omega \ll \omega$.

In the general case, there will be an elliptically polarised outgoing beam, consisting of both plane and circularly polarised components. The power of the two components will be $P_c = |\sigma_{zout}|P$ for the circularly polarised component, and $P_p = (1 - |\sigma_{zout}|)P$ for the plane polarised component. The measured power P_m after the outgoing beam passes through a linear polariser acting as an analyser will be

$$P_m = \{1 + (1 - |\sigma_{zout}|) \cos 2\Omega t\} P/2. \tag{27}$$

Measurement of the variation of the transmitted power therefore allows the determination of the rotation period of the trapped particle, and the degree of (but not the direction of) circular polarisation. The result of a measurement of this type will be as shown in figure 1. This measurement is an average over the beam, and it is not important whether or not the entire beam passes through the particle. In the case where the particle is not rotating, due to some restraining torque, the plane of polarisation of the transmitted light will not be rotating. The degree of circular polarisation can be measured in this case by rotating the linear polariser, which will give the same result where Ω is the rotation rate of the polariser relative to the particle. The orientation of the particle can also be determined from the position of the measured power maxima.

In many cases, the direction of the transmitted polarisation will be known beforehand – such as when the particle is insufficiently thick to change the direction of polarisation (note that a calcite particle approximately $3\mu m$ thick is a $\lambda/2$ plate for 1064 nm light), or when the particle is small and the outgoing light is dominated by light that did not pass through the particle and has not changed in polarisation. If necessary, the direction of circular polarisation can

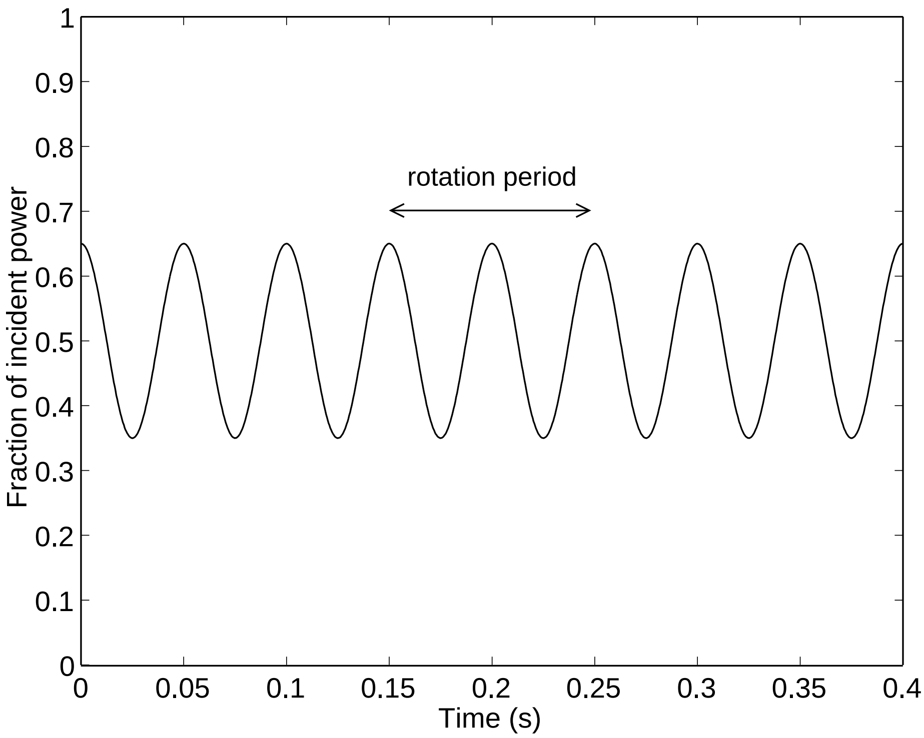

Figure 1: The power which would be measured through a plane polariser after the beam has passed through a birefringent particle is shown for a beam with $\sigma_{zin} = +1$, $\sigma_{zout} = +0.7$ and particle rotation frequency $\Omega = 10$ Hz. The mean power measured through the polariser is half of the power incident on the particle. The frequency of the variation is two times the rotation rate Ω of the particle. The optical torque can be found from the amplitude of the variation and the measured power once the direction of the transmitted polarisation is known. In this case, for a 100 mW trapping beam of wavelength 1,064 nm, an optical torque of 16.9 pN$\cdot\mu$m is being exerted.

