# General relativity notes: Overview

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## I. GRAVITY

General relativity is Einstein's theory of gravity. Gravity has many interesting properties when compared to the other known forces in nature (the electromagnetic force, the weak and strong nuclear forces).

(Following shamelessly cribbed from Hartle Chapter 1) Hand out figure 1.1.

- In Newtonian physics gravity is a universal force between all mass, in special relativity gravity should be a force (at least) between all forms of energy.
- Gravity is unscreened. No negative gravitational charges exist and gravity is always attractive*.
- Gravity is a long range interaction because of the $1/r^2$ law (this fact is something to do with the previous one, screening usually means electromagnetism is not very long range in practice, the other forces tend to die off very rapidly).
- Gravity is the weakest of the fundamental interactions. Compare the (repulsive) electromagnetic force between two protons with their gravitational attraction

$$\frac{Gm_P^2/r^2}{e^2/(4\pi\epsilon_0 r^2)} = \frac{Gm_P^2}{e^2/(4\pi\epsilon_0)} \sim 10^{-36} \tag{1}$$

Since electromagnetic interactions screen charge and lead to the formation of neutral clumps of matter like atoms, gravitational effects are left to dominate physics on the very large length scales associated with astrophysics and cosmology.

### A. When do we need a theory of gravity consistent with special relativity?

Special relativity is important when particle velocities can become comparable with the speed of light.

So for example we could ask when the Newtonian gravitational force associated with a mass $M$ is enough to keep a test mass $m$ in a circular orbit with velocity $c$ at radius $R$?

$$\frac{GMm}{R^2} = \frac{mc^2}{R} \tag{2}$$

That is when

$$\frac{GM}{Rc^2} = 1. \tag{3}$$

One might imagine that this unitless quantity characterizes the importance of relativistic effects in gravitational problems. Convince yourself that all of the fundamental constants you expect to be there are there. Clearly general relativity should be significant for large masses, small separations and by implication high densities.

One might try the estimation differently. Suppose we consider a particle in orbit around a mass $M$ are radius $R$ and wish to give it a vertical velocity component just sufficient for it to escape from orbit. Again using Newtonian formulae we can work out the required vertical velocity by asking that the kinetic energy exactly balances the negative potential energy of the particle due to the gravitational field

$$\frac{1}{2}mv_{\text{escape}}^2 = \frac{GMm}{R}. \tag{4}$$

By our previous criterion we find that the escape velocity is equal to $c$ when

$$\frac{GM}{Rc^2} = \frac{1}{2} \tag{5}$$

So up to an irrelevant constant of order one we find exactly the same unitless parameter controlling when we expect relativistic effects to be significant in describing gravity.

### B. Historical sequence of tests for general relativity

We will have more to say about each of these below and in the course proper.

- 1905 Special relativity
- 1907 Einstein predicts qualitative effects of relativistic theory of gravity; gravitational redshift, deflection of light
- 1916 Final version of general relativity published with explanation of precession of perihelion of Mercury (much other stuff, weak field limit, gravitational waves, corrected prediction for light deflection)
- 1919 Eddington observes deflection of light and claims accuracy to distinguish general relativistic version, Einstein becomes famous with the general public.
- 1960 Pound and Rebka observe gravitational redshift for the first time
- 1968 Shapiro observes time delay in radar echoes from Venus (effect associated with light bending)
- 1979 Walsh, Carswell and Weyman observe first gravitational lenses (light bending again!)
- 1979 Taylor, Fowler and McCulloch observe decay of binary pulsar rotation frequency due to gravitational radiation

### C. Some applications for general relativity

Here is a brief list of important applications of general relativity, we will learn about many of these.

