

MATH4105 General Relativity Problem Sheet 5

I. QUESTIONS

1/ (Hartle problem 5.12) (a) Show explicitly that the straight line path between any two points in flat three-dimensional space ($dS^2 = dx^2 + dy^2 + dz^2$) is the shortest distance between them.

(b) Is the straight line path between two spacelike separated points in flat spacetime the shortest distance between them?

2/ (Hartle problem 5.4) A satellite orbits the Earth in the same direction it rotates in a circular orbit above the equator a distance of 200 km from the surface. By how many seconds per day will a clock on such a satellite run slow compared to a clock on the Earth? (Compute just the special relativistic effects.)

3/ (Schutz problem 1.18) Use the special relativistic velocity addition law to show that if a particle has velocity $v = \tanh \theta$ then in a frame moving at velocity $V = \tanh \Theta$ along the same direction the velocity of the particle is given by

$$v' = \tanh(\theta + \Theta). \quad (1)$$

4/ (Hartle problem 5.20) Consider a particle with four-momentum \mathbf{p} and an observer with four-velocity \mathbf{u} . Show that if the particle goes through the observer's laboratory, the magnitude of the three-momentum measured is

$$|\vec{p}| = \sqrt{(\mathbf{p} \cdot \mathbf{u})^2 + \mathbf{p} \cdot \mathbf{p}} \quad (2)$$