be measured simply by placing a reversed circular polariser (eg a quarter-wave plate followed by a linear polariser appropriately oriented) in the beam path instead of a linear polariser. In the case where the trapping beam has a right-handed helicity (left circularly polarised, with $\sigma_{zin} = +1$), the light emergent from the particle can be described in terms of left and right circularly polarised components P_L and P_R, where $P_L = (1 + \sigma_{zout})P/2$ and $P_R = (1 - \sigma_{zout})P/2$. If the output beam is predominantly left circularly polarised, $\sigma_{zout} > 0$, and $P_L > P_R$. A right circularly polarised beam has $\sigma_{zout} < 0$, and $P_R > P_L$. It is only necessary to determine which of these two components is larger, rather than to measure each one individually since $|\sigma_{zout}|$ is already known. In this way, the direction of circular polarisation can be determined, and σ_{zout} as opposed to merely $|\sigma_{zout}|$ can be found. Once σ_{zout} is known, the optical torque acting on the particle can be found using equation (15). A measurement of this type will be as shown in figure 2.

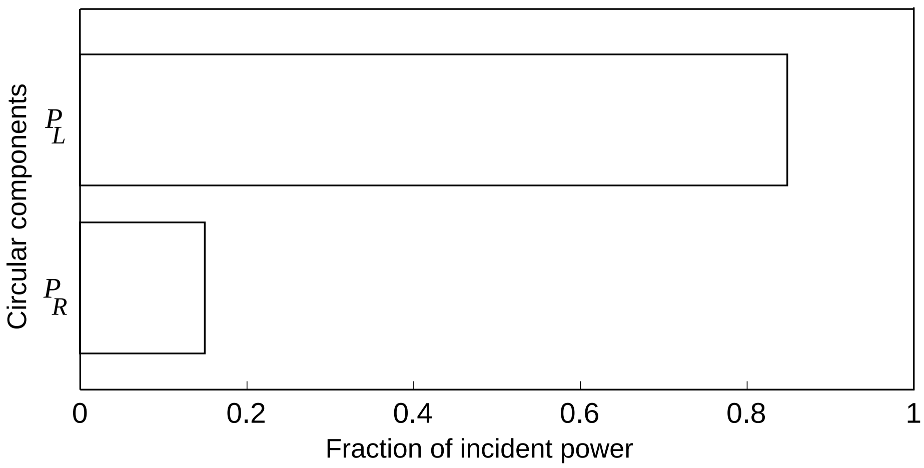

Figure 2: The two circularly polarised components of the transmitteed light can be measured to determine the direction of circular polarisation. P_L is the power of the left circularly polarised component, and P_R is the power of the right circularly polarised component.

It should be noted that this technique is robust. It is not necessary to measure the power of the entire transmitted beam; it is sufficient to measure the portion of the beam that has passed through the trapped particle. Similarly, reflections are not likely to cause significant error. Some of the incident beam will be reflected from the trapped particle; the reflection will depend on the angle of incidence and the refractive indices of the particle and the surrounding medium. For example, for calcite trapped in water, the Fresnel amplitude coefficients for reflection at normal incidence are $(n_{water} - n_{calcite})/(n_{water} + n_{calcite})$, which gives reflected amplitudes of $-0.057E_x$ and $-0.11iE_x$ for linearly polarised components parallel to and normal to the optic axis respectively. In terms of circular components, this becomes $E_L = -0.02E_{L0}$ and $E_R = -0.08E_{L0}$, showing that the torque due to backreflected light will be less than 0.6% of the available torque. Therefore, the reflected light will not cause any significant error.

5 Conclusion

A simple method of measuring the rotation speed and the optical torque applied to a laser trapped birefringent particle has been described. This method can be used even if the viscosity of the medium in which trapping is performed is unknown, and provides a means to measure this viscosity. Thus, this method is suitable for employment in a micro-rheometer, which could be simply constructed by trapping a birefringent probe particle in the fluid of interest. A suitable test particle would be a small fragment of calcite, the exact shape not being critical at the very low Reynolds numbers encountered in these cases, or a more ideal shape could be fabricated from a birefringent polymer [11]. As the optical torque can be controlled by varying the power, the probe particle rotation speed can be varied, allowing, for example, the investigation of non-linear properties of the fluid. Wall effects and the small-scale behaviour of polymer and colloidal suspensions could be investigated, or even the rheological properties of intracellular fluids or membranes in vivo.

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