#### *1. Precision orbits about the earth and the sun and other astronomical objects*

We can ask ourselves whether we or low satellites are very relativistic in our motion about the earth. We get $GM_\oplus/c^2R_\oplus \sim 10^{-9}$, so not so much. However you may actually be driving round with a device that needs to take general relativistic effects into consideration in order to function. It turns out that the Global Positioning System requires sufficient accuracy that it takes into account both special and general relativistic time dilation effects as well as a host of other issues. (Do you know about the GPS system? Military and civilian applications, "smart bombs" navigation software in many taxis)

GPS functions by comparing time signals from a variety of satellite clocks. The signal from three satellites is enough to find our location and from four to track our motion even if we ourselves do not possess an accurate clock. The advertised precision of GPS is 2m, this turns out to require measurement of times and in particular time differences to better than 6ns. Atomic clocks find it easy to keep time that well but we already see that we expect special and general relativistic time dilation effects may be important even on times of a few seconds.

Consider a satellite orbiting the earth every twelve hours at radius $R_s$ from the centre of the earth (so $2\pi R_s/(12\text{hr}3600\text{s/hr}) = v_s$ where $v_s$ is the speed of the satellite). Once again gravity provides the centripetal acceleration and we find

$$R_s \sim 2.7\times10^4\text{km} \sim 4.2R_\oplus \tag{6}$$

$$v_s \sim 3.9\text{km/s}, \quad v_s/c \sim 1.3\times10^{-5} \tag{7}$$

If we compare a clock on this satellite to a clock in the inertial frame that is instaneously co-moving with the earth (warning this observer should be far from the earth so that this frame is really inertial but we are just estimating orders of magnitude here) we find the fractional correction for time dilation is roughly

$$\frac{1}{2}(\frac{v_s}{c})^2 \sim 0.84\times10^{-10} \tag{8}$$

while our figure of merit (which turns out to be the same as something called the gravitational redshift or sometimes the general relativistic time dilation) is roughly

$$\frac{GM_\oplus}{R_sc^2} \sim 1.6\times10^{-10} \tag{9}$$

Notice that the earth is heavy enough that these are roughly the same. This suggests that it would take less than a minute for time dilation errors to accumulate in GPS to the point where the 2m accuracy was compromised. (Check this based on the height of the satellites, remember light travels 30cm in a nanosecond.)

In fact GPS is made up of 24 satellites, each in a 12hr orbit in a total of six different orbital planes. Each satellite carries atomic clocks that can keep time to one part in $10^{-13}$ over a period of a few weeks!

For the sun we have at our distance $GM_\odot/c^2R_\odot \sim 10^{-6}$ so despite the extra distance a larger effect. The most relativistic orbit in the solar system is of course Mercury's, which is about $R_\odot/3$. In Newtonian gravitation the planets orbit the sun in perfect ellipses (is this familiar?). It turns out that in general relativity these orbits precess, we will study this. While there are many effects (such as the presence of the other planets) that lead to small precessions of planetary orbits from Newtonian effects, the precession of Mercury is large enough that it was observed in the 1850's by Le Verrier and by the 1880's astronomers could explain about 531 arcseconds per century by Newtonian effects with about 43 arcseconds per century left unexplained. Einstein kept this discrepancy in reserve as a test for his general theory of relativity and modern measurements (radar ranging, satellite fly-bys) of the precession of Mercury are still completely consistent with general relativity. (Most of the precession observed at the Earth is due to the precession of the equinoxes, what are these anyone?)

Since light is energy, it stands to reason that light should bend in its motion around the sun due to gravity. This effect was predicted by Cavendish and von Soldner based on Newtonian arguments (what would they have been?) and independently by Einstein (who eventually found a factor of two difference to the Newtonian value as a result of the full theory of general relativity.) and observed (roughly speaking) by Eddington in 1919 during a solar eclipse, providing the test of general relativity that convinced many physicists. This effect is now very important to observational astronomy since light rays are bent by mass distributions in the universe, known as gravitational lenses.

### *2. Neutron stars*

Stars support themselves against gravity because the fusion reactions inside them generate enough pressure to keep them in shape. So even for our sun, which is not an especially small star, general relativistic effects are generally of secondary importance. When the nuclear fuel in the stars runs out however they collapse to much smaller objects. Eventually the Pauli exclusion principle for electrons in the solar matter will slow the collapse, providing sufficient pressure to counteract the gravitational attraction. Such objects are called white dwarfs. However if the mass is large enough the gravitational attraction will provide the energy to drive reversed radioactive decays, all the electrons and protons in the atomic matter of the collapsing star will combine to form neutrons and these will continue to collapse until they too suffer from Pauli exclusion principle "pressure" and become stable. A neutron star has about a solar mass and a radius of about 10km resulting in $GM/c^2R \sim 0.1$ so these are very relativisic objects.

### *3. Black Holes*

It is possible for the star to be so massive that nothing will prevent its gravitationial collapse. A black hole is a region of spacetime containing so much mass that even light is not able to escape. We can estimate the required amount of mass by requiring the escape velocity discussed above is equal to $c$. We get just the condition described above $GM/Rc^2 = 1/2$. Although not even light can escape a black hole it is possible for matter falling into a black hole to become so hot that it emits large amounts of X-rays or even gamma-rays. (X-ray binaries, quasars, gamma ray bursts?). Black holes of a few solar masses have been observed in orbit around companion stars but there are also supermassive black holes with masses of up to a billion solar masses. The black hole at the centre of the Milky Way is three million solar masses, it appears that all galaxies above a certain mass may contain black holes.

### *4. Gavitational waves*

Consider two massive objects orbiting each other. As each object moves, in Newtonian gravity, the gravitational field at a distance instantaneously adjusts to the new location. Clearly this feature of Newtonian gravity is not relativistic since information about the new location of the orbitting objects cannot travel faster than the speed of light. Based on electromagnetism we might expect that this disturbance will propagate at the speed of light and involve waves. This is indeed the case and these waves will take energy away from the orbitting pair. (This loss of energy resulting from a finite speed of propagation of gravity is apparently an observation of Laplace.) In 1974, observations of a binary pulsar. Pulsars give of radio frequency radiation (they are rotating neutron stars) but the radiation from this pulsar had a frequency modulation period of eight hours which can be ascribed to motion about some other body (another neutron star in all likelihood). The frequency could be very accurately measured and along with other observations it is possible to estimate the masses for the two orbiting stars to within about 0.1%. Once this data is found it is possible to use general relativity to work out how much energy should be being radiated in gravitational waves and how much the orbit should slow as a result. Tracking of the system over many years is consistent with general relativity (hand out) The observations of the slowing of the orbit are precisely in agreement for this and a range of other binary pulsars. Until now this is the only observation of gravitational waves, albeit indirect. Very soon it may be possible to observe gravitational waves directly in LIGO and/or LISA.

#### *5. Cosmology*

The dynamics of the universe are dominated by gravitational effects. The expansion of the universe, its acceleration, formation of galaxies, dark matter and energy. This is one of the most exciting areas of physics at present and we will try and learn some more about cosmology later in the course.

### D. Course Outline

We will get through the following:

- Special relativity in geometric language.
- The equivalence principle, gravitational redshift and the curvature of time; qualitative features of relativistic gravity.
- The Schwarzchild solution; precession of planetary orbits, deflection and time-delay of light, something about black holes
- Perfect fluids, pressure and the stress energy tensor.
- The Einstein field equations and the weak field limit, gravitational waves.
- The Freedman-Robertson-Walker homogenous and isotropic solutions. The expansion of the universe, its acceleration, cold dark matter and dark energy or the cosmological constant. Big bang observations, BOOMERANG, WMAP, WiggleZ.

If there is time we could say something about the following, expressions of interest are welcome.

- Neutron stars, pulsars, X-ray binary pulsars, accretion disks.
- Gravitational lensing
- More on black holes
- More on cosmology
- Qualitative features of quantum gravity: Hawking radiation
- Rotational motion and Gravity Probe B